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TESI DI LAUREA MAGISTRALE

Design of a new cycle and pedestrian cable-stayed bridge over river Arno: challenges in geometric and structural concept.

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1. Introduction

1.1 Background

All bridge engineering projects have, to some extent, impact on the environment and the people who live in it. They mark urban skylines and rural landscapes. They change the way humans interact with what surrounds them and, most importantly, they affect the quality of life in a society. Bridges have the responsibility to embody a meaning that cannot be attributed to other civil engineering constructions, that is representing, in our collective imagination, something that we all refer to when we speak of linking people, cultures and races. A kind of link that is, at times, just abstract, at others, clear and tangible, as in the great examples of constructions to which history has accustomed us. This huge responsibility towards society leads to an obligation to design bridges worthy of the name, capable of coexisting with present reality and able to help designing the future environment, always in close contact with society's needs. In addition, the increase of a common sensibility towards environmental issues such as gas emissions has led local and national administrations to adopt sustanaibility politics and development plans to ensure territory's conservation.

In this respect, Tuscany regional institutions and municipalities started financing projects dealing with a new cycling-pedestrian infrastructure that connects several tuscan provinces, including a new cycle-path running along the river Arno, linking the city of Florence to the mouth of the river (Pisa province). On December 16th (2009) a protocol was signed, stating all the activities and costs regarding the design, construction, managment and promotion of this new infrastructural network named "Sistema integrato Ciclopista Dell'Arno". Among other specific infrastructure projects within this new network, a pedestrian crossing on river Arno was proposed, linking two distinct municipalities of the Pisa Province: Vicopisano and Cascina.

The first has a total population of 8,253 occupying an area of about 27 km^2. Located between the north bank of the river and the Mount Pisano, Vicopisano is featured with a varying orography (from plains to montainous terrain) is the smaller of the two municipalities, not only from a geographical point but also by means of economical potential.

The second, Cascina, is located on the south bank of river Arno, on a markedly plain terrain, counting a population of 44,133 inhabitants over a total area of 78.61km^2. It takes pride in two of the biggest light industrial areas of the whole Province.

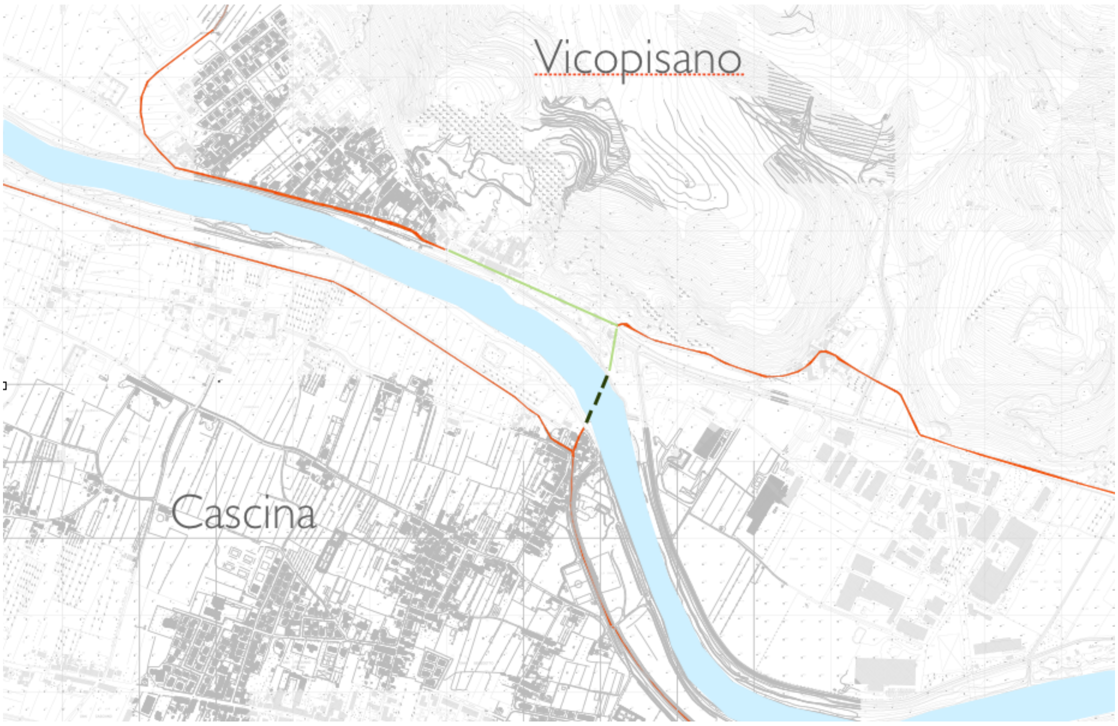


Fig.1 – Urban plan highlighting existing cycle-path (orange) and future network extension (green)

The structure shall then satisfy specific needs, of which the most important is gaping the difference between the two orographies that the river separates, and, metaphorically, between two distant urban realities.

Nonetheless, the bridge is required to blend in with the environment having in mind that, even though that specific location in not subjected to protective restrictions, there are still architectural costraints due to the proximity to the Monti Pisani chain on the Vicopisano side. In addition, the site of the bridge shall be located at the same river's cross section as the one where, during Second World War, American troops built a gangway to allow for the passage of heavy duty vehicles , as shown in Figure 2.

In order to acquire the tools for designing a structure that could fit in with all these requirements and that, at the same time, could reflect an original approach to footbridge concept and design, a background of recently-built pedestrian bridges on river Arno has been created. An investigation of the architectural and structural features of these bridges (crossing the river along its way from the city of Florence to its mouth at Bocca d'Arno, Pisa) has been carried out and the results are reported in the following table (Tab.1).

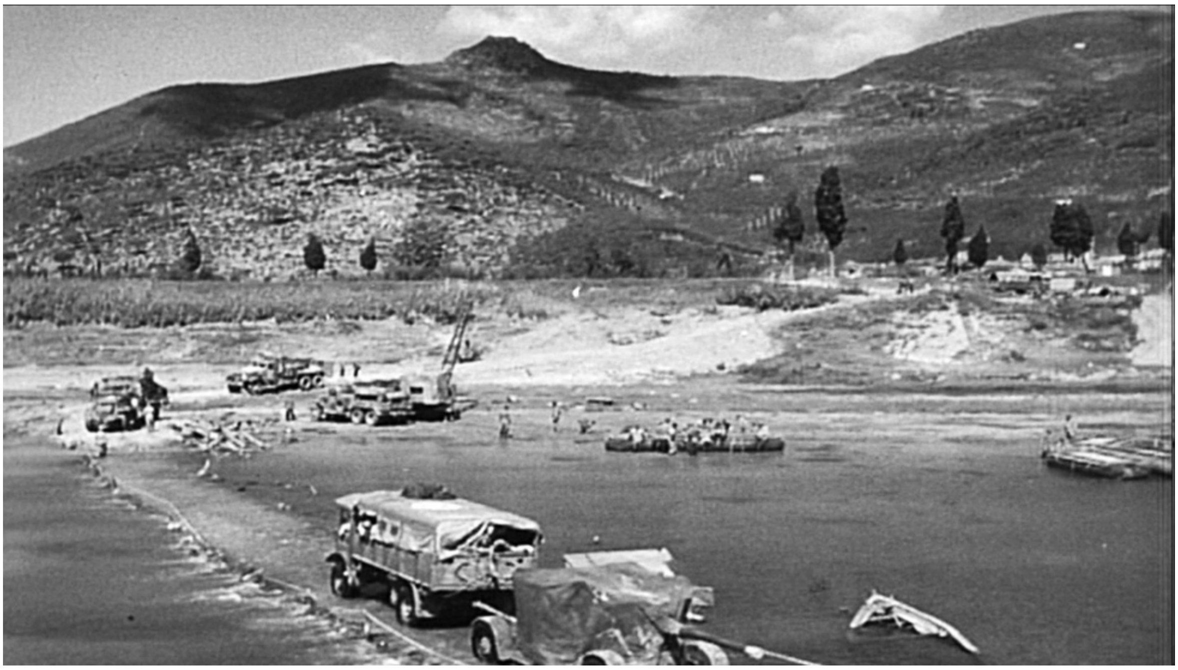


Fig.2 - Passageway for american troops during 1944

Location	Type	Designer	Span [m]	Year of completion	Figure
1) Peretola-Isolotto (Florence)	Road/Pedestrian	De Miranda, Montemagni	206	1975	2 , 3 , 4
2) Lungarno dei Pioppi-Piazzale Kennedy (Florence)	Cycle/pedestrian	Damerini, Scalesse	105	1962	5 , 6
3) Figline Valdarno	Cycle/pedestrian	Florence Province technical bureau	95	2010	7 , 8 , 9
4) Lungarno dei Pioppi-Viale Abramo Lincoln (Florence)	Rail/cycle/pedestrian	Florence City technical bureau	130	2009	10 , 11
5) Parco Cascine-Argingrosso (Florence)	Cycle/pedestrian	Florence City technical bureau	114	Not built yet	12

6) Figline Valdarno	Road/cycle/pedestrian	ACS and BF associates	210	Not built yet	13
7) Scandicci (FI) - Greve river	Cycle/pedestrian	Marrucci, Mazzali	70	1989	14 , 15 , 16 , 17

Tab.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig.7

Fig.8

Fig.9

Fig.10

Fig.11

Fig.12

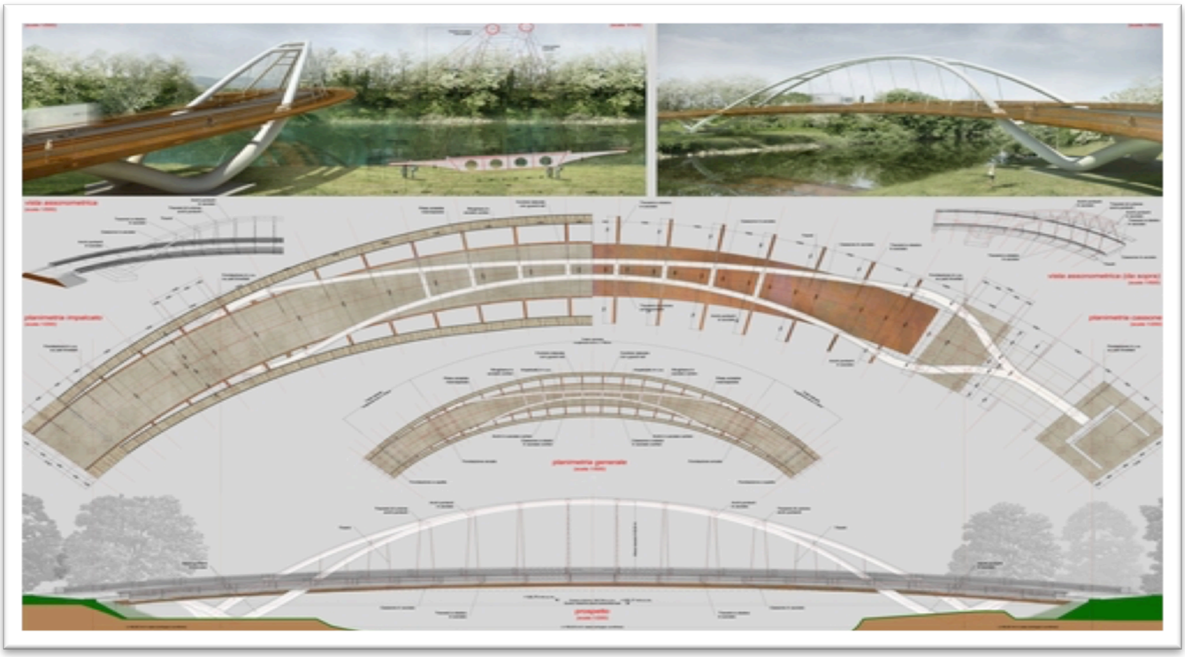


Fig.13

Fig.14

Fig.15

Fig.16

Fig.17

1.2 Purpose

The main aim of this thesis was to propose a suitable design for a new cycle-pedestrian bridge crossing river Arno at the hydraulic section AR 0186 (Fig.20). To achieve this the following objectives were identified:

- A hydraulic study of the relevant cross section of the river to estabilish peak flow levels used to set the minimum clear distance between deck and water level, the location of the masts' foundations and the span of the bridge;

- An architectural study of the location and its environmental constraints, which led to a range of possible solutions in terms of bridge typology and shapes;

- A parametric study of the bridge geometry where the initial shape of the deck, the masts' fixed points location, their spatial orientation, and the cables prestresses under dead loads were modified to get and optimal solution in terms of structural response and architectural soundness;

- An actual design of the footbridge where all the elements were dimensioned to resist code-based loads and to exihibit allowable displacements.

1.3 Outline of the thesis

Here is the outline of the coming chapters in this thesis:

- **Chapter 2** is a general description of the project, its location and the aspects that influenced the pre-design stage, from an architectural and structural point of view;
- **Chapter 3** focuses on the geometric optimization process that has been carried out to get the final solution in terms of geometry and prestress distribution;
- **Chapter 4** the different types of analysis utilized in the design are treated in details;
- **Chapter 5** outlines the actual design process from loads definition to stresses and displacements results;
- **Chapter 6** is dedicated to the study of the dynamic behaviour of the structure with a focus on the footfall-induced vibrations response;
- **Chapter 7** is an introduction to the Annexes where all the analysis and design calculation are reported.

2. Project description

The traditional definition of a bridge structure lies on the basic function of connecting two points, one each side of an obstacle, using the shortest route. While this definition perfectly applies to road and rail bridges, which are forced by the race for ever-increasing speeds to be as straight and as flat as possible, it doesn't entirely apply to footbridges. They can curve and form angles in the horizontal plane, while the vertical plane can include arches, humps, stairs and slopes. The reason is that pedestrian and cycle bridges are coinceived and designed to become part of the recreational environment of an anthropized area, significantly contributing in letting people interact with the urban surroundings and shaping a tool that pedestrians might use not only to cross a river but also to enjoy new viewpoints while walking, standing still or sitting on a bench at the bridge's mid-span. This recreational architecture connotation has been accounted for by using a deck spine line which is curved in both the horizontal and vertical planes, reflecting a planimetric axial-simmetric S shape and a vertical camber of 1.5m (figures 18 and 19). The load bearing system is that of a cable-stayed bridge with an unusual layout: the inner side of the curves is hung from cables while the outer side cantilevers out, offering a great visual effect.

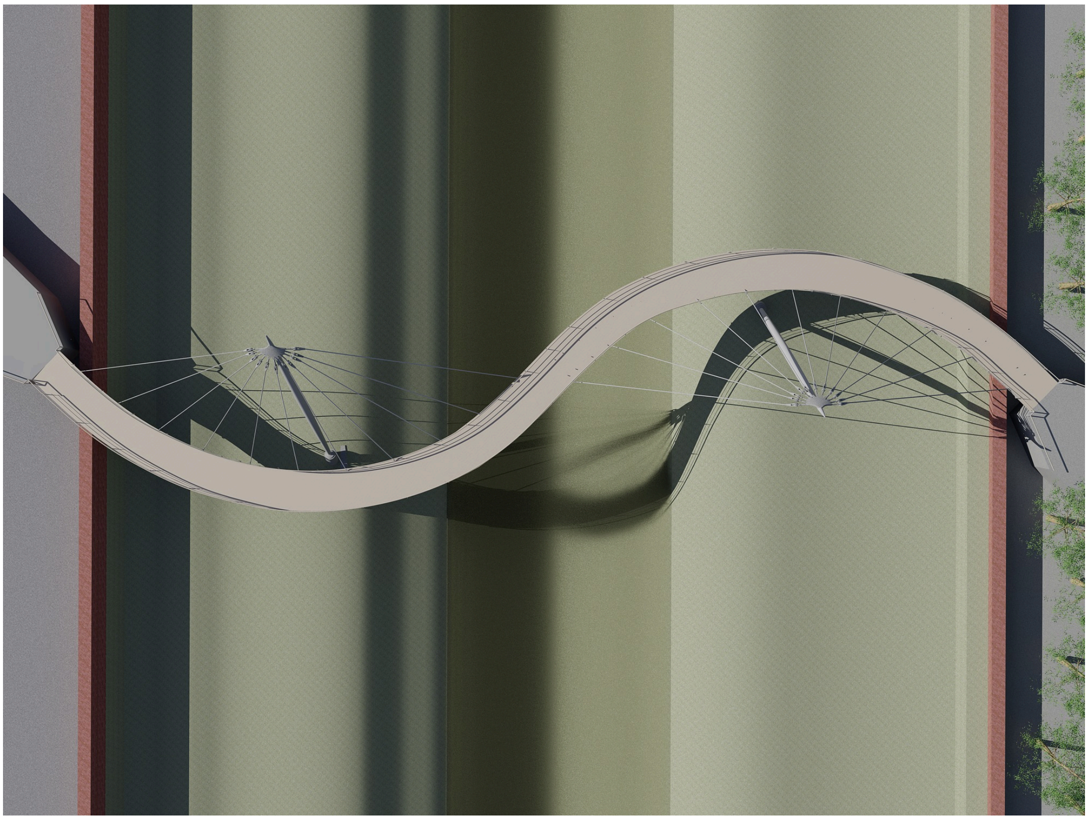


Fig.18 – Plan view of the bridge

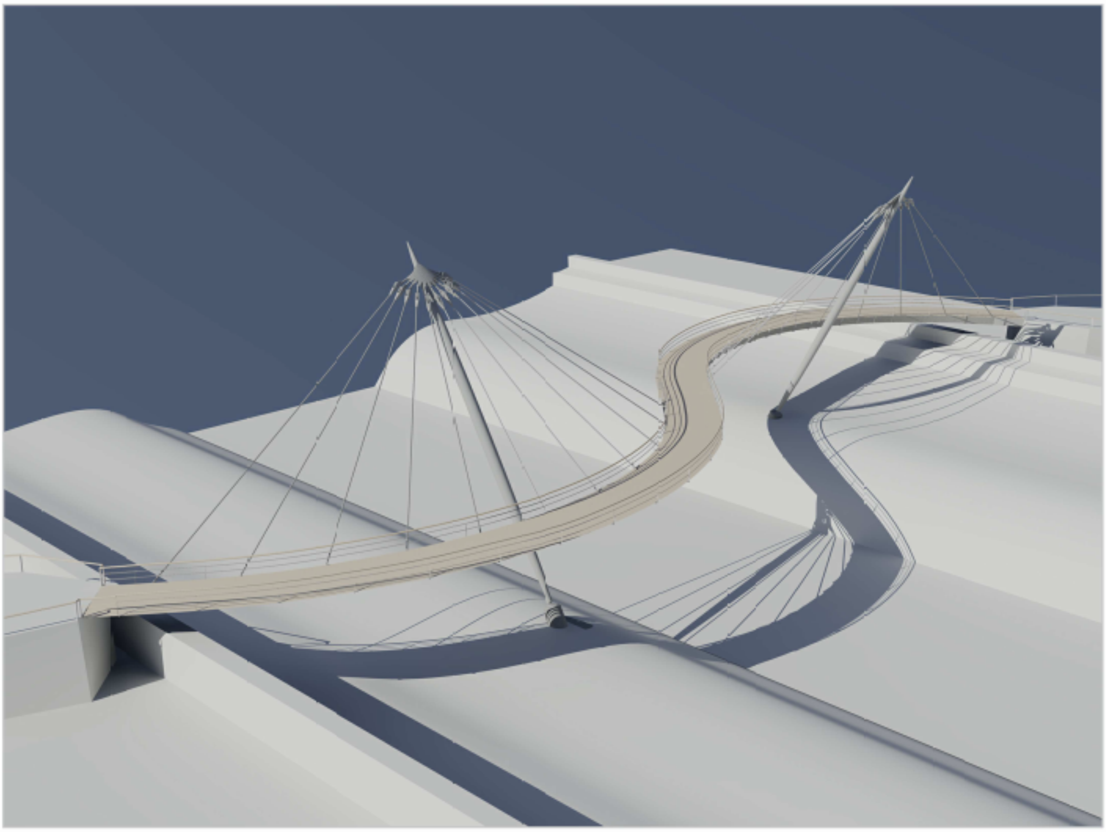


Fig.19 – Top-side view of the bridge

The structural concept behind this support system is very simple yet really efficient. Imagine that we are to design a straight bridge connecting points A and B being only pinned on one side of its cross section. Obviously it is impossible to achieve a static equilibrium since the structure is a mechanism and would certainly flip over. If we could just curve the deck layout line between A and B, we would notice that an equilibrium condition would be possible. Thus, only by changing the deck geometry we could think of a bridge that is only supported on one side. This basic concept has been explored from a mechanical point of view in Chapter 3 and an optimal shape (by means of stresses distribution along the length of the bridge) has been found. The result is an elegant structure that gently twists and turns between the masts giving the impression of an ever-changing path through the riverside nature. In accordance with the trend of conceiving really slender structures, that is becoming popular in nowadays footbridge design, the bridge cross sections vary, along the span, progressively reducing their depth from a maximum of 867 mm to a minimum of 584 mm. The deck consists of a variable spine trapezoidal box-girder section with a constant top panel width of 2m, from which stiffened horizontal plates cantilever out, giving a full width of 4m (between balustrades).

2.1 Boundary conditions

In order to establish whether the proposed design was suitable or not, a global analysis of the environmental and infrastructural characteristics of the location was conducted.

2.1.1 Cycle and footpath network

The **Ciclovia dell'Arno** (or River Arno cycle-path), has a total length of about 270 km and begins at the river's source, on Mount Falterona and ends at Bocca d'Arno (Pisa). Right now (2015), the cycle-path is not yet fully accessible, though the Region of Tuscany has approved and funded a plan to complete the design and construction of this green infrastracture.

Among the segments that everyday are used by cyclists and walkers we find the "Renai" stretch, in the Signa municipality, the Rovezzano path (between Florence and Fiesole municipalities) and all the "lungarni" in the urban areas along the river flow. The result is a path that primarily twists through nature and turistic landscapes, encountering cities and small villages with different features and habits.

The Ciclovia takes inspiration from many of the well-known, cycle-turistic paths scattered across Europe, like the ones on Danube or Drava rivers, or the Mincio cycle-path in Northern Italy. These are good examples of how such recreational infrastructure networks, with services and tourism facilities could help the local areas get a positive economical and social feedback.
The Arno cycle-paths develops entirely in Tuscany, in the Arezzo, Firenze and Pisa provinces, e intersects some of the other national and international cycle ways, among which EuroVelo (connecting Cape North, Norway, to the island of Malta with a central segment between Bologna and Rome) is the most remarkable. The Arno ciclovia spaces from nature landscapes to historical conurbations. In its first segment, crossing the towns of Stia and San Giovanni Valdarno (Arezzo), Figline, Incisa, Rignano and Pontassieve (Florence), users might enjoy incredible outlooks on the valley, while when in Florence and Pisa cyclers are guided straight to the heart of two of the most important art cities in the world. Along the whole cycle-path there are small hotels or rural b&bs, other facilities for small repairs.

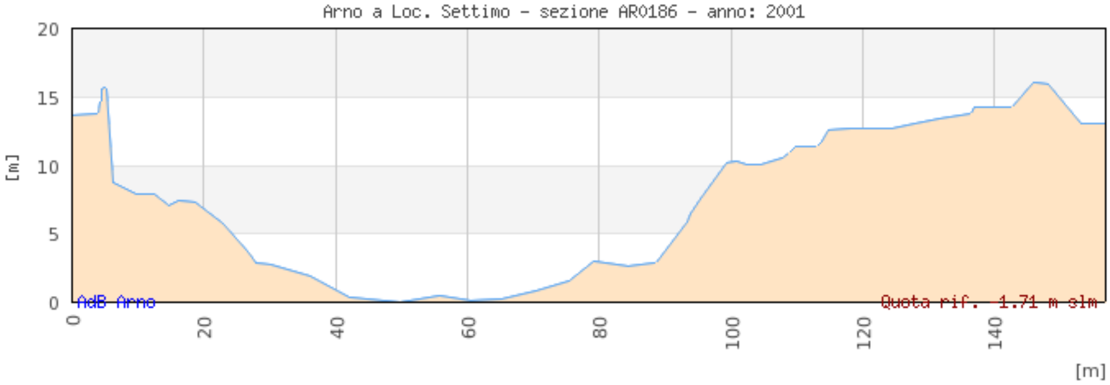
The location of bridge designed for this thesis belongs to those areas where the Ciclovia is not yet accessible. The bridge itself will create a link between the existing pedestrian and cycle ways on the two banks of the river, as shown earlier in Fig.1.

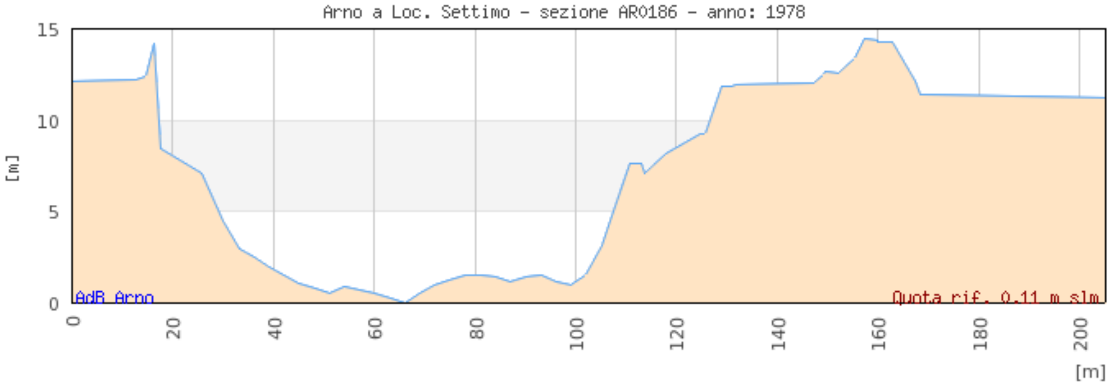
2.1.2 Environmental and landscape restrictions

Since the bridge crosses Arno river, a hydraulic study has been conducted to assess the river cross sectional dimensions and the peak flow level for an adequate return period. The river cross section is identified by the Arno basin authority as AR0186 (Fig.20) The most recent data related to that section date back to 2001 and since no other information is available we will be refering to those values for the design of the footbridge.

Fig.20

As shown in Fig. 21(data related to the year 2001), the distance between the floodplains on the opposite banks of the river is approximately equal to 100m with a vertical height difference of 3-4m. Since an actual design of such a structure would require extensive land survey activities to get more recent data, for the purpose of this thesis a slightly different and more regular river cross section has been selected, keeping the real main dimensional values in order to get a consistent final design. Figure 22 shows the peak flow level values for different return periods.


Arno a Loc. Settimo - sezione AR0186 - anno: 2001
[m]
AdB Arno
Quota rif. -1.71 m slm
[m]


Arno a Loc. Settimo - sezione AR0186 - anno: 1978
[m]
AdB Arno
Quota rif. 0.11 m slm
[m]

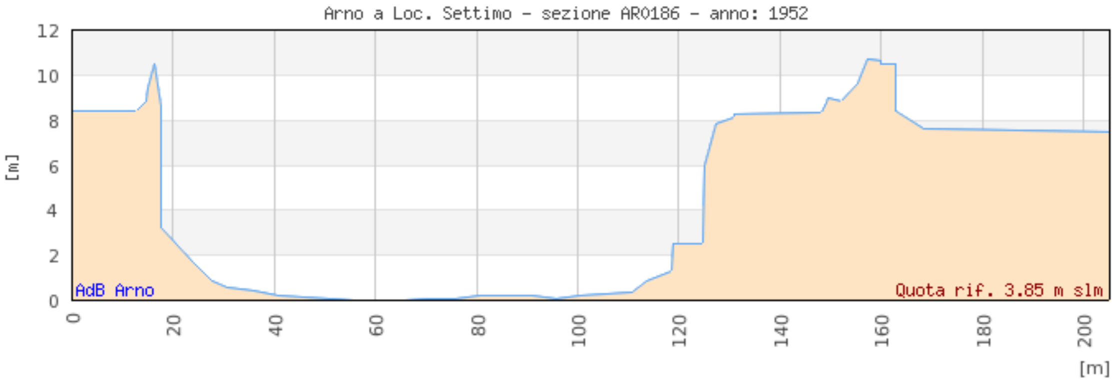

Arno a Loc. Settimo - sezione AR0186 - anno: 1952
[m]
AdB Arno
Quota rif. 3.85 m slm
[m]

Fig.21

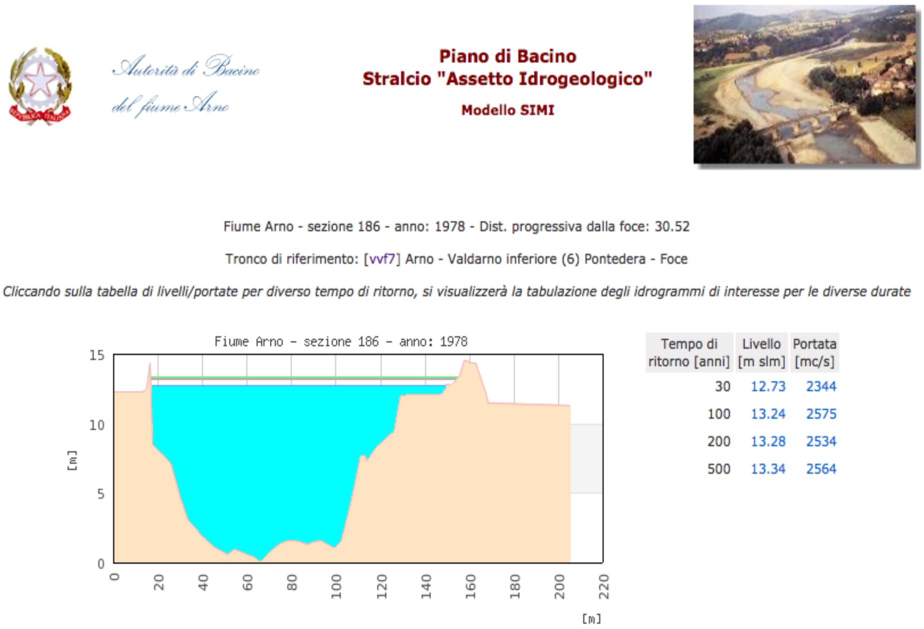


Fig.22

For a return period of 500 years the maximum flow level calculated is 13.34m above sea level. This measure has been taken into account when sizing the height of the location and the height of mast's foundation pile caps in order to get an adequate safety margin for the mast base being reached by water. On the south bank of the crossing (Via Barca di Noce, Cascina municipality) the abutment shall be built within the large open space between the residential houses (Fig. 23,24), used by local people as a gathering point, while the north abutment (Fig. 25) site shall be obtained by cutting some of the spontaneous vegetation, clearing an access way to the bridge. On each side of the river the deck shall overstep the embankment's masonry retaining walls with a clear distance of 1.5m (to allow for the passage of maintenance equipment).

Fig.23

Fig.24 – South bank residential area

Fig.25 – North bank vegetation

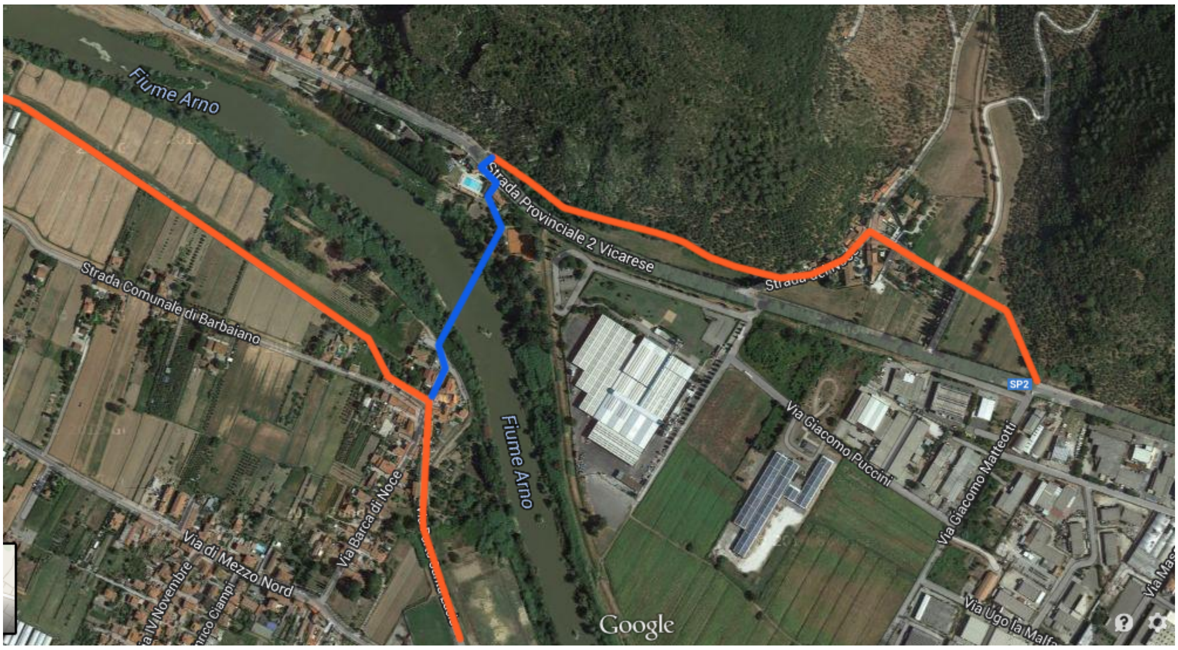


Fig.26 – Global satellite view of the crossing and existing cycle-path highlighted in orange

2.2. Functional and architectural requirements

As mentioned in previous paragraphs, the structure has been conceived to fit into the 110 meters that separate Arno's embankments, in a constant dialogue with the adjacent environment and reflecting a deep architectural inspiration, that is, a construction able to fulfill the recreational needs of the linked municipalities and to become a landmark. The bridge, in fact, has to be capable of enriching both the pedestrians' crossing experience and the surrounding environment. In other words, it shall be, somewhat, spectacular.

Willing to design such a structure, a straight traditional bridge solution would not have been admissible. A selection of horizontal and vertical plane curves has been made, then, depicting spatial dynamic shapes that allows the user to lose the perception of the structure's staticness itself. Here-hence, a spatial curve has been used to model the deck layout line along with a variable cross section, defining a really complex geometry.

This complexity translate into a big structural challenge, especially when the load bearing system (i.e. the mast to which the stay-cables are attached) are designed to be pinned a both ends, leading to an apparently unstable structure. The impression of being in a precarious equilibrium is amplified by the structural glass elements, that in an advanced design stage, might be added to become an integral part of the deck. These would lead to the possibility of having trasparent floor sections to walk on, letting the users feeling the thrill of walking on air. This solution is becoming very popular among the recently designed skywalks looking over the famous canyons and valleys shown is Fig. 27, 28 and 29.

Fig. 27

Fig.28

Fig.29

Since the area on which the bridge will be constructed is not subjected to environmentally protective restrictions nor historical-archaelogical constraints (the closest cultural property is the Noce village, situated 1 km north from the north bank of the river at the bridge location, while the site of the future crossing connects Barca di Noce conurbation to an unused area close to the Uliveto mineral water plant, i.e. an industrial site.

The geometric complexity of the bridge has been treated mathematically and the search for a final geometry that could interface with a optimal structural solution has been conducted and reported in the following chapter.

3. Studies on the global geometry

As mentioned above, the original concept of the deck horizontal alignment was that of a centrally symmetric s-shape suspended on the inner sides of the curves. A geometric optimization method was developed to assess whether the choosen alignment could represent the best solution according to a structural, economic and architectural response.

3.1 Deck geometry definition and optimization

An optimal geometry of the footbridge deck is here researched in order to achieve an initial shape to start the design with. The shape optimization process was conducted following three criteria:

1) Structural efficiency
2) Cost
3) Architectural soundness

Structural efficiency relies on the idea that some shapes represent, compared to others, more advantageous solutions, meaning that they are capable of distributing internal stresses in a more advisable way.

This property is, in general, associated with a reduction of structural members' cross-section and thus a less costly structure. The cost criterion plays an important role in the selection of the optimal shape since we want to design a structure that is competitive from a budget standpoint and could be actually built without facing cost-related issues. Eventually, we want our bridge to be pleasing to the eye and able to become a landmark once built. Thus the architectural aspects deeply influenced the final choice.

3.1.1 Structural efficiency

The idea behind the whole structural optimization process is to achieve a curved geometry that is able to behave in the same way as a bow girder acting as a structural element that only needs to have a hinged support along a single line to keep it from flipping downward and that is capable of converting torsional stresses into bending stresses by means of its own shape.

Driven by this concept, our study begins with the analysis of curved girders with circular directrix., subjected to torsion.

To have a better understanding of the possibilities that such geometries offer an analytical model has been developed to control parametrically how a change in the geometrical features of the curve affects the internal re-distribution of bending and torsional stresses.

A bow girder subjected to a distributed torque and with different restraints has been considered.

3.1.1.1 *Girder with encastres a both ends*

First, a circular girder, with both ends fixed, has been analysed. Refering to Fig.30, let M_θ and T_θ be the bending and torsional moment respectively at any section with angular distance θ from section C, with M_C and T_C taken as bending and torsional moment at section C.

Let **t** be the distributed torque along the girder (constant modulus), and **r** the bow constant radius.

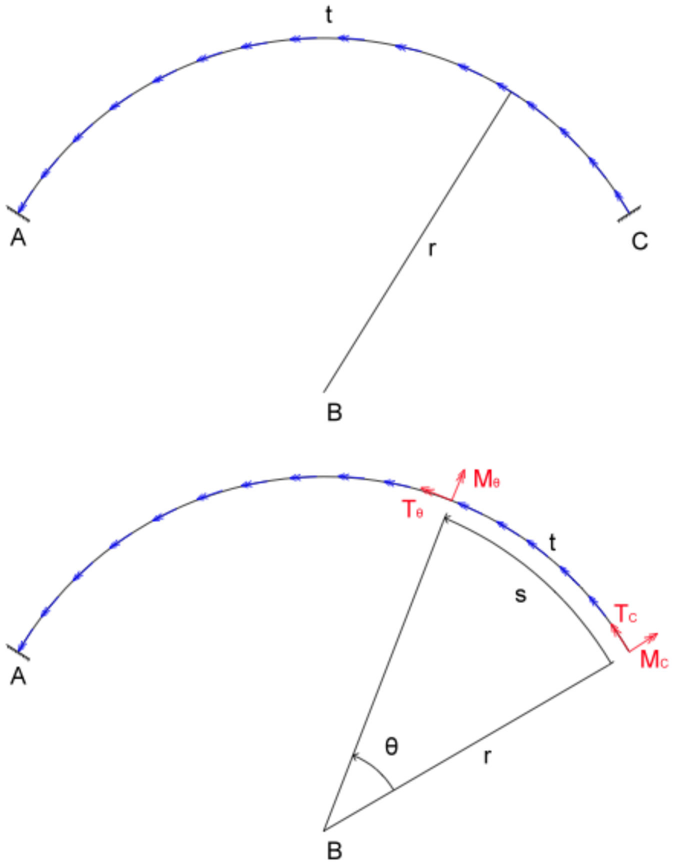


Fig.30

By equilibrium condition of portion $0 < s < r\theta$, we obtain:

$$M_\theta = -M_C \cdot \cos\theta - T_C \cdot \sin\theta - \int_{\theta_C}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi \qquad \text{Eq. 1}$$

$$T_\theta = M_C \cdot \sin\theta - T_C \cdot \cos\theta - \int_{\theta_C}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi \qquad \text{Eq. 2}$$

Total strain energy will be defined by the bending and torsional components: $U = U_b + U_t$, with U_b and U_t being respectively $\int_{\theta_C}^{\theta_A} \frac{M_\theta^2}{2EI} r\, d\theta$ and $\int_{\theta_C}^{\theta_A} \frac{T_\theta^2}{2GJ} r\, d\theta$ **,** where EI = section's flexural stiffness and GJ = section's torsional stiffness.

Since both support points A and C are fixed, we get the following system of differential equations (Syst.1):

$$\begin{cases} \dfrac{\partial U_b}{\partial M_C} + \dfrac{\partial U_t}{\partial M_C} = 0 \\ \dfrac{\partial U_b}{\partial T_C} + \dfrac{\partial U_t}{\partial T_C} = 0 \end{cases} \qquad \text{Syst. 1}$$

Where :

$$\frac{\partial U_b}{\partial M_C} = \int_{\theta_C}^{\theta_A} \frac{M_\theta}{EI} \cdot \frac{\partial M_\theta}{\partial M_C} \cdot r\, d\theta$$

$$\frac{\partial U_b}{\partial T_C} = \int_{\theta_C}^{\theta_A} \frac{M_\theta}{EI} \cdot \frac{\partial M_\theta}{\partial T_C} \cdot r\, d\theta$$

$$\frac{\partial U_t}{\partial M_C} = \int_{\theta_C}^{\theta_A} \frac{T_\theta}{GJ} \cdot \frac{\partial M_\theta}{\partial M_C} \cdot r\, d\theta$$

$$\frac{\partial U_t}{\partial T_C} = \int_{\theta_C}^{\theta_A} \frac{T_\theta}{GJ} \cdot \frac{\partial T_\theta}{\partial T_C} \cdot r\, d\theta \quad .$$

And again :

$$\frac{\partial M_\theta}{\partial M_C} = \cos(\theta) \quad ; \quad \frac{\partial M_\theta}{\partial T_C} = \sin(\theta) \quad ; \quad \frac{\partial T_\theta}{\partial M_C} = -\sin(\theta) \quad ; \quad \frac{\partial T_\theta}{\partial T_C} = \cos(\theta).$$

By substituting these values into Syst.1 equations, we obtain:

$$\begin{cases} r \cdot \displaystyle\int_{\theta_C}^{\theta_A} \frac{M_\theta}{EI} \cdot \cos(\theta) - \frac{T_\theta}{GJ} \cdot \sin(\theta)\, d\theta = 0 \\ r \cdot \displaystyle\int_{\theta_C}^{\theta_A} \frac{M_\theta}{EI} \cdot \sin(\theta) + \frac{T_\theta}{GJ} \cdot \cos(\theta)\, d\theta = 0 \end{cases}$$

Skipping few steps (all calculation steps are reported in the Annex) and substituting the values M_C and T_C in the previous equations, we get:

$$\begin{cases} M_C \cdot C_1 + T_C \cdot C_2 + t \cdot r \cdot C_3 = 0 \\ M_C \cdot C_4 + T_C \cdot C_5 + t \cdot r \cdot C_6 = 0 \end{cases}$$

The constant values $C_{1,\dots,6}$ are defined as follows:

$$C_1 = \frac{2\theta_A - 2\theta_C + \sin(2\theta_C) - \sin(2\theta_A)}{4} + m\left(\frac{\theta_A}{2} + \frac{\sin(2\theta_A)}{4} - \frac{\theta_C}{2} - \frac{\sin(2\theta_C)}{4}\right);$$

$$C_2 = \frac{sin^2(\theta_C) - sin^2(\theta_A)}{2} + \frac{m\left(cos^2(\theta_C) - cos^2(\theta_A)\right)}{2};$$

$$C_3 = \frac{\theta_C}{2} - \frac{\sin(2\theta_C)}{4} + \sin(\theta_C) \cdot (\cos(\theta_C) - \cos(\theta_A)) - \frac{\theta_A}{2} + \frac{\sin(2\theta_A)}{4} + m(\frac{\sin(\theta_C)}{4} + \frac{\theta_c \cdot \cos(\theta_A)}{2} - \sin(\theta_C) + \sin(\theta_A) + \frac{\sin(\theta_C - 2\theta_A)}{4} - \frac{\theta_A \cdot \cos(\theta_C)}{2};$$

$$C_4 = \frac{\left(cos^2(\theta_C) - cos^2(\theta_A)\right)}{2} \cdot (m-1);$$

$$C_5 = \frac{\theta_A}{2} + \frac{\sin(2\theta_A)}{4} + \frac{\theta_C}{2} - \frac{\sin(2\theta_C)}{4} + m\left(\frac{2\theta_A - 2\theta_C + \sin(2\theta_C) - \sin(2\theta_A)}{4}\right);$$

$$C_6 = \frac{cos^2(\theta_C) - cos^2(\theta_A)}{2} + \sin(\theta_C) \cdot (\sin(\theta_C) - \sin(\theta_A)) + m\left(\frac{\cos(\theta_C - 2\theta_A) - \cos(\theta_C) + 2\theta_C \cdot \sin(\theta_C) - 2\theta_A \cdot \sin(\theta_C)}{4} + \cos(\theta_C) - \cos(\theta_A)\right)$$

where m = GJ/EI.

By solving for M_C and T_C, we substitute their values in Eq.1 and Eq.2. obtaining the bending and torsional moments at any section of the girder (i.e. M_θ and T_θ).

A different radius bow girders (with circular directrix) analysis has been carried out using this analytical tool and the results were compared with a finite-element analysis to investigate their reliability.

The variables used are the radius of the bow and its center point location, while the distance between support points is fixed, since our goal is to explore the behaviour of girders with different radii and lengths. Since, the total span of the bridge will be approximately 100 m, a set of differente curvature girders has been choseen, all having a horizontal projection equal to 100 m.

Thus, the i-th girder will have equation:

$$x^2 + \left(y + \sqrt{{r_i}^2 - 25^2}\right)^2 = {r_i}^2 \qquad \text{Eq. 3}$$

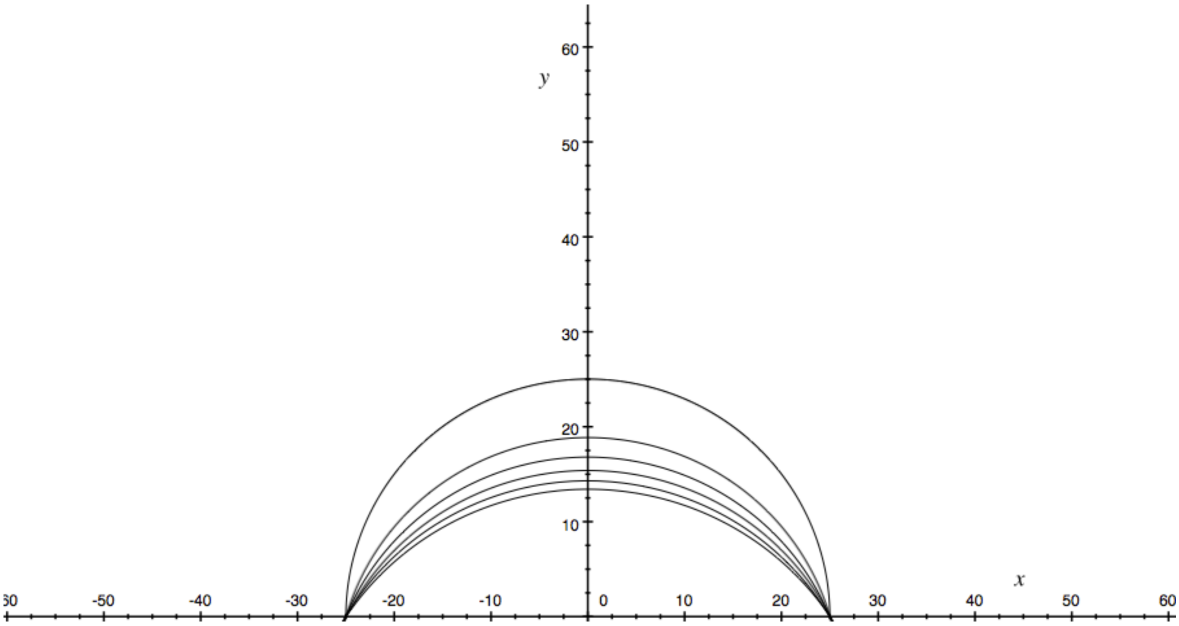


Fig. 31

For testing purposes suppose we consider a generic box steel girder (e.g. radius= 25m, center point at (0,0), equation: $y=(25^2-x^2)^{0.5}$) subjected to a distributed torsional moment of 10 kNm/m with a trapezoidal cross section, as shown in Fig.32.

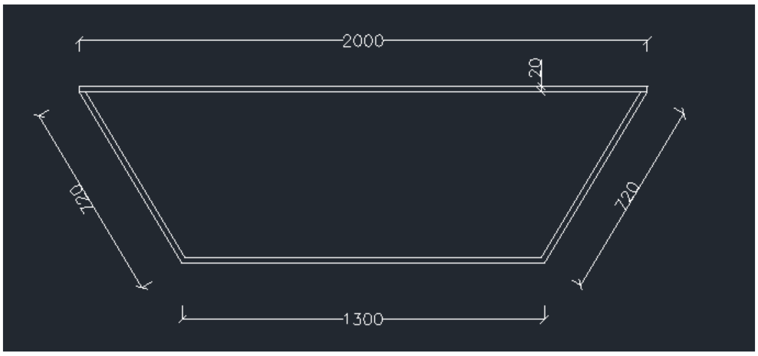


Fig.32

By applying the aforementioned method the following values are obtained:

M_C	250 kNm
T_C	-168.923 kNm
C_1	3.347
C_2	0

C_3	-3.347
C_4	0
C_5	3.347
C_6	2.262

Tab.2

Thus the expressions for bending and torsional moment, at any section with angular distance θ from initial section C (θ_C), are:

$$M_\theta = 250 \cdot \cos(\theta) - 168.923 \cdot \sin(\theta) - r \cdot t(\cos(\theta) - 1)$$

$$T_\theta = -250 \cdot \sin(\theta) - 168.923 \cdot \cos(\theta) + r \cdot t \cdot \sin(\theta)$$

A rough 3D visualization gives an idea of the internal stresses distribution due to bending and torsional actions (all dimensions in kNm):

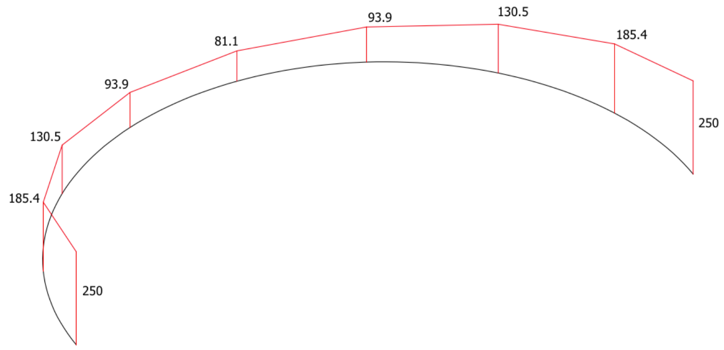


Fig.33 - Bending moment

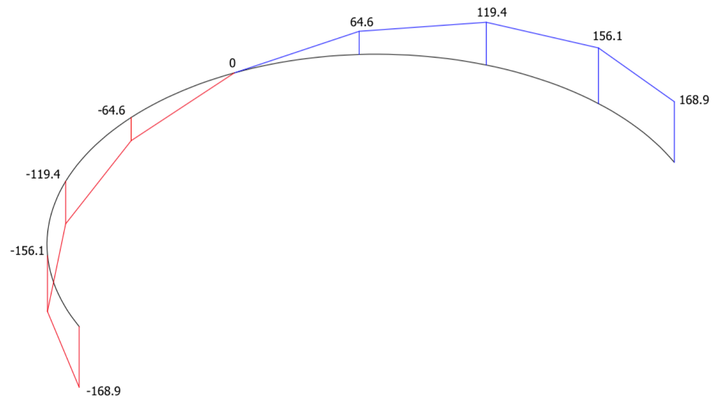


Fig.34 - Torsion

By comparing our results with those obtained with a FEM analysis, we can state that there is just a minimal difference (1.7%) between the values, most likely due to the discretization points automatically generated in the FEM model to linearize curves.

Now that a reliable calculation tool has been developed, we can begin the optimization study. As mentioned before, we want to focus on the structural response of a family of bow girders, described by Eq.3, with different radius and curvature.

Observing the results listed in Tab.3 (or plotted in Fig.35), one can easily understand the relationship between length of the girders and maximum bending and torsional moment (always occuring at supports).

Curve no.	Radius [m]	θ_C [°]	θ_A [°]	$\lvert M(\theta_C)\rvert$ [kNm]	$\lvert T(\theta_C)\rvert$ [kNm]	Length [m]
1	25	0	180	250.00	168.92	78.54
2	26	0	164	204.09	194.98	67.21
3	27	0	158	187.61	203.91	63.91
4	28	0	153	173.08	209.83	61.8
5	29	0	149.5	163.65	214.50	60.28
6	30	0	146.5	155.52	218.28	59.11

Tab.3

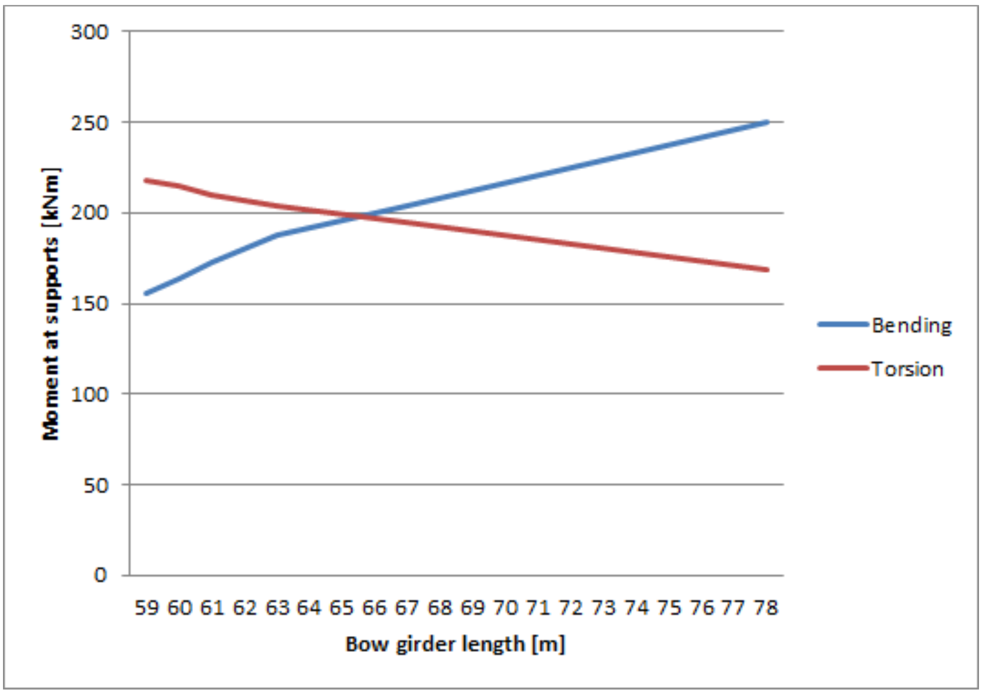


Fig.35

We observe a trend towards the decreasing of maximum torsional moment and the increasing of bending moment when the girder's length, and consequently its curvature, increases. So the smaller is the girder's curvature the closer we get to the case of a straight line beam subjected to pure torsion. Instead, big curvature girders are proven capable to convert torsional stresses into bending stresses by means of their own shape.

This is a powerful concept in terms of structural efficiency since it means that just by choosing a specific curve we can prevent our structure from having issues related to an imbalance between stresses from torsional or bending moments.

In this study-case we chose a girder with a generic steel box cross-section for the sake of simplicity. In the design project of the footbridge eventually we will be dealing with a more complex polygonal cross-section. Thus one can easily understand how revelant is to balance torsion and bending demands at certain points in the structure in order not to need thicker box section plates.

3.1.1.2 *Girder hinged on the inner side*

The second model of this parametric study consists of a curved girder hinged at some points of the inner side of the deck. The same set of different curvature layout lines as in Par. 3.2.1.1 has been choosen, so that the results could be compared.

At the pre-design stage we decided to deal with a simplified model of the actual deck. This led to the decision of studying curve girders with only four vertical support points. This choice can be justified by observing that when considering the actual deck support condition (i.e. the stay-cables' restraints), the amplitude of the maximum moments along the girder will be smaller than those obtained with the simplified model, the latter being a safe-sided solution.

The support points divide the bow length into three equal spans. Being $\mathbf{y_G}$ (see expressions below) the distance between girder's center of gravity and the chord connecting the bow extreme points, **d** the distance between the y_G-axis parallel to the chord connecting the bow extreme points, and the second row of supports and $\boldsymbol{\alpha}$ the angular, as in Fig36, the vertical reactions at the hinges nodes will be expressed as:

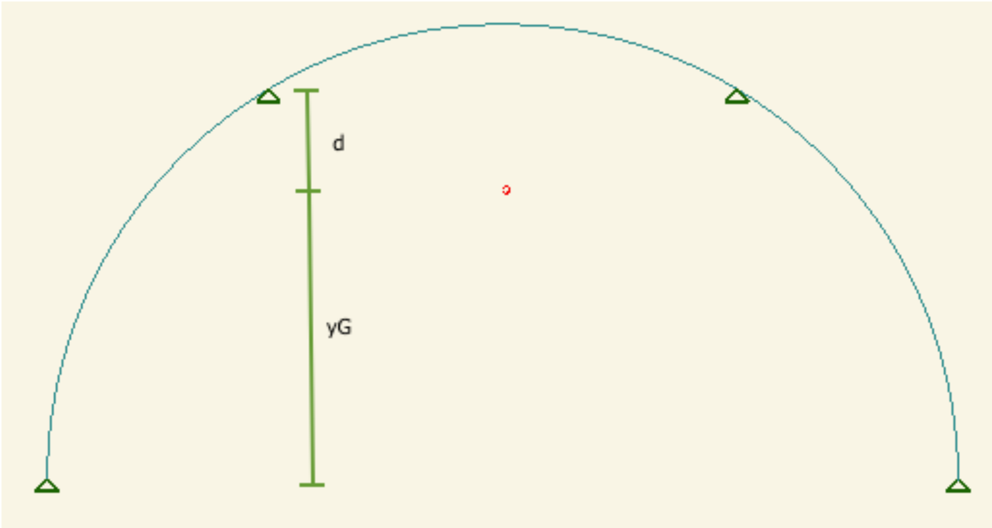


Fig.36

$$|R_i| = \frac{\int_0^\alpha t \cdot r \, d\alpha}{2(y_G + d)}$$

where:

$$y_G = r \cdot \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}$$

$$d = r \cdot [\sin(\beta) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}}]$$

with $\boldsymbol{\beta}$ = angular distance between two consecutive supports, $\boldsymbol{t}$ = distributed torques (as in 3.2.1.1).

These expressions are only valid for $\alpha = \pi \, rad$; all the formulae for girder with an angular aperture different from the half turn are reported in the Annex.

Given that the structure we are considering is symmetric, from a geometric and a loading distribution standpoint, we shall take advantage of that by using only half of it and deriving the results for the second half by reflection. Thus, the bending and torsional moments have been calculated as follows (reactions are taken positive if pointing out of the paper):

If $0 \leq \theta \leq \beta$, then: $M(\theta) = -R_1 \cdot r \cdot \sin(\theta - \theta_0) + \int_{\theta_0}^{\theta_1} t \cdot r \cdot \sin(\theta - \varphi) \, d\varphi$

$$T(\theta) = R_1 \cdot r \cdot [1 - \cos(\theta - \theta_0)] + \int_{\theta_0}^{\theta_1} t \cdot r \cdot \cos(\theta - \varphi) \, d\varphi$$

If $\beta \le \theta \le \frac{\pi}{2}$, then:

$$M(\theta) = -R_1 \cdot r \cdot \sin(\theta - \theta_0) - R_2 \cdot r \cdot \sin(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta_1} t \cdot r \cdot \sin(\theta - \varphi)\, d\varphi$$

$$T(\theta) = R_1 \cdot r \cdot [1 - \cos(\theta - \theta_0)] + R_2 \cdot r \cdot [1 - \cos\,(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta_1} t \cdot r \cdot \cos(\theta - \varphi)\, d\varphi$$

Being: $\boldsymbol{\theta_0}$ the initial angle, $\boldsymbol{\theta_1}$ the final angle (or $\boldsymbol{\theta_0 + \alpha}$), $\boldsymbol{R_1}$ = - $\boldsymbol{R_2}$ = vertical reaction.

For a 25m radius girder with $\alpha = \pi\, rad$, we can now get the bending and torsional moment diagrams plotted versus the angular abscissa θ (Fig.37,38):

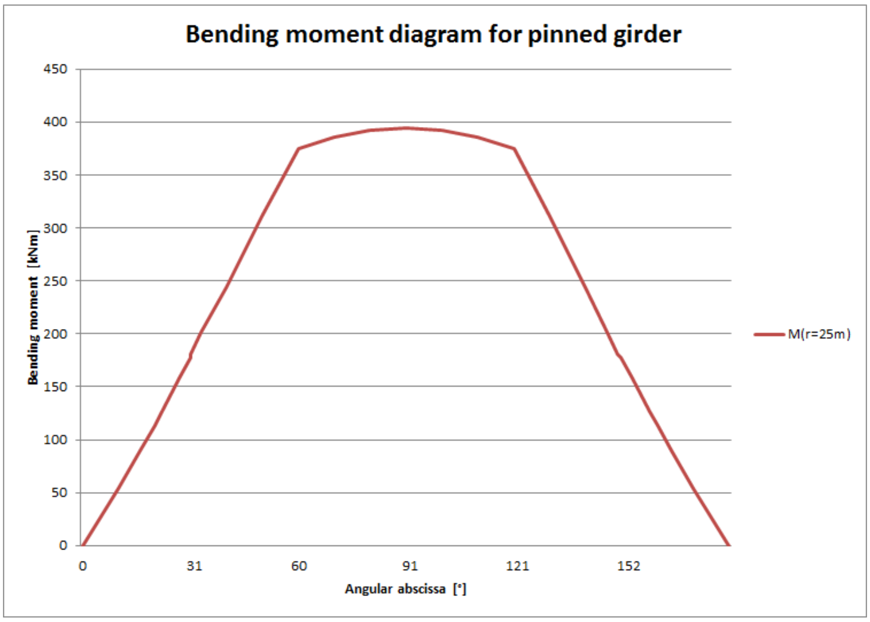


Fig.37

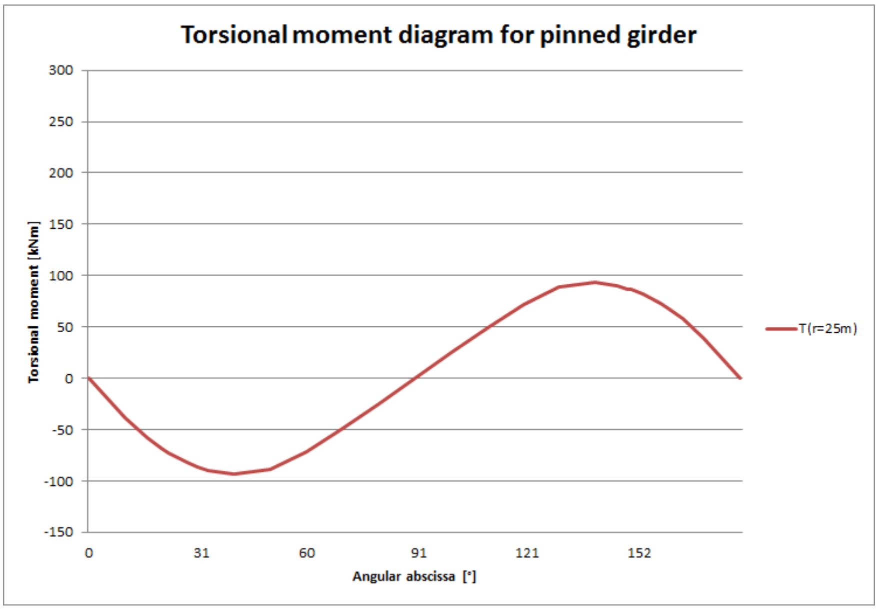


Fig.38

3.1.1.3 *Results comparison*

In order to understand the behaviour of circular girders subjected to two restraint conditions that are relevant for the actual structure we are willing to design, a comparison between the two responses has been made. The following tables (Tab.4 to Tab.9) show the bending and torsional moment values at different locations along the span, for girders with variable radius and angular opening (Fig.31).

R=25m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
0	0	0	250	-168,923
10	54	-39,026	220,667	-166,357
16	89,254	-57,727	203,439	-162,379
20	113,91	-68,096	192,225	-158,736
22	126,344	-72,631	186,72	-156,623
27	158,403	-82,034	173,311	-150,511
30	177,831	-86,325	165,539	-146,292
30,5	181,106	-86,94	164,265	-145,549
33,5	200,859	-90,031	156,765	-140,862
40	244,046	-93,16	141,418	-129,403
50	310,441	-88,393	120,597	-108,582
60	375	-72,169	103,708	-84,462
70	385,633	-49,366	91,264	-57,775
80	392,145	-25,063	83,643	-29,333
90	394,338	0	81,077	0
100	392,145	25,063	83,643	29,333
110	385,633	49,366	91,264	57,775
120	375	72,169	103,708	84,462
130	310,441	88,393	120,597	108,582
140	244,046	93,16	141,418	129,403
146,5	200,859	90,031	156,765	140,862
149,5	181,106	86,94	164,265	145,549
150	177,831	86,325	165,539	146,292
153	158,403	82,034	173,311	150,511
158	126,344	72,631	186,72	156,623
160	113,91	68,096	192,225	158,736
164	89,254	57,727	203,439	162,379
170	54	39,026	220,667	166,357
180	0	0	250	168,923

Tab.4

R=26m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
16	0	0	204,092	-194,976
20	28,17	-18	190,627	-190,601
22	42,688	-25	184,018	-188,064
27	80,101	-42,357	167,916	-180,726
30	103,224	-51,174	158,584	-175,659
30,5	107,122	-52,525	157,055	-174,767
33,5	130,74	-59,913	148,049	-169,14
40	183,042	-71,623	129,622	-155,38
50	265,197	-77,901	104,621	-130,379
60	347,195	-69,818	84,341	-101,417
70	394,426	-48,927	69,399	-69,373
80	400,88	-24,841	60,248	-35,222
90	403,053	0	57,167	0
100	400,88	24,841	60,248	35,222
110	394,426	48,927	69,399	69,373
120	347,195	69,818	84,341	101,417
130	265,197	77,901	104,621	130,379
140	183,042	71,623	129,622	155,38
146,5	130,74	59,913	148,049	169,14
149,5	107,122	52,525	157,055	174,767
150	103,224	51,174	158,584	175,659
153	80,101	42,357	167,916	180,726
158	42,688	25	184,017	188,064
160	28,17	18	190,627	190,601
164	0	0	204,092	194,976

Tab.5

R=27m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
22	0	0	187,614	-203,913
27	40,381	-21,794	170,155	-195,956
30	66	-33,132	160,037	-190,463
30,5	70,469	-34,892	158,379	-189,496
33,5	96,471	-44,661	148,614	-183,394
40	154,341	-61	128,634	-168,474
50	246,009	-73,3	101,526	-141,367
60	338,406	-69,414	79,538	-109,964
70	407,477	-50,037	63,336	-75,22
80	414,077	-25,405	53,414	-38,19

90	416,3	0	50,073	0
100	414,077	25,405	53,414	38,19
110	407,477	50,037	63,336	75,22
120	338,406	69,414	79,538	109,964
130	246,009	73,3	101,526	141,367
140	154,341	61	128,634	168,474
146,5	96,471	44,661	148,614	183,394
149,5	70,469	34,892	158,379	189,496
150	66	33,132	160,037	190,463
153	40,381	21,794	170,155	195,956
158	0	0	187,614	203,913

Tab.6

R=28m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
27	0	0	173,085	-209,833
30	27,609	-13,941	162,25	-203,95
30,5	32,28	-16,123	160,474	-202,915
33,5	60,688	-28,353	150,018	-196,381
40	124,196	-49,654	128,623	-180,404
50	225,516	-68,051	99,596	-151,377
60	328,492	-68,576	76,05	-117,751
70	420,917	-51,289	58,702	-80,546
80	427,682	-26,04	48,077	-40,895
90	429,96	0	44,499	0
100	427,682	26,04	48,077	40,895
110	420,917	51,289	58,702	80,546
120	328,492	68,576	76,05	117,751
130	225,516	68,051	99,596	151,377
140	124,196	49,654	128,623	180,404
146,5	60,688	28,353	150,018	196,381
149,5	32,28	16,123	160,474	202,915
150	27,609	13,941	162,25	203,95
153	0	0	173,085	209,833

Tab.7

R=29m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
30				
30,5	0	0	163,649	-214,502
33,5	30,589	-14,387	152,596	-202,674
40	99,19	-39,952	129,979	-190,706
50	209,19	-62,715	99,294	-160,022
60	321,666	-68,014	74,404	-124,475
70	434,847	-52,71	56,064	-85,146
80	441,801	-26,767	44,833	-43,23
90	444,143	0	41,051	0
100	441,801	26,767	44,833	43,23
110	434,847	52,71	56,064	85,146
120	321,666	68,014	74,404	124,475
130	209,19	62,715	99,294	160,022
140	99,19	39,952	129,979	190,706
146,5	30,589	14,387	152,596	202,674
149,5	0	0	163,649	214,502

Tab.8

R=30m				
	Pinned at cable supports		Fixed at both ends	
Θ [°]	Bending moment [kNm]	Torsion [kNm]	Bending moment [kNm]	Torsion [kNm]
33,5	0	0	155,525	-218,278
40	71,816	-30	131,744	-200,52
50	187,696	-59,781	99,48	-168,256
60	306,987	-68,995	73,309	-130,88
70	460,17	-58,297	54,026	-89,527
80	467,86	-29,598	42,217	-45,454
90	470,45	0	38,24	0
100	467,86	29,598	42,217	45,454
110	460,17	58,297	54,026	89,527
120	306,987	68,995	73,309	130,88
130	187,696	59,781	99,48	168,256
140	71,816	30	131,744	200,52
146,5	0	0	155,525	218,278

Tab.9

For both the restraint condition considered we observe that:

- Bending moment at supports (fixed girder) decreases when radius is increasing (arc length decreases), while at midspan section increases;

- Torsional moment at supports (fixed girder) increases when radius is increasing (arc length decreases);

- Bending moment at the midspan section (pinned girder) increases when radius is increasing (arc length decreases);

- Torsional moment at any section (pinned girder) decreases when radius is increasing (arc length decreases);

These results are made clear in the following graphs (Fig.39 to Fig.41):

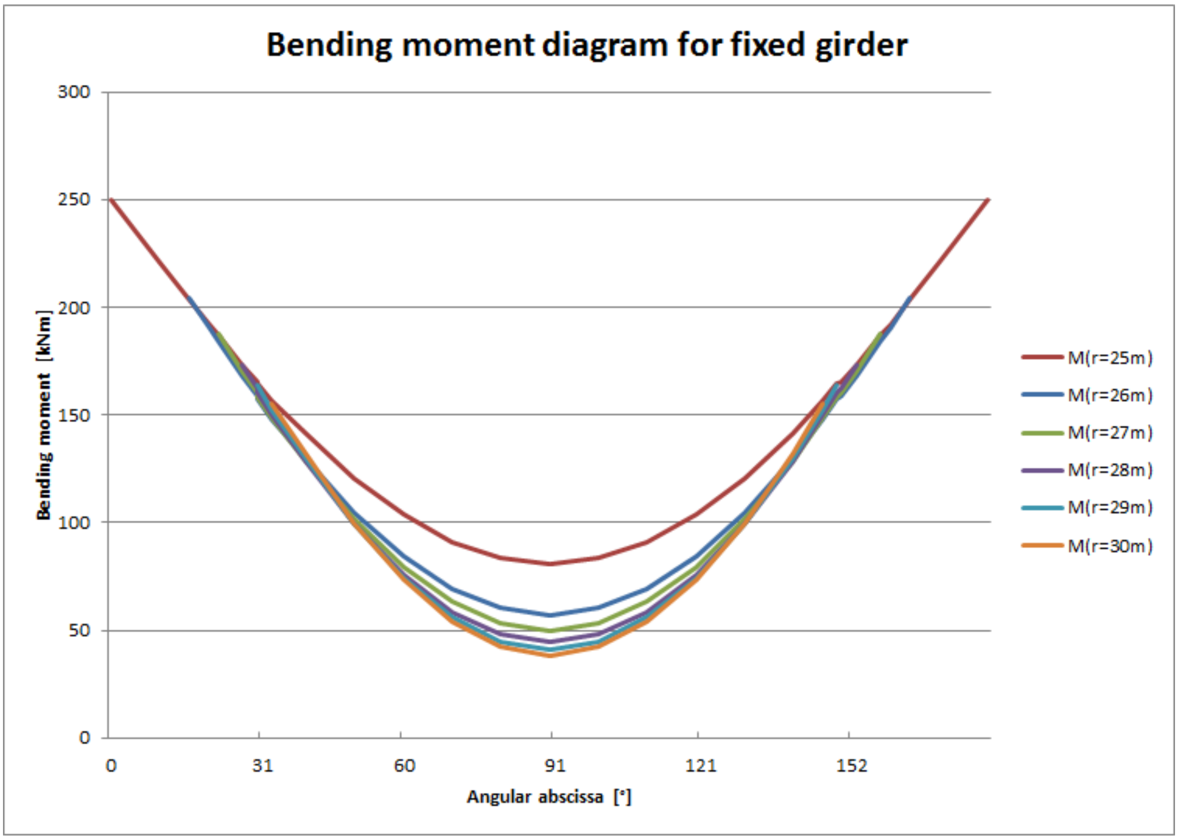
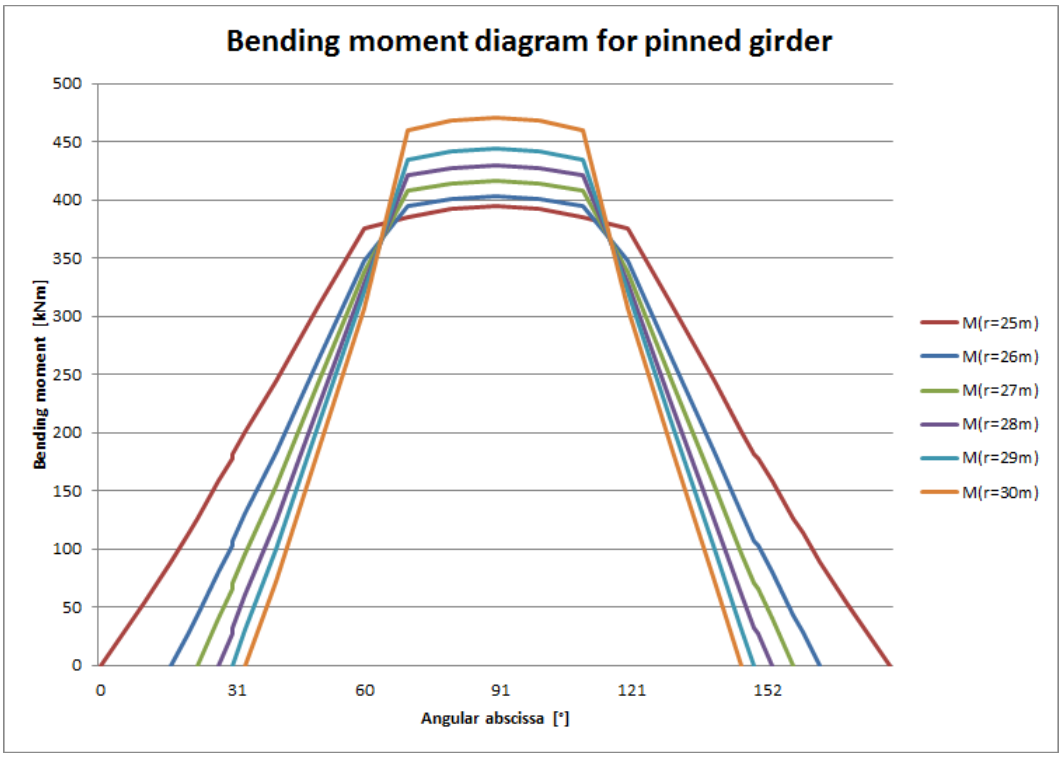
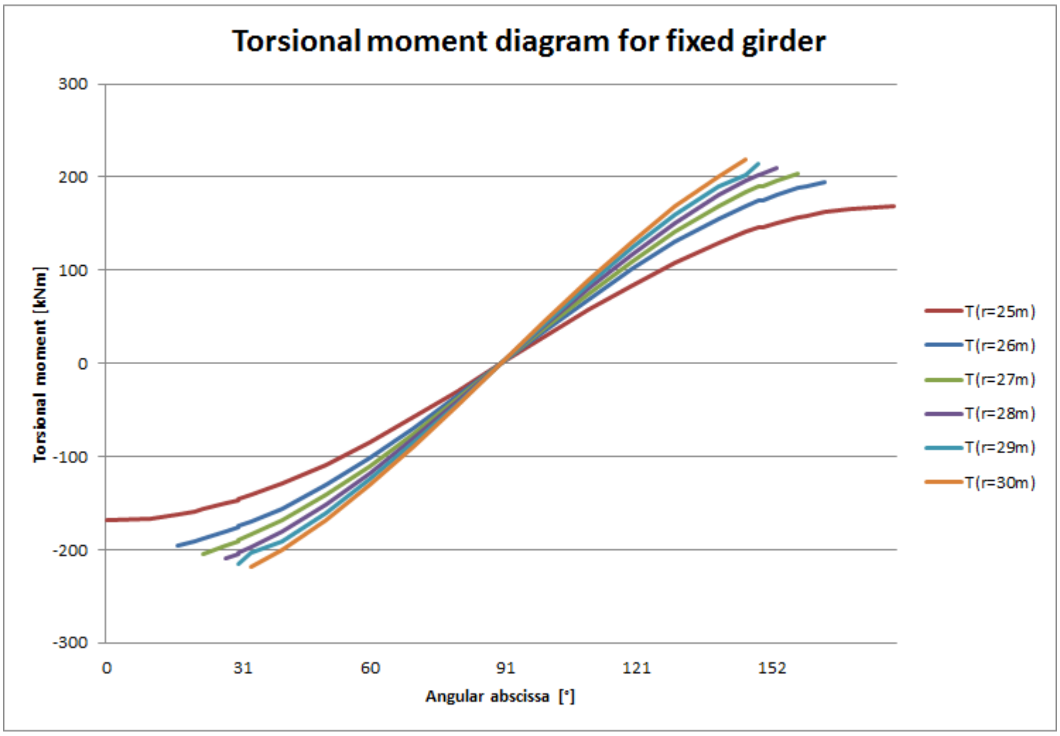


Fig.39



Bending moment diagram for pinned girder
500
450
400
350
300
250
200
150
100
50
0
Bending moment [kNm]
0
31
60
91
121
152
Angular abscissa [°]
M(r=25m)
M(r=26m)
M(r=27m)
M(r=28m)
M(r=29m)
M(r=30m)

Fig.40



Torsional moment diagram for fixed girder
300
200
100
0
-100
-200
-300
Torsional moment [kNm]
0
31
60
91
121
152
Angular abscissa [°]
T(r=25m)
T(r=26m)
T(r=27m)
T(r=28m)
T(r=29m)
T(r=30m)

Fig.41

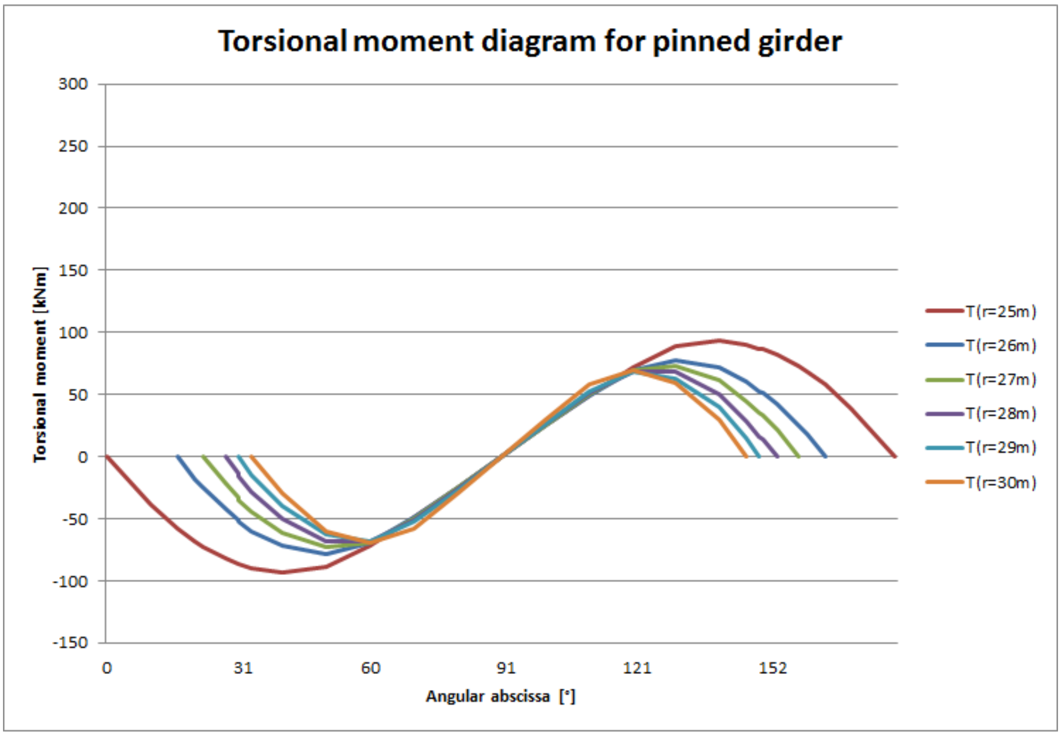


Fig.42

The data collected so far describe the behaviour of a single-curve girder. One could point out that this results do not adhere to the real structure we are designing since it is a double curve (s-shaped) girder. This can be easily rebutted by observing that the structure is centrally symmetric (by means of geometry and loading) when it is subjected to a uniform vertical load exerted by a crowd of pedestrians that are located at the regions of the deck relevant for the maximum torsional response (i.e. the parts that cantilever out from the spine box-girder, Fig**), meaning that we can analyse just one half of the structure, substituting the midspan section with a fixed node.

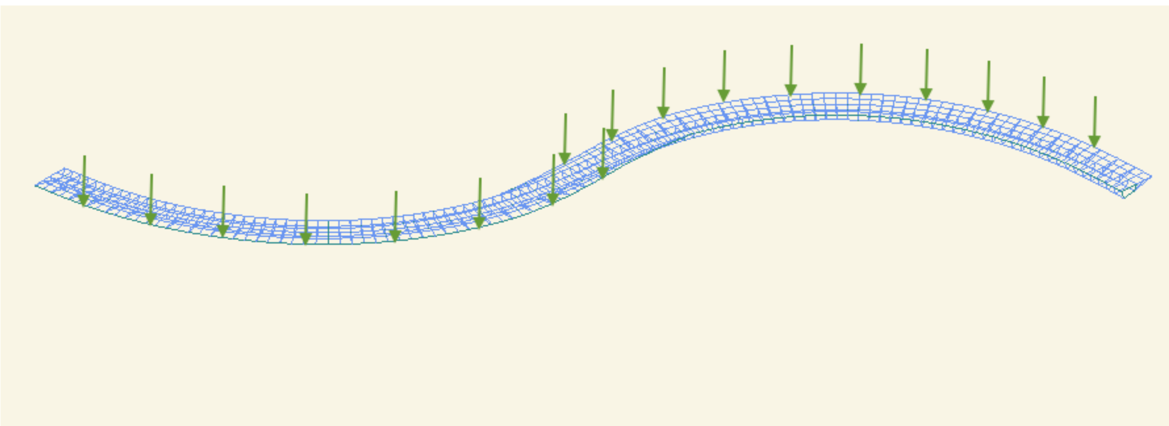


Fig.43

Thus, the solution found for a single-curve girder with fixed ends is representative of the actual behaviour of the structure when subjected to this specific load distribution. In addition it can be stressed that this particular loading condition represents the worst possible configuration in terms of torsional stresses induced in the box-girder cross sections since an opposite direction of the torques would be partly counterbalanced by the cables.

However, this loading condition only applies when searching for the maximum bending and torsional moments at the midspan section. To collect data relative to the end and quarter-span sections one shall use either an anti-symmetric load distribution and free the midspan node from all restraints. Thus, the pinned model treated a 3.2.1.2 or the following cantilever scheme apply:

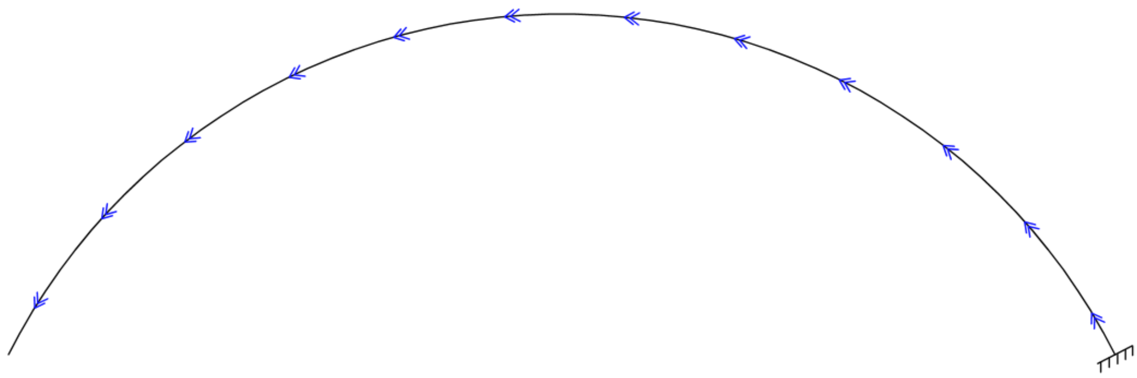


Fig.44

Representing one half of the actual structure subjected to an anti-symmetric loading:

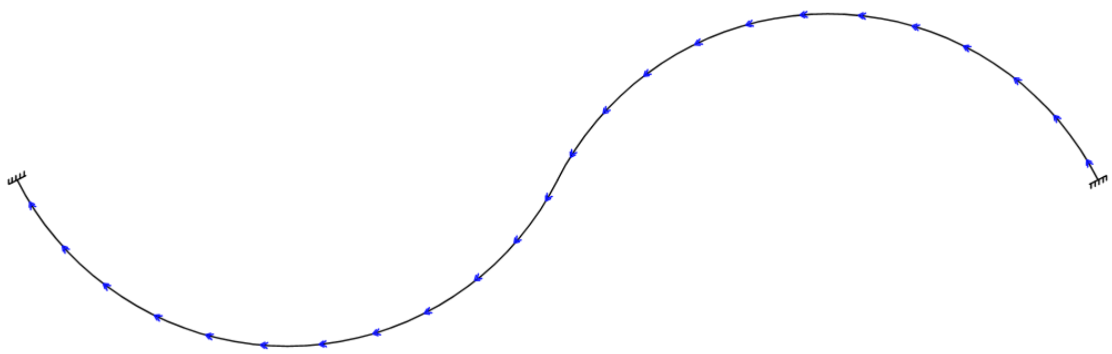


Fig.45

The numeric results for the cantilever beam model are shown in the following graphs (Fig.46,47):

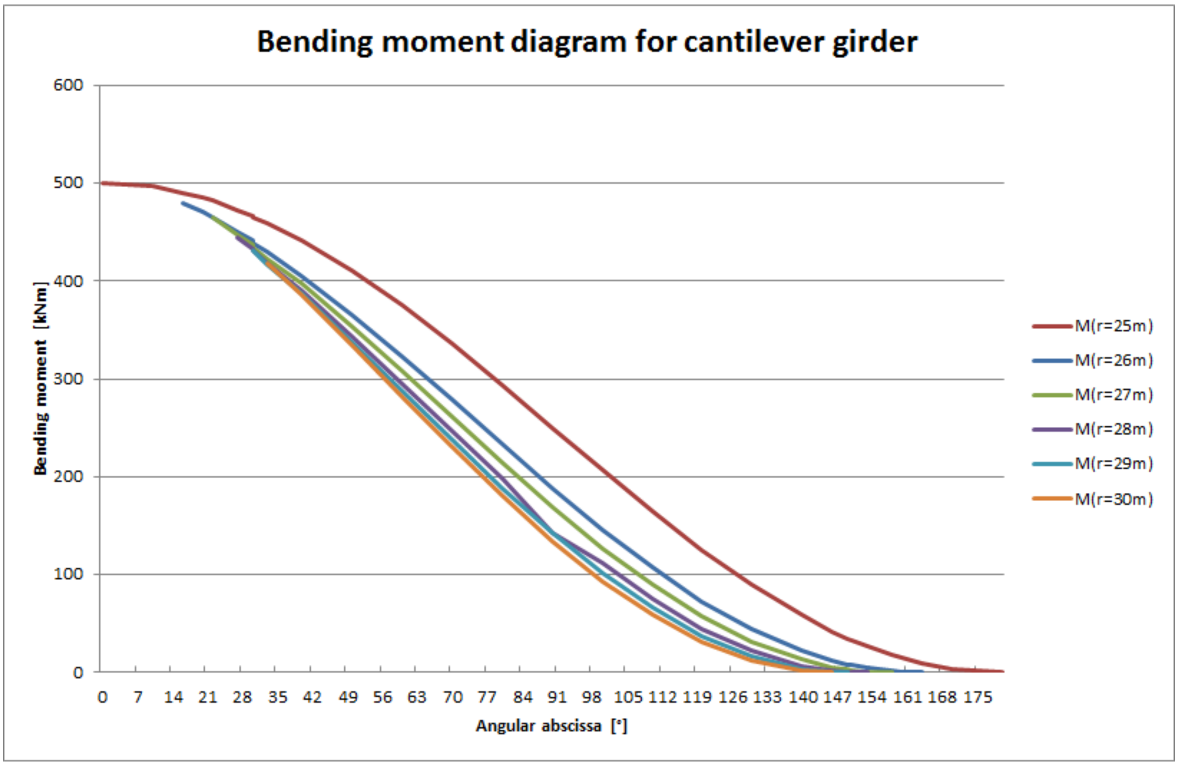


Fig.46

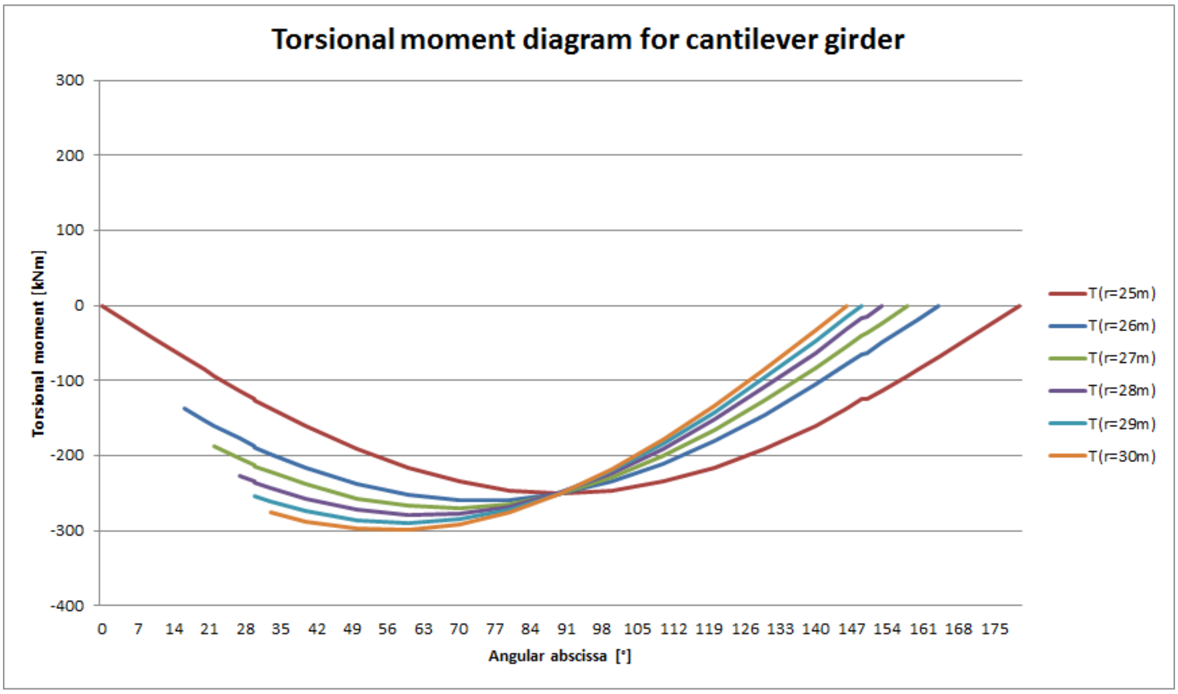


Fig.47

As the radius increases (curvature is reducing) the bending moment at the encastre becomes smaller, while the torsional moment increases (same behaviour as in the fixed-fixed model). In general we observe a worsening of the torsional condition in the first half of the girder, while a decrease of its demand is registered as we get closer to the free-end section.

The following figures give an idea of where the application of a model instead of another is appropriate to get the maximum response. For instance, in the "Maximum bending moment at quarter-span section" graph we can point out that the maximum response will occur when the pinned girder model is used (blue curve has the higest value for $\theta = 90°$).

On this basis an envelope curve has been developed upon which the optimization process has been built.

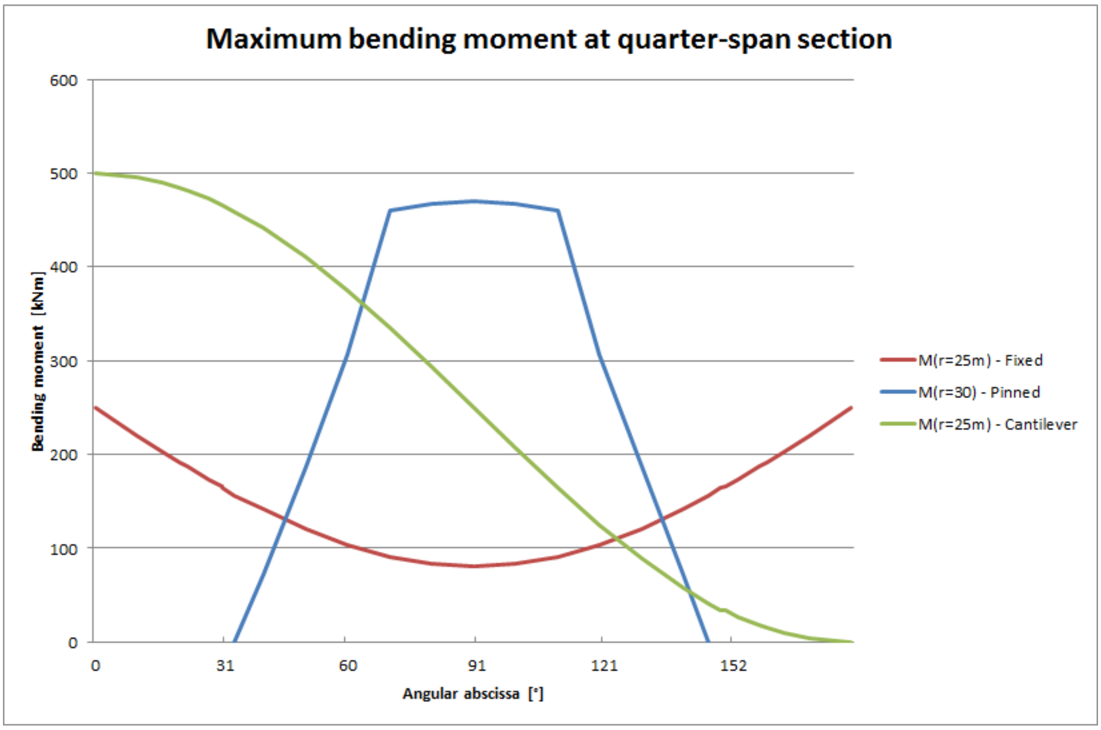
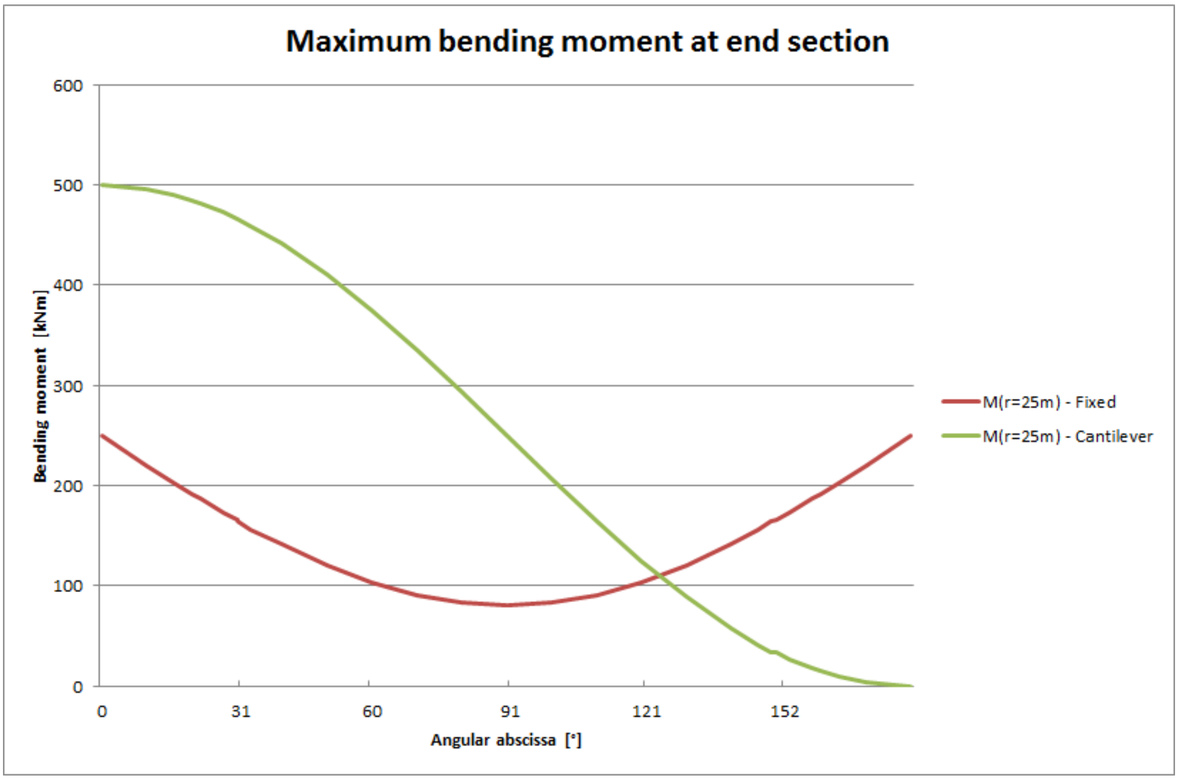
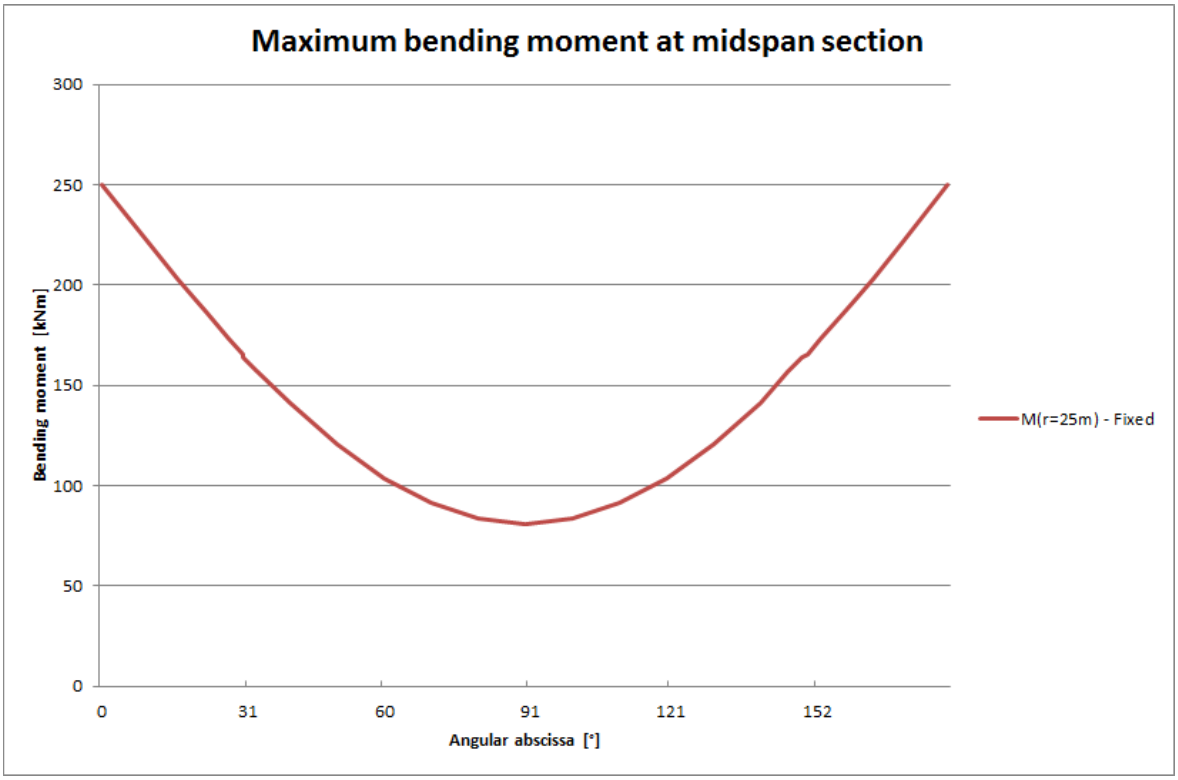


Fig.48



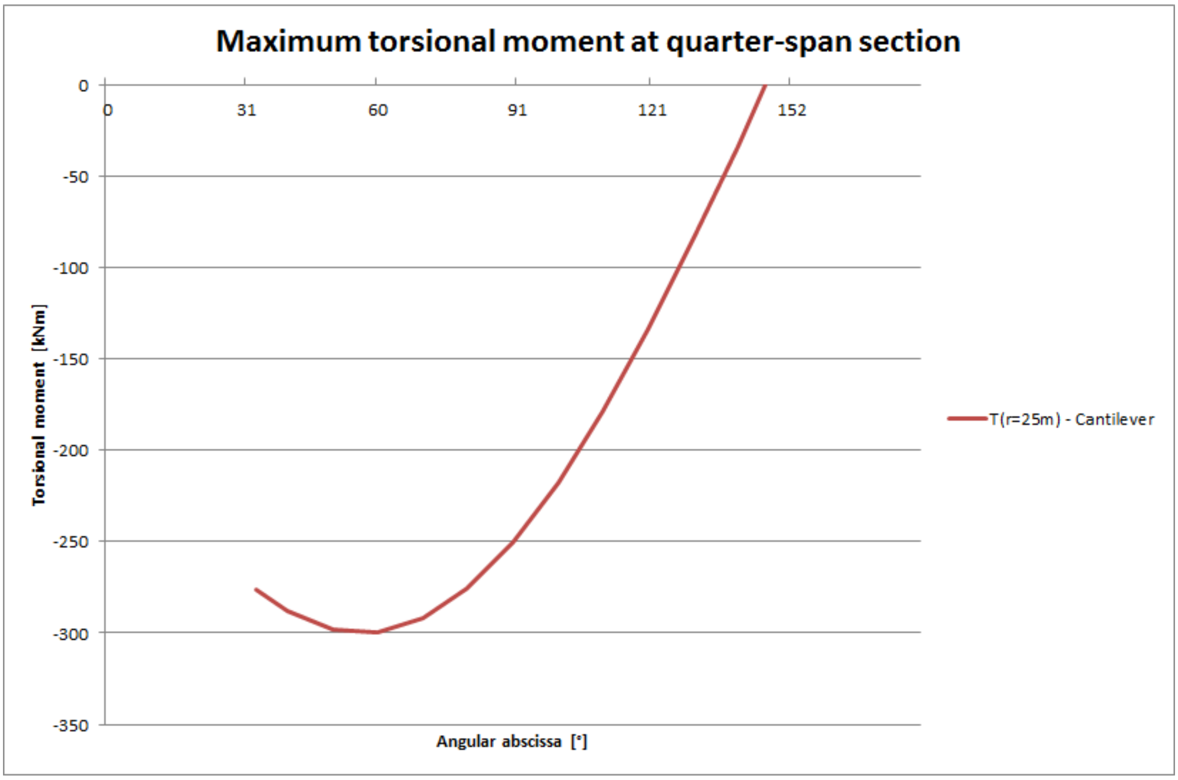
Maximum bending moment at end section
600
500
400
300
200
100
0
Bending moment [kNm]
0
31
60
91
121
152
Angular abscissa [°]
M(r=25m) - Fixed
M(r=25m) - Cantilever

Fig.49



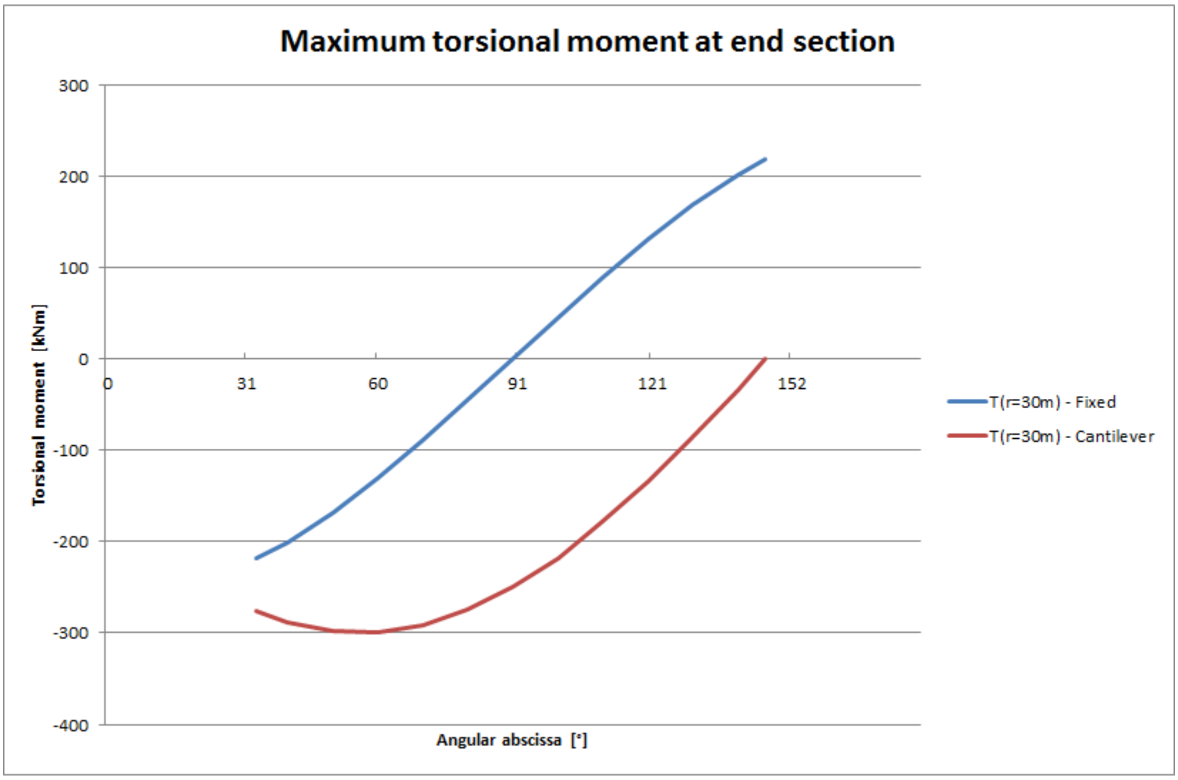
Maximum bending moment at midspan section
300
250
200
150
100
50
0
Bending moment [kNm]
0
31
60
91
121
152
Angular abscissa [°]
M(r=25m) - Fixed

Fig.50



Maximum torsional moment at quarter-span section
Torsional moment [kNm]
0
-50
-100
-150
-200
-250
-300
-350
0
31
60
91
121
152
Angular abscissa [°]
T(r=25m) - Cantilever

Fig.51



Maximum torsional moment at end section
Torsional moment [kNm]
300
200
100
0
-100
-200
-300
-400
0
31
60
91
121
152
Angular abscissa [°]
T(r=30m) - Fixed
T(r=30m) - Cantilever

Fig.52

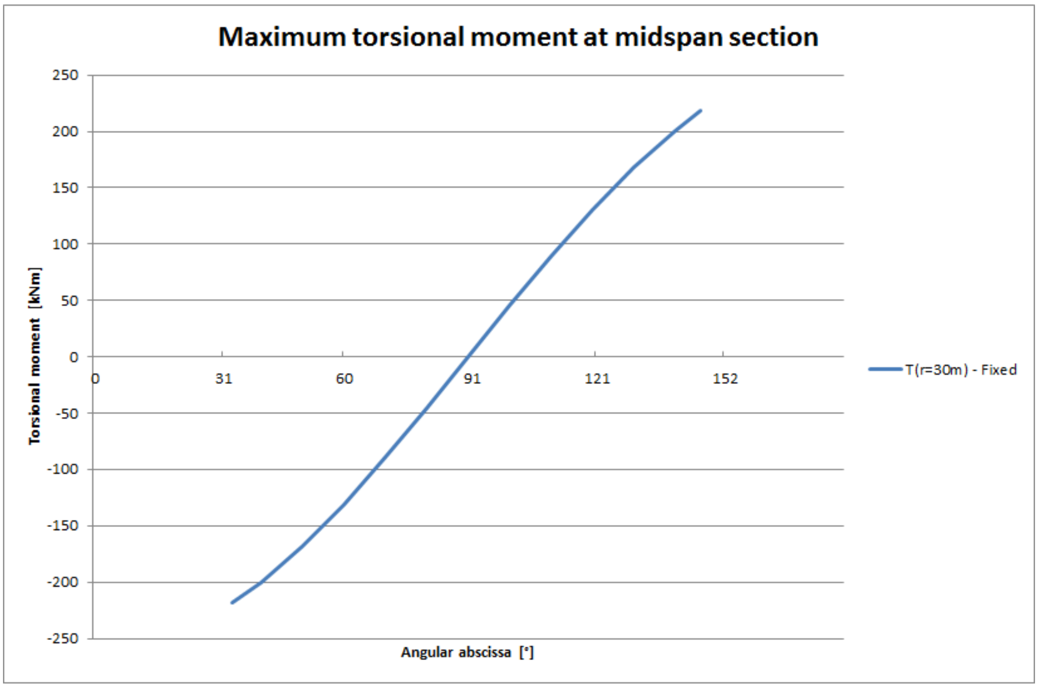


Fig.53

3.1.1.4 *Optimal solution*

In order to get an optimal solution to the problem we are dealing with an envelope curve has been plotted where each dot represents a maximum value obtained by comparing the three above-mentioned models at the relevant locations along the girder length. These quantities are functions of the girder radius as shown below.

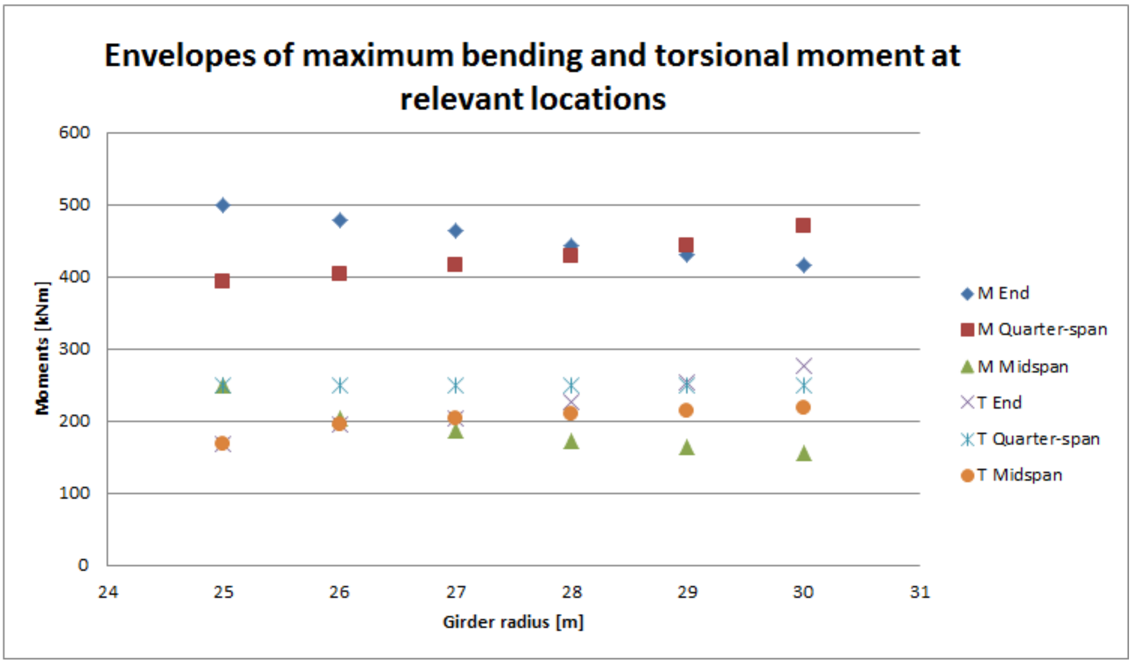


Fig.54

R [m]	M End max [kNm]	M Quarter-span max [kNm]	M Midspan max [kNm]	T End max [kNm]	T Quarter-span max [kNm]	T Midspan max [kNm]
25	500	394.338	250	168,923	250	168,923
26	480	403.053	204,092	194,976	250	194,976
27	464.222	416.3	187,614	203,913	250	203,913
28	444.58	429.96	173,085	226,525	250	209,833
29	430.595	444.143	163,649	253,64	250	214,502
30	417.219	470.45	155,525	276,151	250	218,278

Tab.10

The optimal solution will correspond to the value of radius for which the mean relative distance between the maximum response value and the actual value will be the greatest.

The absolute maximum values for each of the quantities above are listed in the following table:

M End [kNm]	M Quarter-span [kNm]	M Midspan [kNm]	T End [kNm]	T Quarter-span [kNm]	T Midspan [kNm]
500	470,45	250	276,151	250	218,278

Tab.11

R [m]	Mean distance from maximum
25	38,7825
26	39,6303
27	39,8195
28	38,4827
29	34,725
30	29,5427

Tab.12

The maximum mean distance corresponds to the 27m-radius girder.

Thus one can conclude that the optimal shape for the sigle curve deck is the one with equation:

$$x^2 + \left(y + \sqrt{27^2 - 25^2}\right)^2 = 27^2 \qquad for\ y \geq 0 \quad \wedge \ -25 \leq x \leq 25$$

The actual shape of the bridge deck in the horizontal plane will be derived by joining the two curves by half-turn rotating one of them to get an S-shape, as in Fig.55:

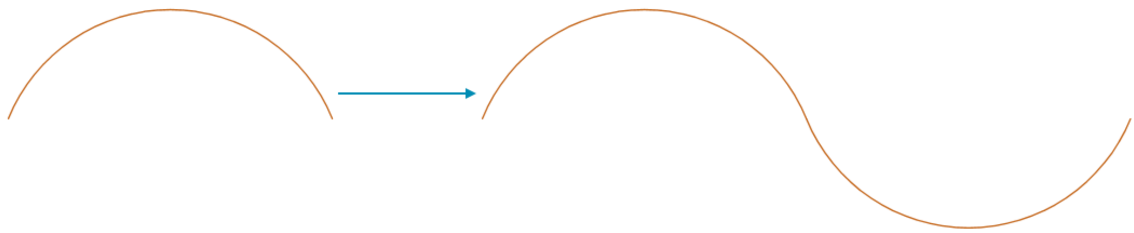


Fig.55

The optimal shape, solely considering the structural response, has now been selected.

Nonetheless, as said before, the optimal shape criterion is just one of the principles according to which the deck geometry is chosen.

3.1.2 Cost

It is easy to observe that we cannot ignore the cost parameter in our evaluation of the optimal solution. A focus on the details related to the cost analysis lies outside the goals of this research since it necessarily belongs to a more advanced design stage. On the other hand we need a tool that allows us to understand how the total cost of the structure qualitatively varies as a function of the parameters that come into play, in order to start off on the right foot.

At a pre-design stage, factors that mainly influence the total cost are the bridge's length and its cross-sectional dimensions. As the first factor changes (producing consequently a change in curvature) variations in flexural and torsional stresses are observed. As a result, the total cost will change as well. As outlined in Par. 3.1, the solution that provides the minimum stress demand is the one corresponding to a 27m-radius girder. Assuming that the bridge deck will, conservatively, have a constant section (this assumption will not apply in the actual design stage) and supposing a deck unit volume cost of 1, we get a cost function that is constant and equal to the volume itself, (Cost = 1*Length*cross-sectional area). Refering to the cross-section shown in Fig.32, and having in mind that the bending and torsional stresses distribution varies as a function of the girder's radius (and so of its length) as illustrated in the previous paragraph, we need to get a valid cross-sectional area to calculate the cost. To do we designed that cross section keeping the perimeter constant (**p** = 4,740 mm) for all the different radius girder, while the box panels have been given a variable thickness (equal for all of them). Thus the area becomes a single variable function of the box thickness. The design moments are taken from the envelope curves plotted at Par. 3.1.1.4. This means that for each girder the maximum bending and torsional responses have been considered, no matter how the beam behaves globally (conservative choice). Thus, as outlined later, the solution found at Par.3.1 representative of a mean optimum response, most likely, will not correspond to minimum box thickness. All calculations are reported in the Annex.

In order to determine the thickness, athe stress/resistance parameter R as been defined, being:

$$R = R_b \cdot R_t$$

where: $\boldsymbol{R_b} = \frac{M_{b,Ed}}{f_{yd} \cdot W}$ and $\boldsymbol{R_t} = \frac{\sqrt{3} M_{t,Ed}}{f_{yd} \cdot (a_1 + a_2) \cdot d \cdot t}$, being $\mathbf{a_1}$ the top deck panel width, $\mathbf{a_2}$ the bottom deck panel width and **d** the section's depth.

Keeping **R** constant and equal to 0.069 (an initial value that satisfies the 25m-radius girder torsional and bending demand), the values of the required box thickness **t** for the relevant family of girders are listed in Tab.**, along with the required cross-sectional area.

R [m]	Deck length [m]	t [mm]	Area [mm^2]	Cost [1•m^3]
25	157	5	2.370•10^4	3.768
26	135	4.8	2.275•10^4	3.105
27	128	4.65	2.204•10^4	2.816
28	124	4.45	2.109•10^4	2.604
29	121	4.5	2.133•10^4	2.541
30	118	5.2	2.465•10^4	2.95

Tab.13

We realize that our analysis is based on the extremely conservative hypothesis of keeping the cross-sectional height (or similarly the stresses) constant throughout the deck line, but one needs to remember that this is just a preliminary research for an initial deck geometry. This issue will be fixed in the actual design stage.

It clearly appears that the length parameter weighs on the total cost a lot more than the reduction of the required cross-sectional thickness. Thus, the solution that better satisfies the cost criterion is represented by the curve with the a 29m radius (highlighted in red in Tab.13)

3.1.3 Architectural soundness

Since we are willing to conceive a structure that reflects a deep architectural inspiration, that is a construction able to fulfill the recreational needs of the abovementioned municipalities, this third optimization criterion is far from being less important. It is based on the principle of selecting curves depicting spatial dynamic shapes that allows the user to lose the perception of staticness even if he is walking on a solid bridge. We want our bridge to be capable of enriching both the pedestrians' crossing experience and the surrounding environment.

Thus, we are looking for shapes that give the user ever-changing viewpoints on the structure itself while he crosses Arno river. To achieve this goal we need to use extremely fluid geometries, both in plan and elevation.

We will use a cosine function ($z = 1.5 \cdot \cos\,(\frac{\pi}{100}x)$) for the X-Z plane in order to create a tridimensional deck curve, getting a camber at bridge midspan measuring 1.50 m (Fig.57).

Architectural dynamism will be fulfilled using a variable spine box section (the bottom panel angle varies as a linear function of the longitudinal abscissa) to which cantilever panels are attached, creating overlooks on the right side of the deck, first, and then on the opposite side as pedestrians reach the midspan section. Non-vertical masts complete the picture with a sense of precariousness.

Since the optimal solution according to the structural efficiency criterion is the 27m-radius girder while the optimal solution that satisfies the cost criterion is the 29m-radius one, we now have the opportunity to choose a third curve which represent a compromise between those two and that at the same time satisfies the architectural requisite.

Such a curve will be described by the following analytical function:

$$f_0(x) = \begin{cases} 0.025 \cdot (x+25)^2 - 16; & -50 < x \ \le -25 \\ 16 \cdot \sin\left(\frac{\pi}{50}x\right); & -25 \le x \le 25 \\ -0.025 \cdot (x-25)^2 + 16; & -50 < x \ \le -25 \end{cases}$$

Fig.55 compares this last solution with the ones previously found, that are defined by f_{c1} and f_{c2} functions, being:

$$f_{c1}(x) = \begin{cases} -\sqrt{27^2 - (x+25)^2} + \sqrt{27^2 - 25^2}; -50 < x \le 0 \\ \sqrt{27^2 - (x-25)^2} - \sqrt{27^2 - 25^2};\ 0 \le x < 50 \end{cases}$$

$$f_{c2}(x) = \begin{cases} -\sqrt{29^2 - (x+25)^2} + \sqrt{29^2 - 25^2}; -50 < x \le 0 \\ \sqrt{29^2 - (x-25)^2} - \sqrt{29^2 - 25^2};\ 0 \le x < 50 \end{cases}$$

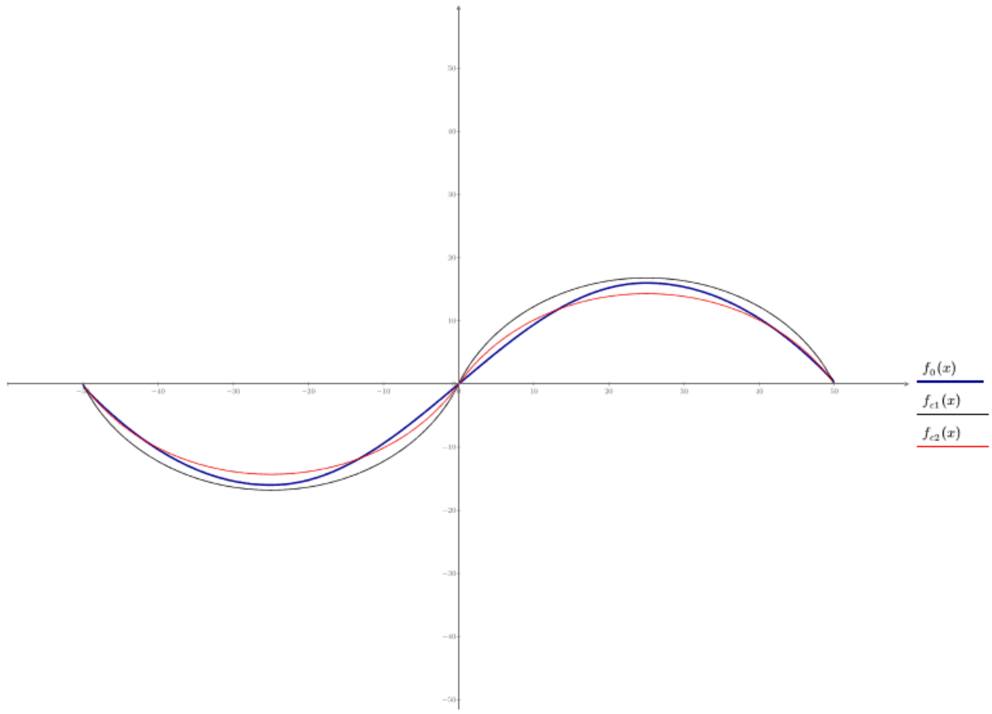


Fig.55

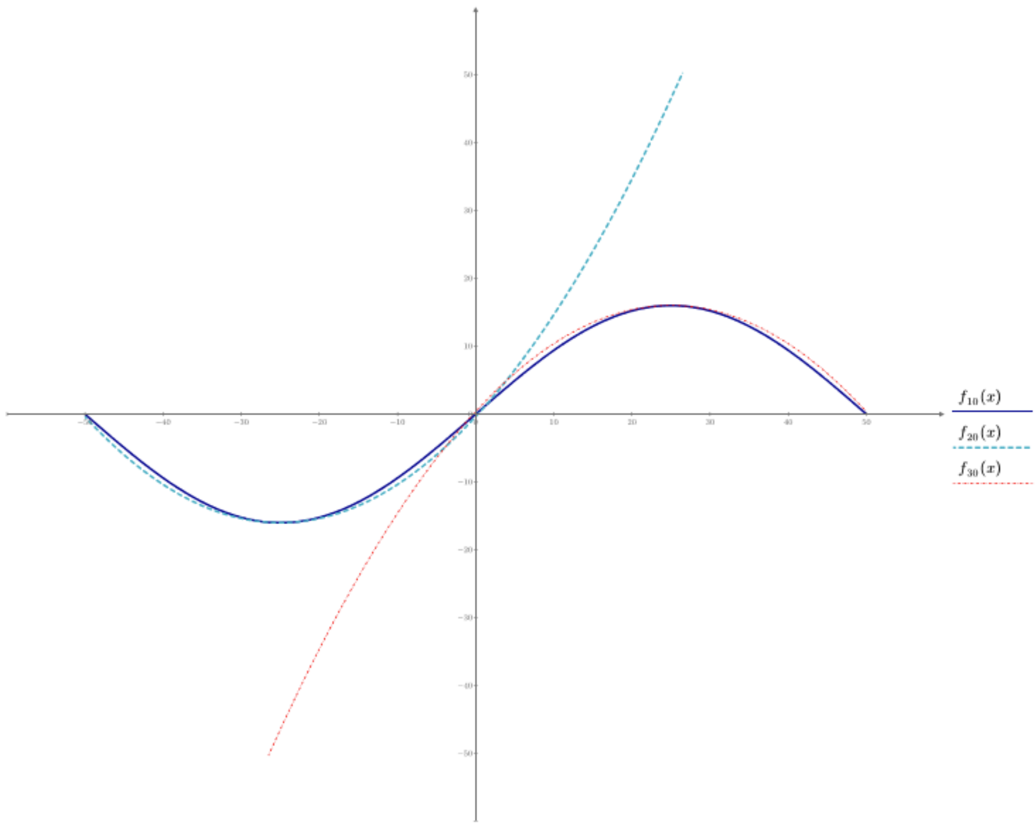


Fig.56

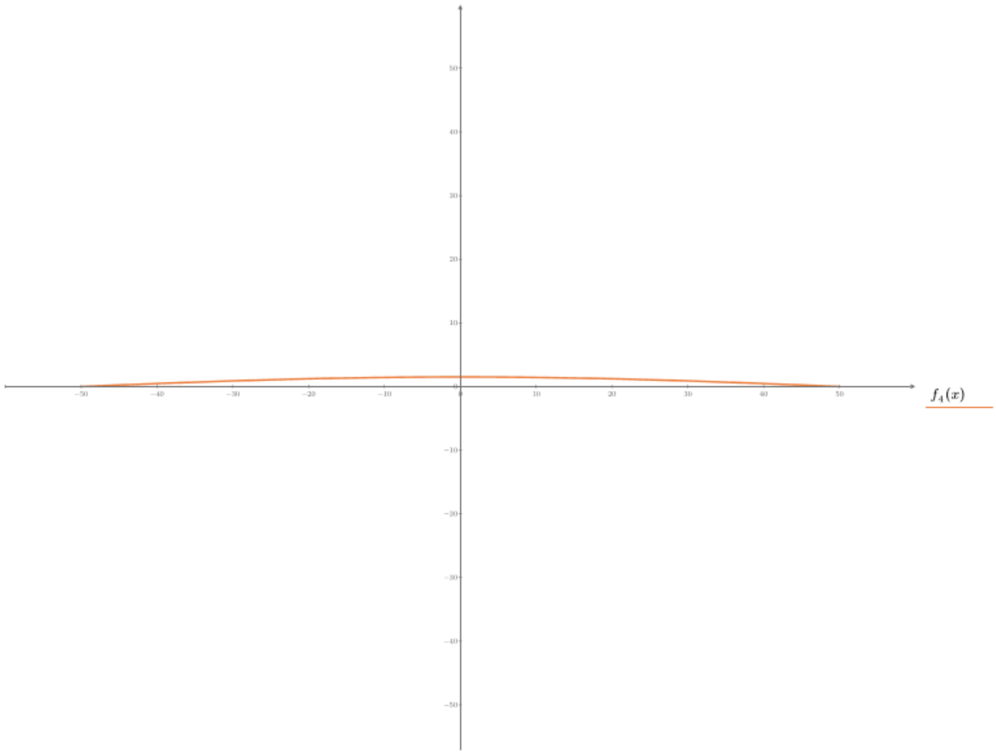


Fig.57

In the actual design stage, the bridge alignment has been modified a little bit to limit the stay-cables length and the masts' height that otherwise would have been excessive, though keeping the initial alignment morphology. Our design curve is the one defined by the following function:

$$y(x) = \begin{cases} 0.0175 \cdot (x+25)^2 - 11; & -50 < x \leq -25 \\ 11 \cdot \sin\left(\frac{\pi}{50}x\right); & -25 \leq x \leq 25 \\ -0.0175 \cdot (x-25)^2 + 11; & -50 < x \leq -25 \end{cases} \qquad \text{Horizontal plane}$$

$$z = 1.5 \cdot \cos\left(\frac{\pi}{100}x\right); \qquad -50 < x < 50 \qquad \text{Vertical plane}$$

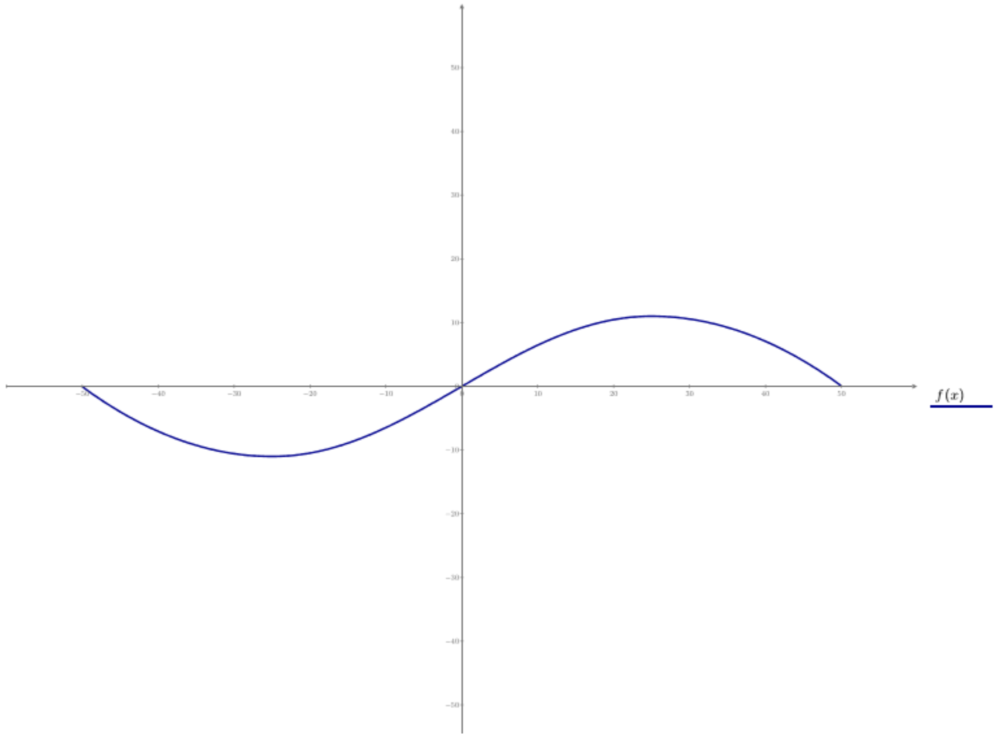


Fig.58

3.2 Suspension system geometry

In order to find an initial position of the masts' top nodes in the unloaded structure configuration, a study has been conducted with the aim of finding the optimal set of coordinates for the relevant nodes so that the cables' system configuration would be associated with the minimum strain energy.
First of all, a continuous beam model of the bridge with a roller support at each cable-to-deck joint was built and the resulting vertical reactions $\mathbf{R_i}$ were obtained.
The optimal solution would, then, have to satisfy the following system of equations (Syst.2):

$$\begin{cases} N_i = \dfrac{R_i}{\sin\left(\alpha_i\,(x,y,z)\right)} \\ \displaystyle\sum_i N_i \cdot \Delta l_i = min \end{cases} \qquad \text{Syst. 2}$$

Where:

- N_i is the axial stress in the i^{th} stay-cable under bridge dead loads;
- $\alpha_i(x,y,z)$ is the angle, in the vertical plane, between the mast's top node and the hinged node and it is a function of the mast top node's coordinates;
- Δl_i is the i th cable elongation under bridge dead loads.

A solution to such a system guarantees that the axial stresses in the cables are in equilibrium with the loads applied and the system's total strain energy in minimized.

The system above can be modified having in mind that $\Delta l_i = \frac{N_i}{EA} \cdot l_i$ (being E the axial stiffness of the cable and A its cross-sectional area). By substituting the first equation into the second one, and re-writing the elongation expression as just stated, we get:

$$f(x, y, z) = \sum_i \left(\frac{R_i}{\sin(\alpha_i(x, y, z))}\right)^2 \cdot \frac{l_i}{E \cdot A}$$

being l_i the length of th i^{th} cable. Since we are aiming to minize the total strain energy we shall search for the values of (x,y,z) that minimize the function *f*. A fixed value of z equal to 15 m (distance between the mast top and the deck top panel) was choosen according to the architectural needs.

The coordinates of the mast top node minimizing *f* were found (all the calculation details are reported in the Annex):

$$(x, y, z) = (17.265m, 17.696m, 15m)$$

An initial geometry for the suspension system is now achieved. A set of cables' prestresses will be specified in order to counterbalance the mast's selfweight which was not accounted for in the form-finding process. This aspect is treated in a later chapter.

3.3 Masts' tilt angles

Mast initial tilt angle was found following a simple concept. Since our goal is to achieve an initial geometry of the cable system that is, as much as possible, close to the deformed shaped when dead loads are acting on the structure, if we could make the top mast nodal net force's tilt angle (obtained from a linear model where this node is hinged) equal to the mast's angle then, under dead loads, the structure will experience displacements only due to the axial deformation of the mast. However one should notice that in the current analysis we are neglecting the mast selftweight. This means that our results will not reflect the foreseen behaviour unless a set of cables' prestresses are defined, so that this selfweight can be counterbalanced.

The mast initial tilt angle will then be equal to 56°, as extensively explained in the Annex.

4. Analysis

4.1 FE model

The structure was modeled using the GSA Oasys software by Arup.

Since the bridge is characterized by a deep geometric and structural complexity, simplified FEM models were not considered in the analysis, since it was deemed that only a complete structural model would describe the real behaviour of the bridge. The following figures picture the model.

The deck structure consists of Quad-4 and Triangle-3 shell elements. The orthotropic panels of the spine box-girder were given an equivalent thickness to account for the increased inertia due to the longitudinal stiffeners. The restraints consist of an hinge and a roller at each of the origin and end sections (Fig.63), other than the 5m-spaced supports guaranteed by the stay-cable. That was the only choice allowable since the main load bearing system is ill-conditioned and made stable only by using the deck stiffness itself.

The masts were modeled as beam elements, pinned at the base node and attached to the cable system at the top node. They were split into 22 elements to realize a variable-radius CHS section (Fig.64).

The cables were modeled using tie elements, i.e. elements with zero stiffness under compressive forces.

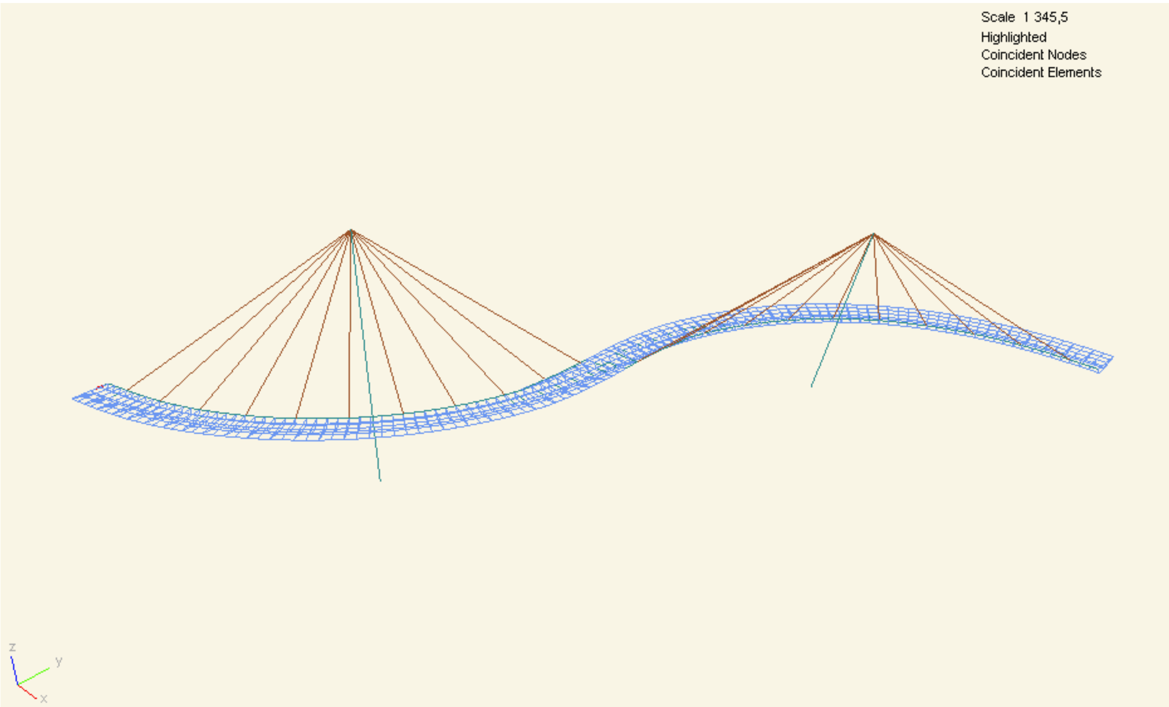


Fig.59

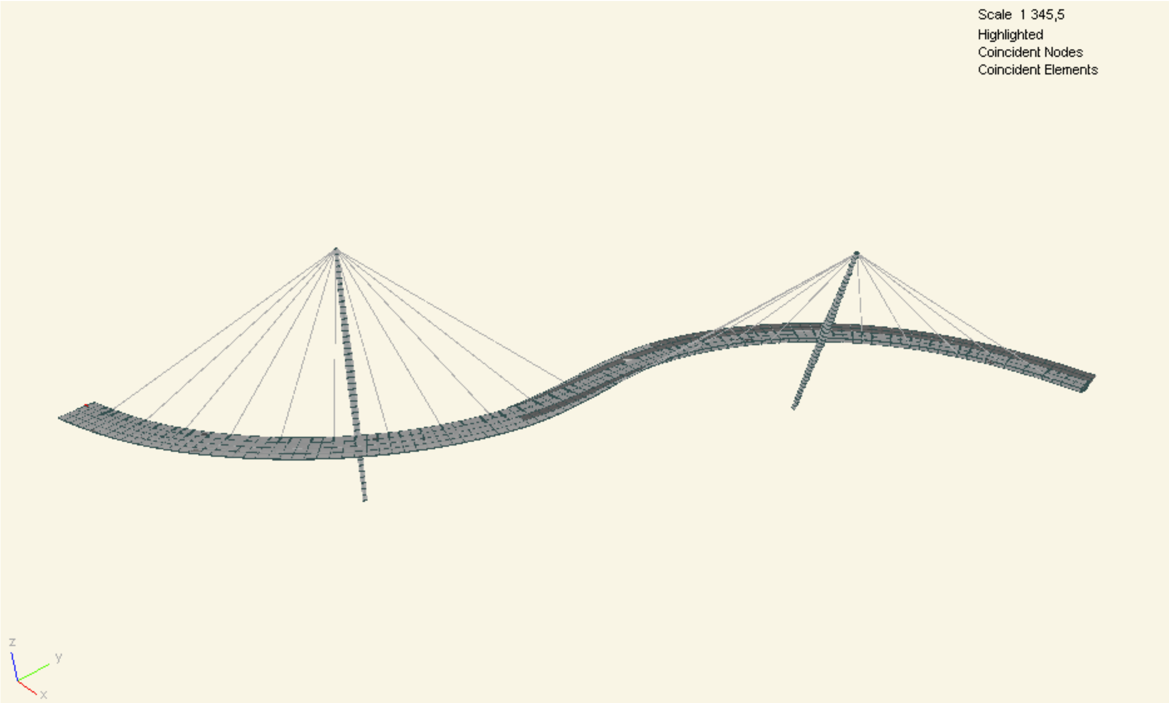

Scale 1 345,5
Highlighted
Coincident Nodes
Coincident Elements

Fig.60

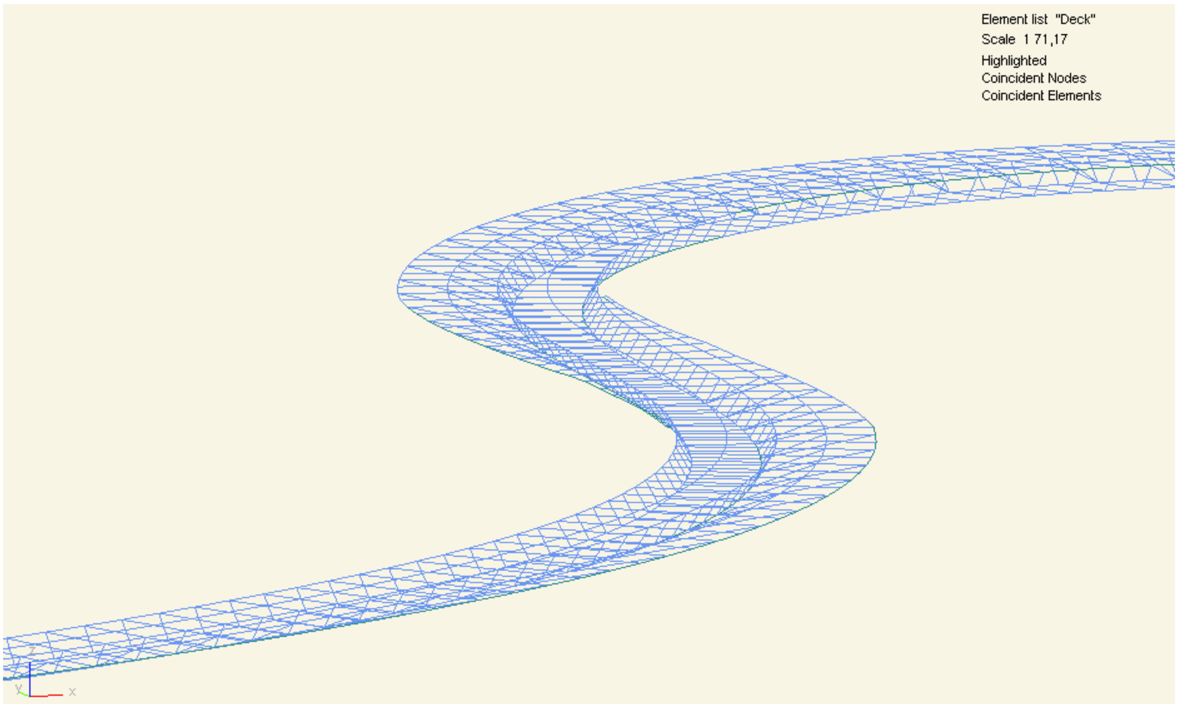

Element list "Deck"
Scale 1 71,17
Highlighted
Coincident Nodes
Coincident Elements

Fig.61

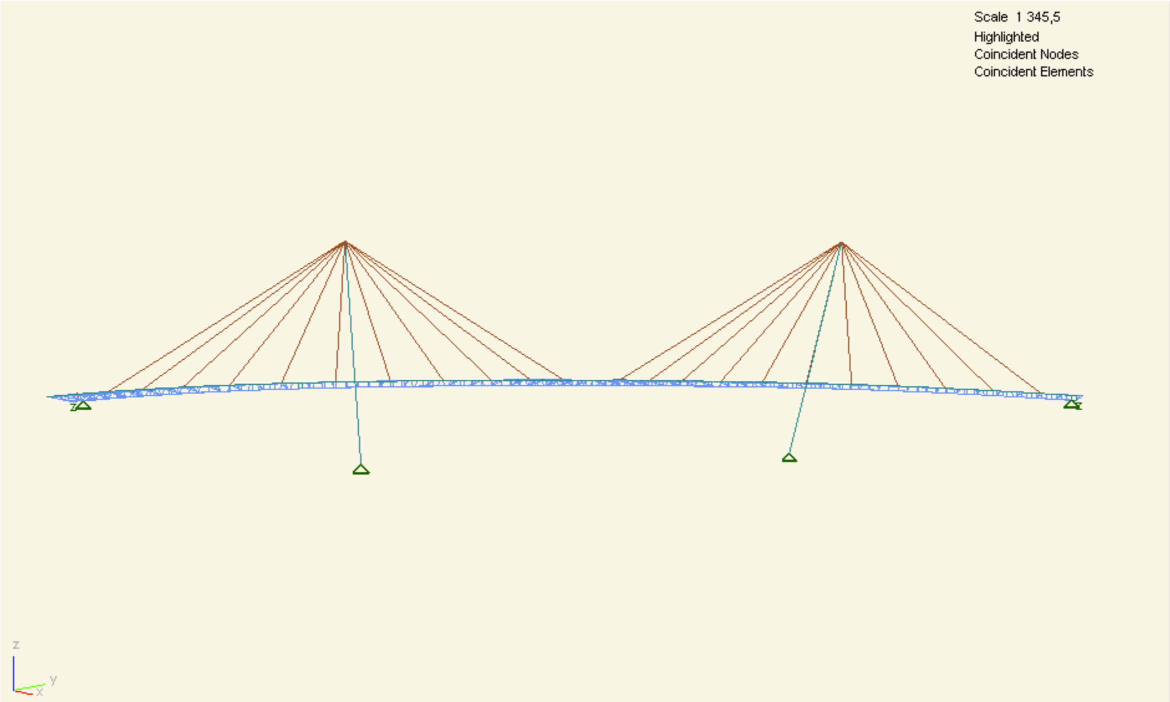

Scale 1 345,5
Highlighted
Coincident Nodes
Coincident Elements

Fig.62

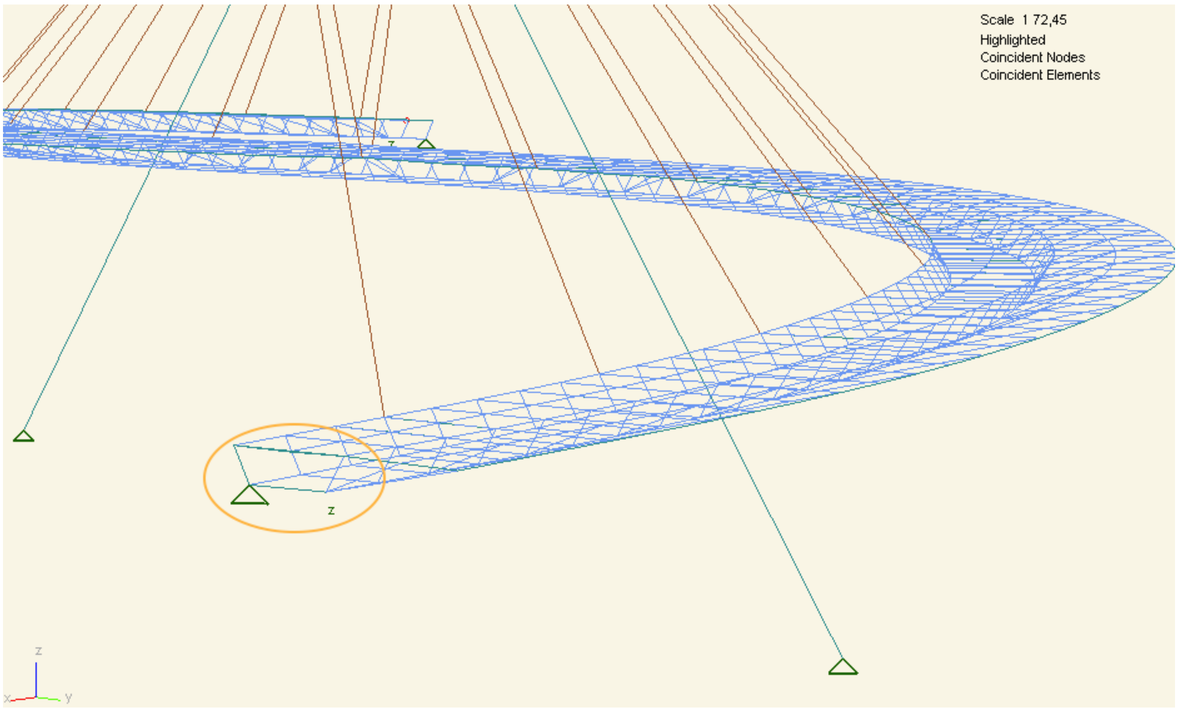

Scale 1 72,45
Highlighted
Coincident Nodes
Coincident Elements

Fig.63

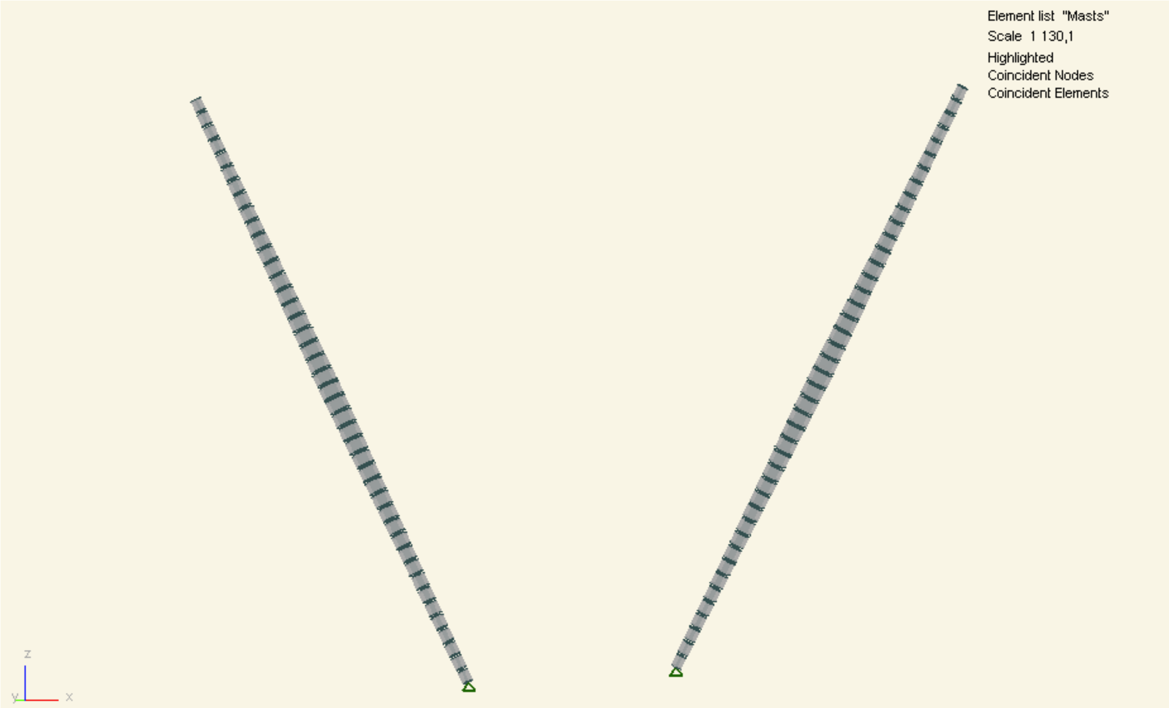


Fig.64

The bridge layout line was drawn using the GSA alignment option and following the optimal curve defined in Chapter 3. The deck nodes were then forced to follow the vertical plane cosine function by using the “transform geometry” command.

In addition, specific FE models were built, both in GSA and Ansys, for the analsysis and design of the mast foundation and the top-mast joint (see Chapter 5).

The mast is founded on four 52cm-radius drilled shafts attached to a 330cmx330cmx150cm pile-cap. To verify the hand calculations, model shown in Fig.64 was used. It consists of 56 thick-shell Quad-4 elements attached to four beam elements representing the piles. Spring restraints were then added to the lateral and bottom surfaces of the shafts and the cap, to model the soil.

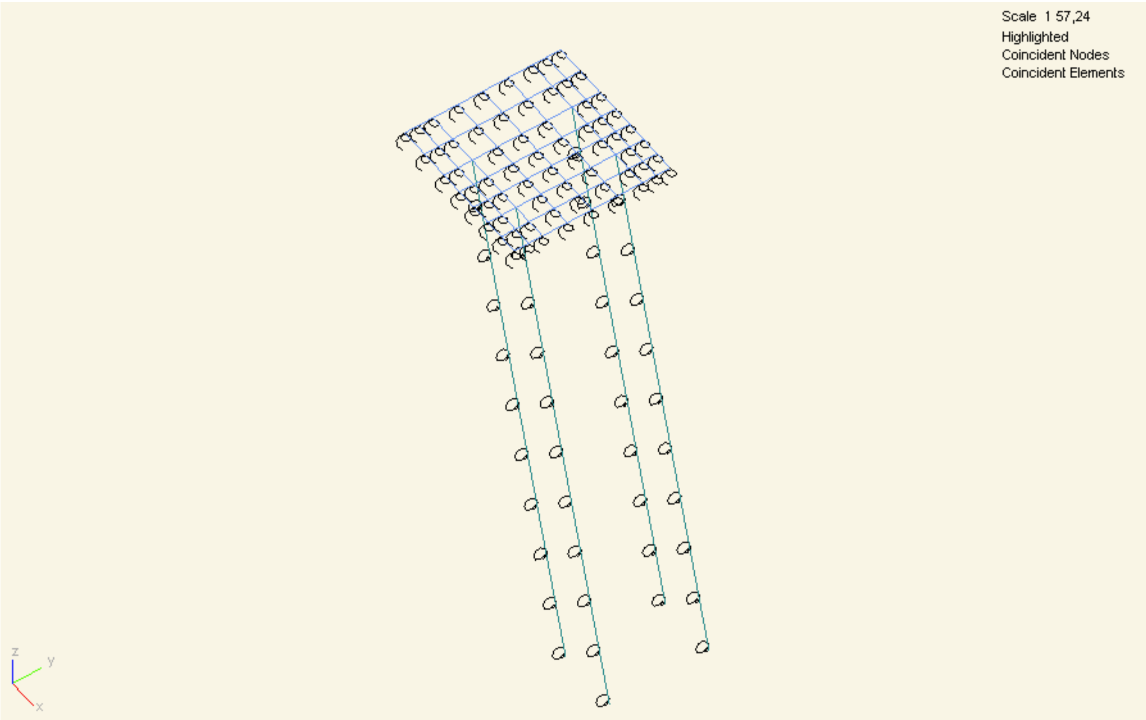

Scale 1 57,24
Highlighted
Coincident Nodes
Coincident Elements

Fig.65

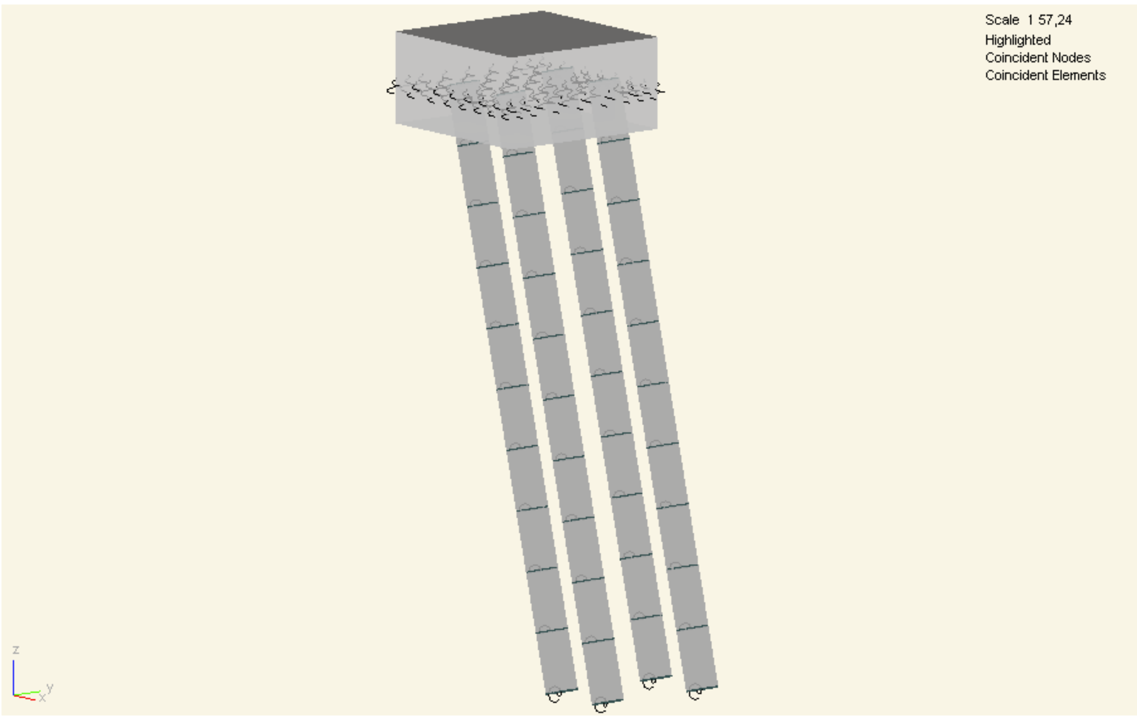

Scale 1 57,24
Highlighted
Coincident Nodes
Coincident Elements

Fig.66

4.2 Types of analysis

Different types of structural analysis were used during the design of the bridge with the aim of getting a better understanding of the structure's physics and behaviours.

Form-finding analysis

A form-finding analysis with the "ignoring form-finding properties" option was used to achieve the initial geometry of the structure. The analysis consists of a static non-linear analysis where the resulting displaced nodal coordinates overwrite the original geometry, and element distortions and tensile forces are stored in an additional load case. The structure in the deformed shape plus the additional load case is in equilibrium with the dead loading. So if the deformed model is analysed with the same applied loading as before plus the new load case, negligible further movement should occur. An exception to this occasionally arises where distributed element loading has been used as the equivalent nodal loads of the element loads will be generated based on the deformed geometry. They are different from the equivalent nodal loads used in the form-finding analysis, that were generated based on the original undeformed geometry.

The process is summarised in the following steps:

- Create a model with the geometry found in the pre-design stage (described in Chapter 3);
- Apply the dead loads to the structure's elements;
- Run a form-finding analysis;
- Get a set of cables prestresses in equilibrium with the deformed shape under dead loads;
- Modify the prestresses to account for the masts' self-weight and the error generated by using distributed dead loads;
- Get the desired dead-load geometry.

Non-linear static analysis

The structure is inherently non-linear due to the fact that it is cable-supported and mechanically ill-conditioned (the masts are, in fact, pinned at the bottom). This double non-linearity made the use of a non-linear static analysis necessary. All the resistance and serviceability members verifications are based on the results obtained from such analysis (as shown in Chapter 5).

GSA's non-linear solver is called GsRelax. The solution technique used in GsRelax solver is the Dynamic Relaxation. Dynamic relaxation is an analysis method for non-linear statically loaded structures direct integration dynamic analysis technique. In dynamic relaxation analysis it is assumed that the loads are acting on the structure suddenly, therefore the structure is excited to vibrate around the equilibrium position and eventually come to rest on the equilibrium position. In order to simulate the vibration, mass and inertia are needed for each of the free nodes. In dynamic relaxation analysis, artificial mass and inertia are used which are constructed according to the nodal translational stiffness and rotational stiffness. If there is no damping applied to the structure, the oscillation of the structure will go forever, therefore, damping is required to allow the vibration to come to rest at equilibrium position. All the solution control parameters can be set by the user depending on the accuracy of the expected results.

Since the structure is mechanically unstable, a non-linear static analysis with load increments was, also, run to investigate what value of the service loads would produce unaccetpable displacements of the relevant elements.

P-Δ buckling analysis

The buckling analysis of the masts was conducted by using GSA member-buckling option, since a modal buckling analysis cannot be used for such a non-linear structure. This type of analysis is useful for estimating the degree of restraint offered by the whole structure under a particular load condition. This is particularly relevant to non-linear structures where the degree of restraint offered by other parts of the model varies with load case.

The program analyses the structure with 100% imposed loads. This is called the first step analysis. If this converges, the structure is well conditioned; the number of cycles required for convergence under applied loads is stored as a measure of the stability of the structure. The program stores the equilibrium element forces and adds a small disturbance moment to the member under investigation since buckling may not occur without a small disturbance to the member.

The next step is to re-analyse the model with the imposed loads from the initial analysis but with a factored axial force in the element(s) under consideration. This is repeated until the element(s) in question buckles.

Buckling is deemed to occur when the analysis fails to converge within 5 times the number of cycles that achieved the convergence in the first step analysis.

P-Δ modal analysis

A P-delta modal analsysis was used to find the foobridge's mode shapes and eigenfrequencies on which all the dynamic analysis is based.

Modal analysis is by definition only applicable to a linear model, so if the model contains non-linear elements (in our case ties) these need to be linearized. For a straightforward modal analysis this is done by treating these elements as bars (able to take both compression and tension). In the case of a P-delta analysis the stiffness matrix is modified by the P-delta effects and this modified stiffness is used in the eigensolver. If the non-linear elements are inactive following the P-delta pass they are then excluded from the stiffness matrix for the modal analysis, if they are included they are treated as bars.

If the geometric stiffness acts to stiffen the structure the result will be that the natural frequencies are increased, while if the geometric stiffness reduces the stiffness of the structure the result will be that the natural frequencies are lowered.

Footfall analysis

Footfall induced vibration analysis has been used to evaluate the response of the structure subjected to the actions of human footfalls. The structural responses include nodal accelerations, velocities and response factors. The human footfall loads are considered as periodical loads which are represented by a number of harmonic loads according to Fourier series theory.

As this type of analysis utilizes modal dynamic analysis results, the modal P-delta dynamic analysis outputs were used.

All the details related to the procedure used by the program will be treated in Chapter 6.

Harmonic analysis

In addition to the footfall analysis, a harmonic analysis has been carried out to be able to control the accuracy of the footfall results. A harmonic forcing load defined by the Sètra guidelines on footfall induced vibrations was used to simulate a distributed crowd walking on the bridge. This was made necessary by the fact that GSA forcing load only reflects the vibrational behaviour of the structure when it is subjected to a single pedestrian moving along the bridge. Since we wanted to model a more realistic stream of walkers, this second approach was utilized to get both vertical and horizontal accelerations. These outputs were then compared to those obtained from the footfall analysis.

Influence lines analysis

Since GSA-Bridge does not allow for the use of 3D alignments, an influence lines analysis was conducted referring to a horizontal plane deck layout line in order to assess what parts of the deck were to be loaded so that a specific maximum response woud have been obtained.

An influence lines-surfaces analysis is inherently only applicable to linear systems, so, to be able to use it with the actual structure we are dealing with, modifications of the model were made, i.e. the structure was linearized by preventing the top-mast node from moving. A set of relevant nodal displacements and members forces influence effects were then defined. Accordingly the deck was loaded to get the maximum responses.

To validate the extension of these results to the actual non-linear structure the following procedure was followed. A dead loads form-finding (ignoring form-finding properties) was run to get the bridge configuration onto which the live loads are applied. Then, the top-mast node restraints were modified into a pin and live loads were expanded to the relevant regions of the deck found from the influence analysis. The displacements and forces were registered and then compared to the ones obtained by applying the live loading to the free-to-move masts model once a dead load+prestress case had been run. These results were sufficiently similar so that the error made in using the linearized model was deemed to be small enough. Since the actual structure has a vertical camber (following the cosine function defined in a previous chapter), the deck nodes were moved into the final configuration by using a "transform geometry" operation and live loads were applied to the relevant parts of the deck.

Response spectrum analysis

A response spectrum analysis has been run in order to determine the response of the structure to the seismic excitation. Spectra used to characterise the earthquake were selected according to the Italian code NTC 2008 in relation with the relevant limit states. Specific results that were matter of concern are stresses induced in the members by the dynamic forces and the relative displacements between the mast and the deck nodes.

5. Design

The design and verification of the structural elements were conducted by using the limit states semi-probabilistic method and adopting the following recognized standards:

6. UNI EN 1536:2010.
7. UNI EN 1537:2013.
8. UNI EN 1990:2006.
9. UNI EN 1991:2004.
10. UNI EN 1992:2005.
11. UNI EN 1993:2005.
12. UNI EN 1997:2005.
13. UNI EN 1090-1:2012.
14. UNI EN 1090-2:2011.
15. British National Annexes to Eurocodes .
16. BS 5400.
17. BS 6841.
18. Sètra/AFGC footfall guideline.

5.1 Preliminary design

The preliminary design of the structure was conducted using the methods treated in Chapter 3.

As mentioned above, given the geometric and structural complexity of the bridge , simplified FE model were neglected, since it was deemed that they would have not been capable of describing the structure's physical behaviour appropriately. However, a pre-design of the main cross section elements, cantilever panels, masts cross section and details, stay-cables and sockets was carried out using the general structural analysis and design methods.

Hand calculations have been reported in the Annex.

5.2 Loads

5.2.1 Permanent loads

The permanent loads include loads that are relatively constant over time, including the weight of the structure itself and immovable fixtures like balustrades, benches, pipes, wires and lighting systems.

Gravity loading

Elements' self-weight is automatically calculated by the program, depending on the material choosen.

Concrete slab

The top deck is provided with a 12cm thick C30/37 concrete slab that has the double function of enhancing the inertial properties of the cross section thus allowing for a reduction of the number of the costly longitudinal stiffeners, and to prevent the top deck steel plate from wobbling, making pedestrian feel uncomfortable. A surface load of 3 kN/m² was choosen (Fig.67).

Decking

A wood decking was choosen for the bridge floor top layer, weighing 0.4 kN/m² (safe-sided value) (Fig.68).

Balustrade

A cable-net balustrade was choosen, modeled as a 1kN/m load distributed along the edges of the deck width (Fig.69).

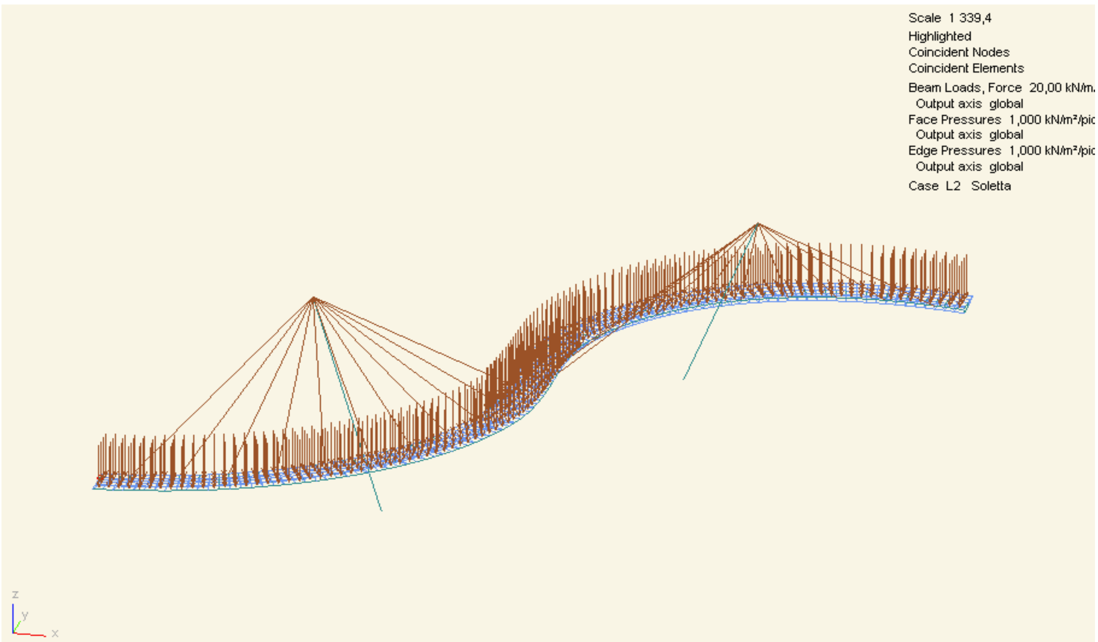
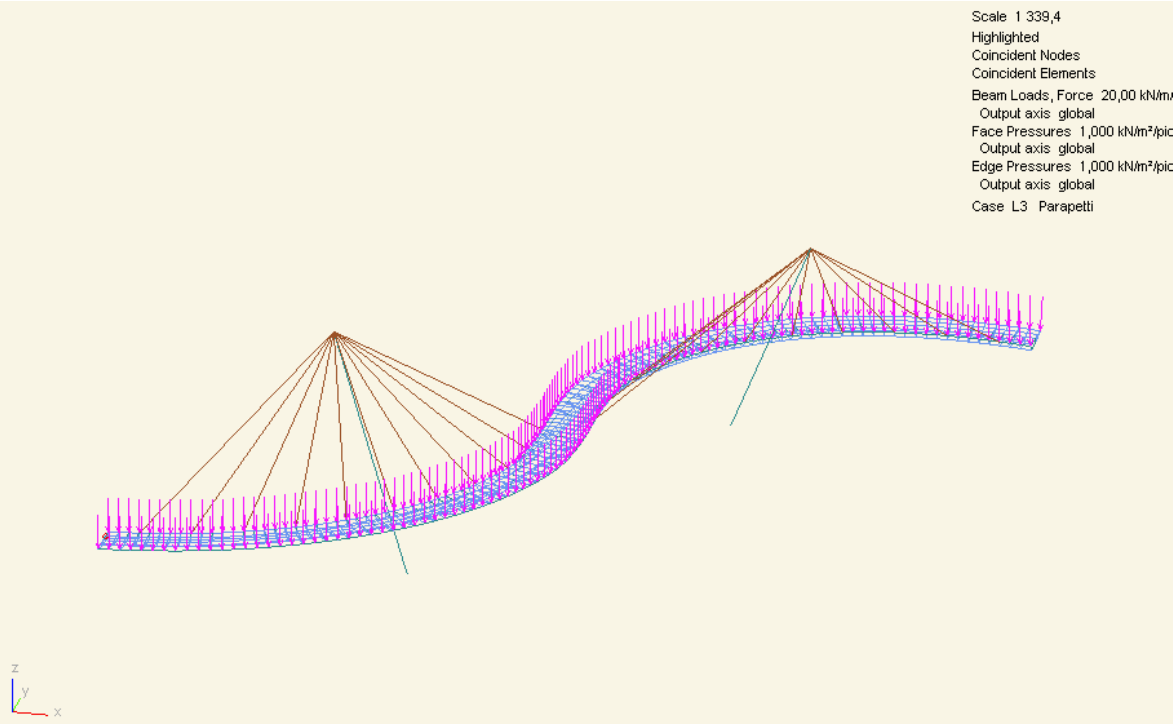


Fig.67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Highlighted
Coincident Nodes
Coincident Elements
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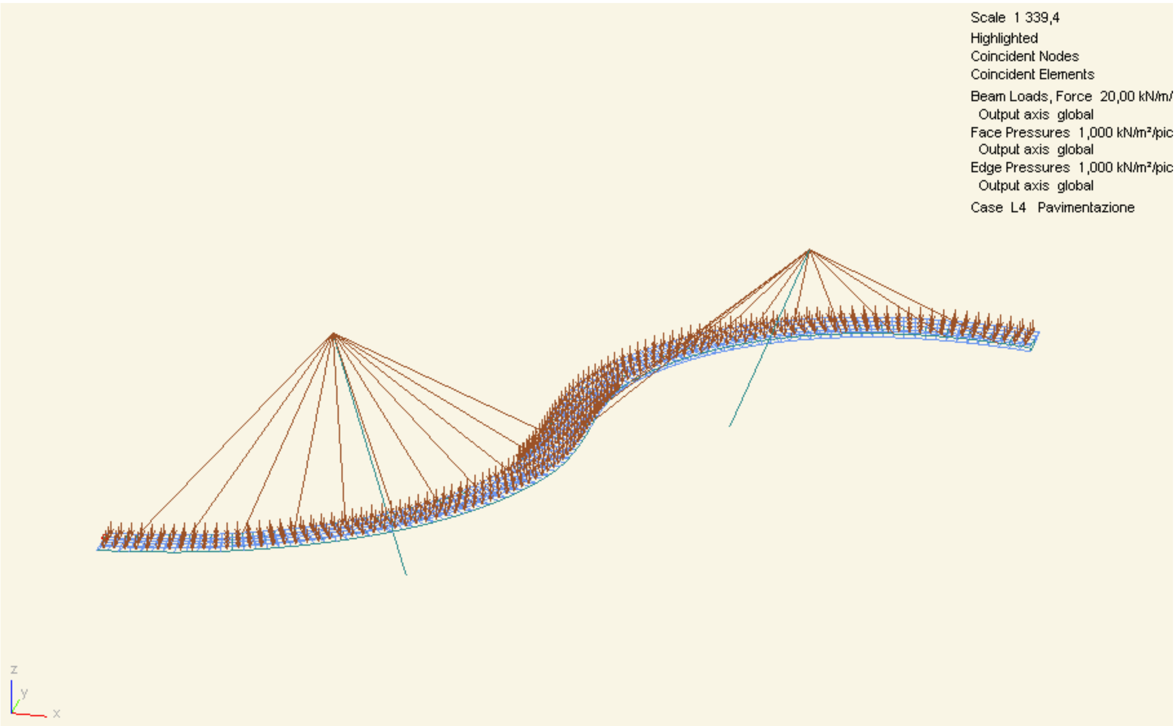
Output axis global
Face Pressures 1,000 kN/m²/pic
Output axis global
Edge Pressures 1,000 kN/m²/pic
Output axis global
Case L3 Parapetti
z
y
x

Fig.68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Coincident Elements
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Face Pressures 1,000 kN/m²/pic
Output axis global
Edge Pressures 1,000 kN/m²/pic
Output axis global
Case L4 Pavimentazione
z
y
x

Fig.69

5.2.2 Traffic static loads

Three different vertical load models can be envisaged for footbridges:

- a uniformly distributed load representing the static effects of a dense crowd;

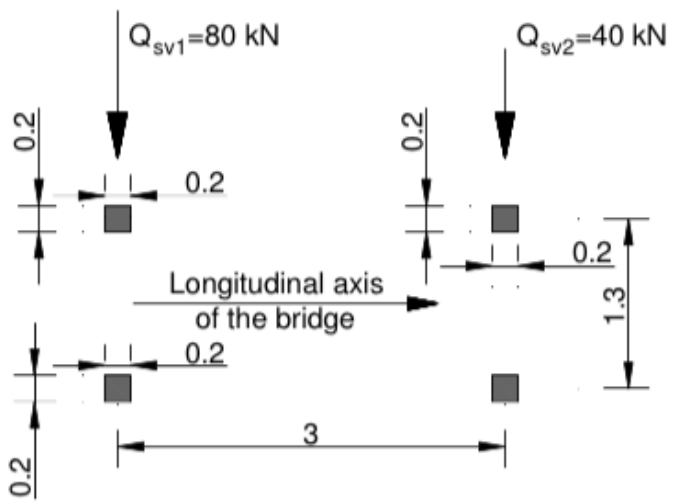
- a concentrated load, representing the effect of a maintenance load;

- a standard vehicle, that shall be used whenever there is the possibility an emergency vehicle could cross the bridge.

The first load model sould be applied to the unfavourable parts of the influence surface both longitudinally and transversally.
The recommended value is a function of the loaded length L:

$$2.5\text{kN/m}^2 \leq q_{fk} = 2 + 120/(L+30) \leq 5.0 \text{ kN/m}^2$$

The second load model only needs to be applied when the service vehicle is not taken into account and thus it has been disregarded.
The third load model shall be utilized when service vehicles for maintenance or emergencies (e.g. ambulance, fire) or an accidental vehicle must be considered. Since no permanent obstacle prevents such a vehicle being driven onto the bridge deck, the following model has been adopted:



In addition a horizontal force Q_{flk} acting along the bridge deck axis at the pavement level should be taken into account, whose characteristic value is equal to the greater of the two following:
- 10% of the toatl load corresponding to the uniformle distributed load q_{fk};
- 60% of the total weight of the service vehicle.
It has to be applied on a square surface of sides 10 cm and it is assumed to act simultaneously with the corresponding distributed vertical load.
Vertical and horizontal loads should then be combined in groups according to the table:

Load type		*Vertical forces*		*Horizontal forces*
Load system		*Uniformly distributed load*	*Service vehicle*	
Groups of loads	*gr1*	F_k	*0*	F_k
	gr2	*0*	F_k	F_k

Each of these groups, which are mutually exclusive, should be considered as defining a characteristic action for combination with non-traffic loads.

5.2.3 Traffic dynamic loads

Dynamic loads are treated extensively in Chapter 6.

5.2.4 Non traffic variable loads

5.2.4.1 Wind load

Wind actions on bridges are specified in EN1991-1-4.
Although these specifications are applicable only to girder bridges spanning up to 200 m with a constant cross section and one or more spans, the rules stated can be extended to other bridge types provided that wind-structure interaction is not relevant.
Nontheless since we are dealing with a cable-supported lightweight footbridge that is significantly curved both in the horizontal and the vertical plane, the wind-structure interaction cannot be disregarded.
To investigate the susceptibility of the structure to aerodynamic excitation reference will be made to the BD 49/01 “Design Manual for Roads and Bridges : Volume 1” document.

Bridges are prone to several forms of aerodynamic excitation which produce motions in isolated vertical bending or torsional modes or, more rarely, in coupled vertical bending-torsional modes. Depending on the nature of the excitation the motions that shall be considered in design are as follows:

. (1) limited amplitudes which could cause unacceptable stresses or fatigue damage,

. (2) divergent amplitudes increasing rapidly to large values, which must be avoided;

. (3) non-oscillatory divergence due to a form of aerodynamic torsional instability which must also be avoided.

The aerodynamic susceptibility parameter, P_b, shall be derived in order to categorise the structure using the equation:

$$P_b = \frac{\rho \cdot b^2}{m} \cdot \frac{16 \cdot V_r^{\,2}}{b \cdot L \cdot f_B^{\,2}} = 1.309$$

where (for numeric values of factors refer to Annex*):
- ρ is the density of air ;
- b is the overall width of the bridge deck;
- m is the mass per unit length of the bridge;
- V_r is the hourly mean wind speed;
- L is the length of the relevant maximum span of the bridge;
- f_B is the natural frequency in bending.

Since the P_b value is greater than 1, the bridge will be very susceptible to aerodynamic excitation according to BD 49-Part 3, 2.1 (c). Thus its stability shall be verified by means of specific studies, or through wind tunnel tests on scale models.
Such types of studies lie outside the goal of the actual thesis and wind actions will be analised through a semplified model.

Wind force acting on the deck

In general, wind is considered blowing in two horizontal directions, *x* and *y*, being *y* the longitudinal axis of the bridge and *x* the transversal axis, originating forces in *x*, *y* and *z* direction. Forces induced by wind blowing in direction *x* can be considered not simultaneous with forces induced by wind blowing in direction *y* and vice versa; on the contrary, wind forces acting in *z* direction should be considered acting simultaneously with the corresponding *x* or *y* force. Wind force in the x (transversal) direction can be calculated using the following expression:

$$F_{W,x} = \frac{1}{2} \cdot \rho \cdot v_b^{\,2} \cdot C \cdot A_{ref,x}$$

where:
- v_b is the basic wind velocity, defined as a function of wind direction and time of year at 10m above ground of terrain category II;
- C is the wind load factor;
- $A_{ref,x}$ is the reference area, i.e. the lateral surface of the deck increased vertically by 0,3 m to take into account the effect of the open parapets that are to be installed.

The basic wind velocity shall be calculated from the following expression:

$$v_b = c_{dir} \cdot c_{season} \cdot v_{b,0}$$

where:

- $v_{b,0}$ is the fundamental value of the basic wind velocity;
- c_{dir} is the directional factor;
- c_{season} is the season factor.

Following the same principle, wind force acting in the vertical direction (z) can be calculated with the same formula:

$$F_{W,z} = \frac{1}{2} \cdot \rho \cdot {v_b}^2 \cdot C \cdot A_{ref,z}$$

where C is now function of the exposure factor and the force factor relative to che z component ($c_{f,z} = \pm 0.9$), while $A_{ref,z}$ is equal to the bridge's width multiplied by its length.

Wind force acting along the longitudinal direction (Y), can be set equal to $0.25F_{W,x}$.

Calculation procedures and numerical results are reported in the relevant Annex.

Wind force acting on other structural elements

Wind action on the masts to which the stay-cables are anchored can be modeled as a distributed load acting along the mast axis, using the following formula:

$$F_{W,mast} = c_s \cdot c_d \cdot c_f \cdot q_p(z_e) \cdot \frac{A_{ref,mast}}{l_{mast}}$$

where:
- c_s is the size factor;
- c_d is the dynamic factor;
- c_f is the force coefficient relative to circular cylinder sections, set equal to the product of $c_{f,0}$ (function of Reynolds number and the solidity ratio φ);
- q_p is the peak velocity pressure of the wind;
- z_e is the reference height for external wind action;
- $A_{ref,mast}$ is the lateral surface of the mast;
- l_{mast} is the mast's length.

The calculation procedure can be found in the relevant Annex.

The same procedure can be utilized for assessing the wind action on the suspension system. Cables-wind interaction shall be treated by means of specific aerodynamic studies. Nonetheless such type of research lies outside the goal of the actual design project as mentioned earlier.

5.2.4.2 Thermal actions

The bridge is subjected to daily and seasonal temperature effects that can't be neglected since changes in temperatures may cause additional deformations and stresses, and may, in some cases, significantly affect ultimate and serviceability limit states of the structure. Reference is made to EN1991-1-5.

Uniform component

The uniform temperature component depends on the maximum and minimum temperature, $T_{e,\max}$ and $T_{e,\min}$, that the bridge can attain during its working life.

Once determined the maximum and minimum shade air temperatures (Fig.**) of the site characterized by 50 years return period, $T_{\max}$ and $T_{\min}$, the uniform temperature components $T_{e,\max}$ and $T_{e,\min}$ can be determined according to the diagram below (Fig.**), where $T_{e,\max}$ and $T_{e,\min}$, in °C, are expressed in terms of $T_{\max}$ and $T_{\min}$, in °C. These values are associated with the bridge Category 1 (steel box girder bridges) and thus $T_{e,\max}$ value can be reduced by 3 °C.

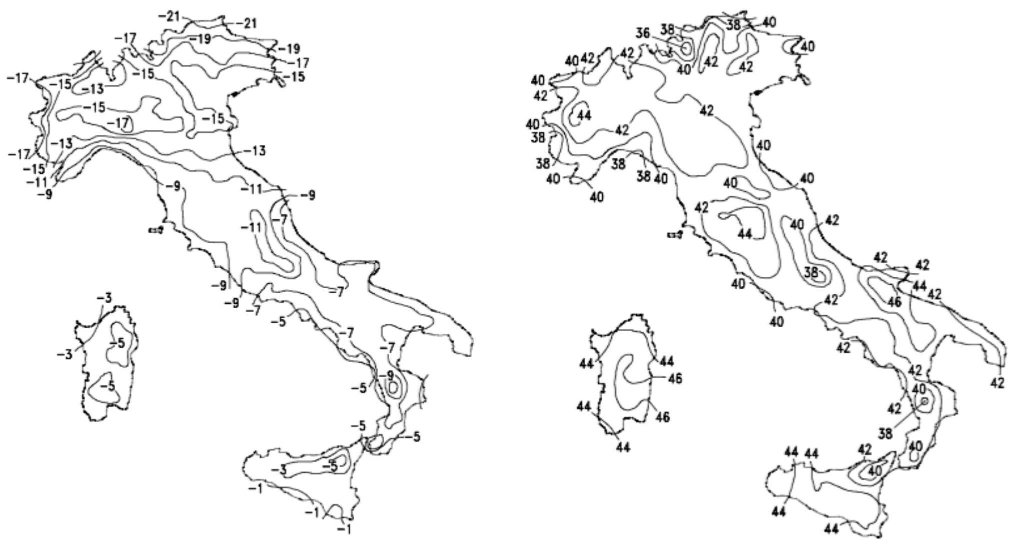


Fig.70

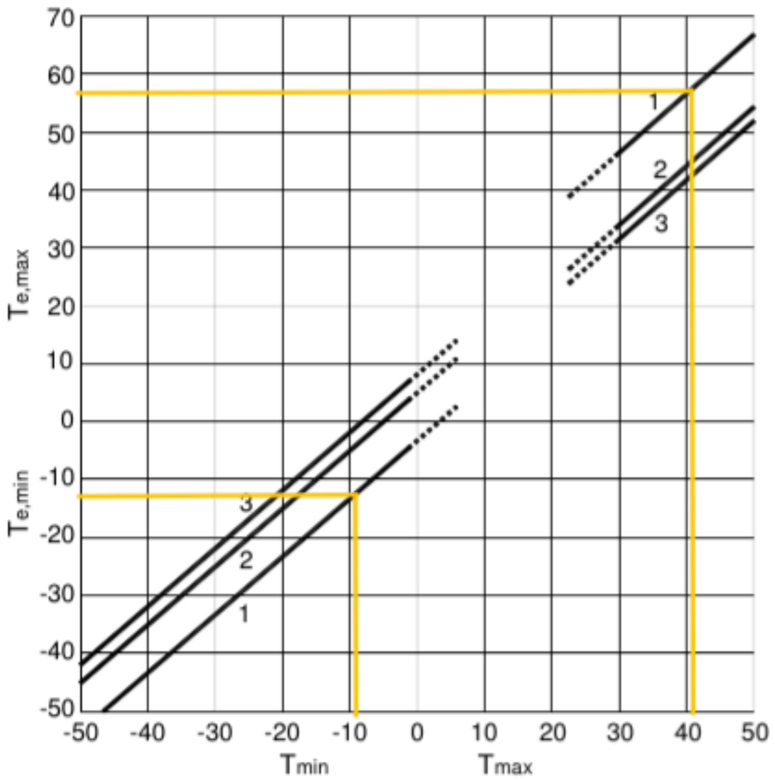


Fig.71

Maximum and minimum shade air temperatures will then be respectively 42°C and -9°C, thus leading to $T_{e,min}$ = -13.5°C and $T_{e,max}$ = 56.5 – 3 = 53.5°C. The initial temperature of the bridge (at the time when it is restrained) T_0 is taken equal to 10°C. The temperature variations to be considered when the bridge expands or contracts, respectively will then be:

$\Delta T_{N,exp} = T_{e,max} - T_0 = 53.5 - 10 = 43.5$ °C $\qquad \Delta T_{N,con} = T_0 - T_{e,min} = 10 - (-13.5) = 23.5$ °C

When assessing bearing displacements, temperature variations shall be increased by 20 °C (i.e. $\Delta T_{N,exp,bear}$ = 43.5 + 20 = 63.5 °C and $\Delta T_{N,con}$ = 23.5 +20 = 43.5 °C).

Linear component

Vertical temperature variations occur when top and bottom of the deck experience different heating or cooling conditions. These variations correspond to maximum heating, when the top surface is warmer than bottom surface, and maximum cooling, when the bottom surface is warmer than the top surface. The vertical temperature profiles can be defined under two different hypothesis, according as non-linear temperature profile ΔT_E is disregarded or not: in the former case a simplified equivalent linear profile can be considered, while in the latter one a non linear profile, including ΔT_E, is taken into account.

Let's consider that the non-linear component is negligible (an accurate study of the the temperature-structure interaction lies outside the goal of the actual thesis). An equivalent linear temperature component is then adopted whose extreme values, corresponding to the upper and lower plate of the deck, are listed in the table below (Type1 - Steel deck bridge, 50mm surfacing):

Type of deck	*Top warmer than bottom $\Delta T_{M,heat}$ [°C]*	*Bottom warmer than top $\Delta T_{M,cool}$ [°C]*
Type 1: Steel deck	18	13
Type 2: Composite deck	15	18
Type 3: Concrete deck		
- concrete box girder	10	5
- concrete beam	15	8
- concrete slab	15	8

Then getting: $\Delta T_{M,heat}$ = 18 °C $\Delta T_{M,cool}$ = 13 °C

5.2.4.3 Snow load

Snow loads on bridges should be determined according the general procedure of Eurocode EN 1991-1-3. Since the bridge is neither a roofed one nor situated in a particular geographic area, snow loads should not be combined with traffic actions.
Snow load can be determined as:

$$s = \mu \cdot C_e \cdot C_t \cdot s_k = 0.92 \frac{kN}{m^2}$$

where:
- s_k is the characteristic ground snow load with the annual probability of exceedence set to 0,02 (i.e. a probability of not being exceeded on the unfavourable side during a "reference period" of 50 years).
- C_e is the exposure factor;
- C_t is the thermal factor;
- μ is the shape factor.
Numerical values can be found in the relevant Annex.

5.2.5 Load combinations

Load combinations have been deduced from UNI EN 1990 (Section 6, Annexes A1 and A2). Permanent loads G and pretension of cables P has been considered as a whole permanent action and, thus, multiplied by γ_{G1}, as stated in UNI EN 1993-1-11 (Par. 5.3).

5.2.5.1 ULS load combinations

ULS load combination is the fundamental combination and has the following expression:

$$\gamma_{G1} \cdot (G_1+P) + \gamma_{G2} \cdot G_2 + \gamma_{Q1} \cdot Q_{k1} + \gamma_{Q2} \cdot \psi_{02} \cdot Q_{k2} + \gamma_{Q3} \cdot \psi_{03} \cdot Q_{k3} + \ldots$$

where:

γ_{G1} is the partial safety factor for permanent loads;

γ_{G2} is the partial safety factor for permanent loads from non-structural elements;

γ_{Q1} is the partial safety factor for variable actions;

G_1 defines the permanent and prestress loads;

G_2 defines the permanent loads of non-structural elements;

Q_{ki} defines the variable loads.

5.2.5.2 SLS load combinations

The following limit states were used:

- SLS characteristic (rare) combination:

$$G_1 + G_2 + P + Q_{k1} + \psi_{02} \cdot Q_{k2} + \psi_{03} \cdot Q_{k3} + \ldots$$

- SLS frequent combination:

$$G_1 + G_2 + P + \psi_{11} \cdot Q_{k1} + \psi_{22} \cdot Q_{k2} + \psi_{23} \cdot Q_{k3} + \ldots$$

- SLS quasi-permanent combination:

$$G_1 + G_2 + P + \psi_{21} \cdot Q_{k1} + \psi_{22} \cdot Q_{k2} + \psi_{23} \cdot Q_{k3} + \ldots$$

5.2.5.3 Seismic combination

$$E + G_1 + G_2 + P + \psi_{21} \cdot Q_{k1} + \psi_{22} \cdot Q_{k2} + \psi_{23} \cdot Q_{k3} + \ldots$$

where E is the seismic action.

Combination coefficients and partial safety factors values are listed in the tables below.

Table A2.2 – Recommended values of ψ factors for footbridges

Action	Symbol	ψ_0	ψ_1	ψ_2
Traffic loads	gr1 Q_{fwk} gr2	0,40 0 0	0,40 0 0	0 0 0
Wind forces	F_W	0,3	0,2	0
Thermal actions	T	0,6[(1)]	0,6	0,5
Snow loads	Sn (during execution)	0,8	-	0
Construction loads	Q_c - Working personal, staff and visitors with small equipment (Q_{ca}) - Storage of construction material, precast elements, etc. (Q_{cb}) - Heavy equipment etc. (Q_{cc}) - Cranes, lifts, vehicles etc. (Q_{cd})	 1,0 1,0 1,0 1,0	 - - - -	 1,0 1,0 1,0 -
1) The recommended ψ_0 value for thermal actions may in most cases be reduced to 0 for ultimate limit states EQU, STR and GEO. See however the design Eurocodes				

NOTE 4 For footbridges, the infrequent value of variable actions is not relevant.

Table A2.4(B) - Design values of actions (STR/GEO) (Set B)

Persistent and transient design situation	Permanent actions		Leading variable action (*)	Accompanying Variable actions (*)	
	Unfavourable	Favourable		Main (if any)	Others
(Eq. 6.10)	$\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$	$\gamma_{Q,1}Q_{k,1}$		$\gamma_{Q,i}\psi_{0,i}Q_{k,i}$

Persistent and transient design situation	Permanent actions		Leading variable action (*)	Accompanying variable actions (*)	
	Unfavourable	Favourable		Main	Others
(Eq. 6.10a)	$\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$		$\gamma_{Q,1}\psi_{0,1}Q_{k,1}$	$\gamma_{Q,i}\psi_{0,i}Q_{k,i}$
(Eq. 6.10b)	$\xi\gamma_{Gj,sup}G_{kj,sup}$	$\gamma_{Gj,inf}G_{kj,inf}$	$\gamma_{Q,1}Q_{k,1}$		$\gamma_{Q,i}\psi_{0,i}Q_{k,i}$

(*) Variable actions are those considered in Tables A2.1 to A2.3.

NOTE 1 The choice between 6.10, or 6.10a and 6.10b will be in the National Annex.

NOTE 2 The γ and ξ values may be set by the National Annex. The following values for γ and ξ are recommended when using expressions 6.10, or 6.10a and 6.10b:
$\gamma_{Gj,sup} = 1,35$[1)]
$\gamma_{Gj,inf} = 1,00$
$\gamma_{Q,1} = 1,35$ when Q_1 represents unfavourable actions due to road or pedestrian traffic (0 when favourable)
$\gamma_{Q,1} = 1,45$ when Q_1 represents unfavourable actions due to rail traffic, to groups of loads 11 to 31 (except 16, 17, 26[3)] and 27[3)]), LM71 and SW/0, and real trains, when considered as individual leading traffic actions (0 when favourable)
$\gamma_{Q,1} = 1,20$ when Q_1 represents unfavourable actions due to rail traffic, to groups of loads 16 and 17 and SW/2 (0 when favourable)
$\gamma_{Q,i} = 1,50$ for other traffic actions and other variable actions [2)]
$\xi = 0,85$ (so that $\xi\gamma_{Gj,sup} = 0,85 \times 1,35 \cong 1,15$).
$\gamma_{Gset} = 1,20$ in case on linear elastic analysis, and 1,35 in case of non linear analysis, where actions due to uneven settlements are unfavourable ; 1,00 where they are favourable.
See also EN 1991 to EN 1999 for γ values to be used for imposed deformations.

[1)]This value covers : self-weight of structural and non structural elements, ballast, soil, ground water and free water, removable loads, etc.
[2)]This value covers : variable horizontal earth pressure from soil, ground water, free water and ballast, traffic load surcharge earth pressure, traffic aerodynamic actions, wind and thermal actions, etc.
[3)]For rail traffic actions for group of loads 26 and 27 $\gamma_{Q,1} = 1,20$ may be applied to individual components of traffic actions associated with SW/2 and $\gamma_{Q,1} = 1,45$ may be applied to individual components of traffic actions associated with LM71 and SW/0 etc.

NOTE 3 The characteristic values of all permanent actions from one source are multiplied by $\gamma_{G,sup}$ if the total resulting action effect is unfavourable and $\gamma_{G,inf}$ if the total resulting action effect is favourable. For example, all actions originating from the self weight of the structure may be considered as coming from one source ; this also applies if different materials are involved. See however A2.3.1(2).

NOTE 4 For particular verifications, the values for γ_G and γ_Q may be subdivided into γ_g and γ_q and the model uncertainty factor γ_{Sd}. A value of γ_{Sd} in the range 1,0 - 1,15 may be used in most common cases and may be modified in the National Annex.

NOTE 5 Where actions due to water are not covered by EN 1997 (*e.g.* flowing water), the combinations of actions to be used should be specified for the particular project.

Coefficients for ULS combination

Combination	Self-weight +prestress	Non-structural permanent	Traffic	Wind	Thermal	Snow
ULS 1.1.1 - 1.1.46, 1.2.1 - 1.2.46	1.35	1.35	1.35		0.9	
ULS 2.1 - 2.9	1.35	1.35		1.5		1.2
ULS 3.1.1 - 3.1.9	1.35	1.35		0.45		1.5
ULS 3.2.1	1.35	1.35			1.2	1.5
ULS 4.1.1	1.35	1.35			1.5	1.2
ULS 4.2.1.1.1-4.2.1.23.1, 4.2.1.1.2-4.2.1.23.1	1.35	1.35	0.54		1.5	
ULS 5.1	1.35	1.35	1.35			
ULS 5.2	1.35	1.35		1.5		
ULS 5.3	1.35	1.35			1.5	
ULS 5.4	1.35	1.35				1.5

Tab.14

As a rule, traffic loads on footbridges are considered not to act simultaneously with significant wind or snow. Wind and thermal actions should not be taken into account as simultaneous.

Coefficients for SLS characteristic combination

Combination	Self-weight +prestress	Non-structural permanent	Traffic	Wind	Thermal	Snow
SLS 1.1.1-1.1.46, 1.2.1-1.2.46	1	1	1		0.6	

SLS 2.1-2.9	1	1		1		0.8
SLS 3.1.1-3.1.9	1	1		0.3		1
SLS 3.2.1	1	1			0.6	1
SLS 4.1.1	1	1			1	0.8
SLS 4.2.1.1.1-4.2.1.23.1, 4.2.1.1.2-4.2.1.23.1	1	1	0.4		1	

Tab.15

Coefficients for SLS frequent combination

Combination	Self-weight +prestress	Non-structural permanent	Traffic	Wind	Thermal	Snow
SLS 1.1.1-1.1.46, 1.2.1-1.2.46	1	1	0.4		0.5	
SLS 2.1-2.9	1	1		0.2		
SLS 3.1.1-3.1.9						
SLS 3.2.1	1	1			0.5	
SLS 4.1.1	1	1			0.6	
SLS 4.2.1.1.1-4.2.1.23.1, 4.2.1.1.2-4.2.1.23.1	1	1	0.4		0.6	

Tab.16

Coefficients for SLS quasi-permanent combination

Combination	Self-weight +prestress	Non-structural permanent	Traffic	Wind	Thermal	Snow
SLS 1.1.1-1.1.46, 1.2.1-1.2.46	1	1			0.5	
SLS 2.1-2.9						
SLS 3.1.1-3.1.9						
SLS 3.2.1	1	1			0.5	
SLS 4.1.1	1	1			0.5	
SLS 4.2.1.1.1-4.2.1.23.1, 4.2.1.1.2-4.2.1.23.1	1	1			0.5	

Tab.17

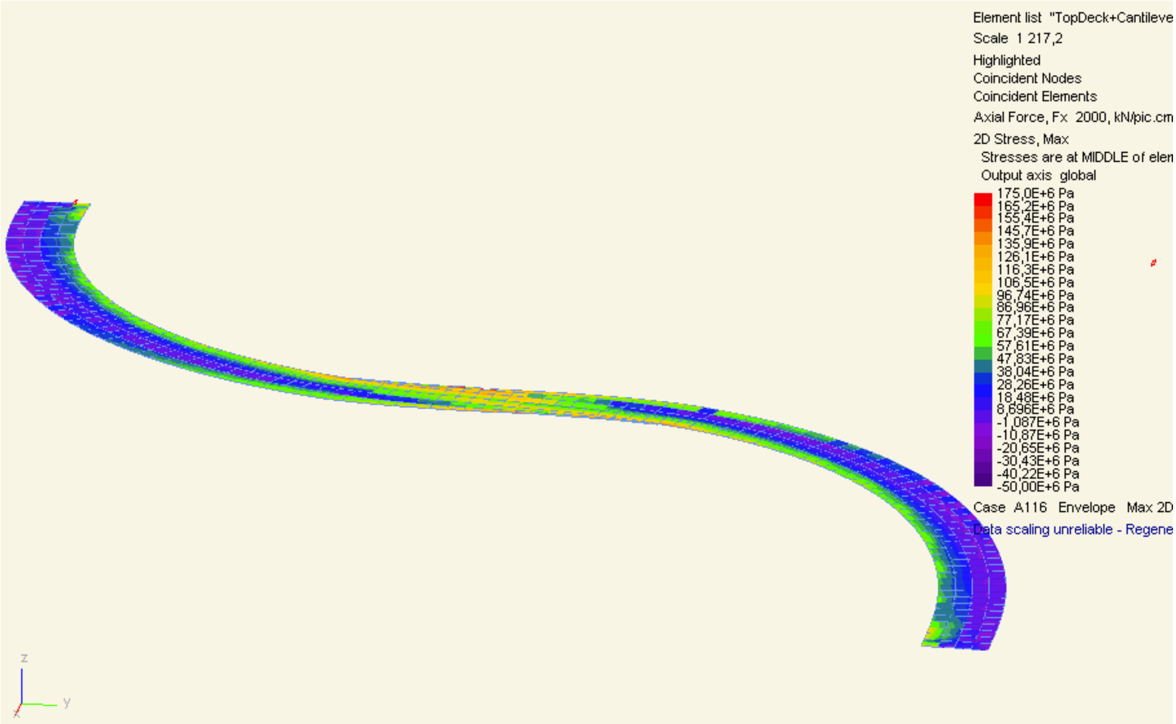
5.3 Stresses and displacements

5.3.1 Ultimate Limit State (ULS)

In order to verify the structural members resistance, the fundamental and seismic load combinations were applied to the FE model and the most significant outputs of the analysis were reported below.

The following figures and tables' contents are listed (all stresses were obtained from a ULS combinations envelope):

- Fig.72: Maximum axial stress [Pa] in the top-deck panels;
- Fig.73 : Minimum axial stress [Pa] in the top-deck panels;
- Fig.74 : Maximum Von Mises stress [Pa] in the top-deck panels;
- Fig.75 : Maximum axial stress [Pa] in the lateral-deck panels;
- Fig.76 : Minimum axial stress [Pa] in the lateral-deck panels;
- Fig.77 : Maximum Von Mises stress [Pa] in the lateral-deck panels;
- Fig.78 : Maximum axial stress [Pa] in the bottom-deck panels;
- Fig.79 : Minimum axial stress [Pa] in the bottom-deck panels;
- Fig.80 : Maximum Von Mises stress [Pa] in the bottom-deck panels;
- Fig.81 : Maximum compression in the masts [kN-Pa];
- Fig.82 : Maximum tension in the cables [kN];
- Tab.18 : Maximum axial stress, minimum axial stress, maximum shear stress, maximum Von Mises stress and average stress in the deck cross section panels [Pa];
- Tab.19 : Maximum tension in the cables [kN];
- Tab.20 : Maximum compression in the masts [kN];
- Tab.21 : Maximum diaphragms stresses [kN];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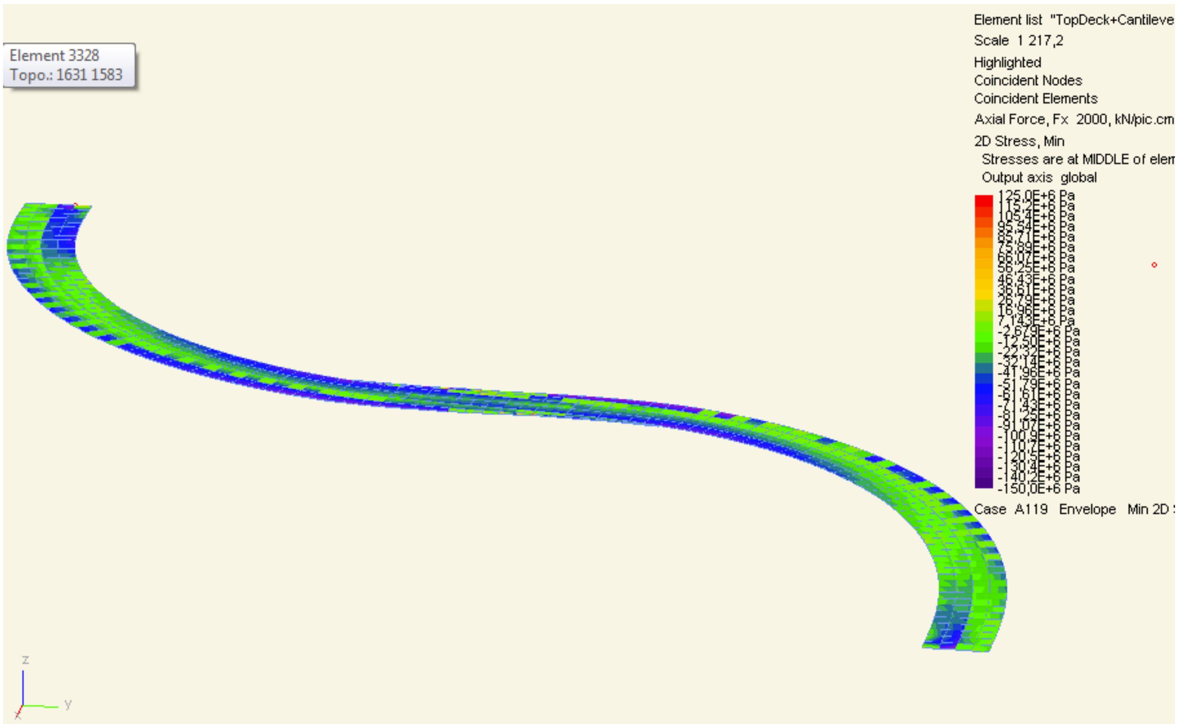
z
y

Fig.72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2D Stress, Min
Stresses are at MIDDLE of elem
Output axis global
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115,2E+6 Pa
105,4E+6 Pa
95,54E+6 Pa
85,71E+6 Pa
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-110,7E+6 Pa
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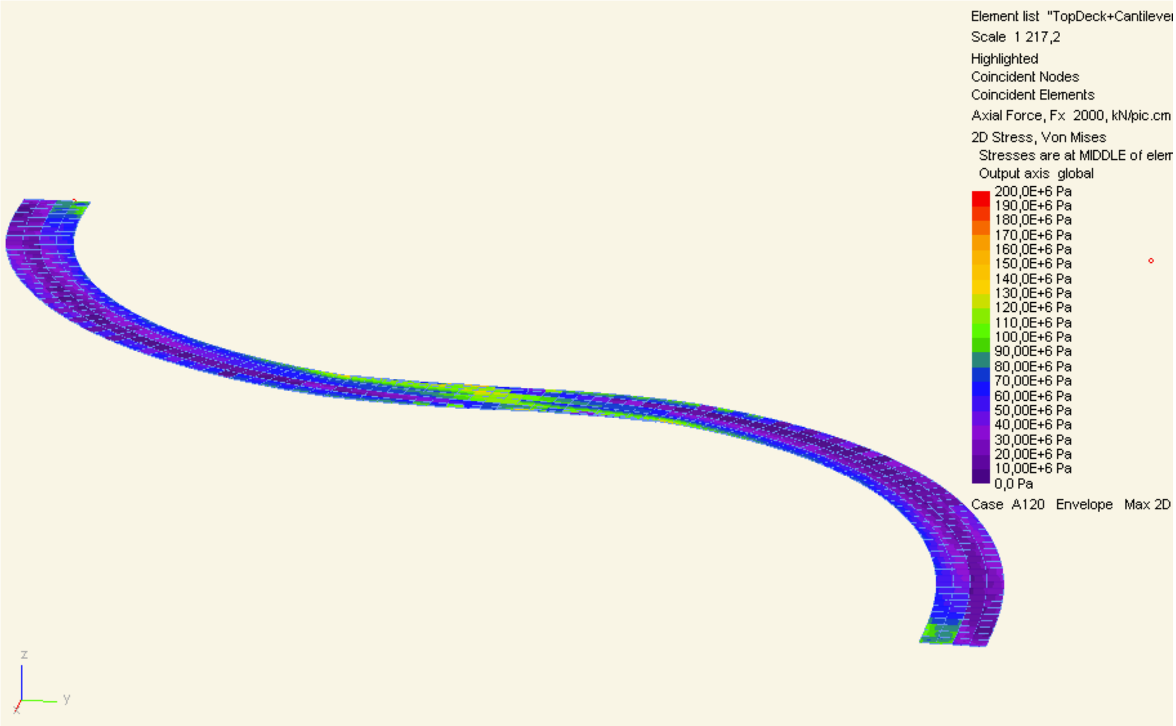
-140,2E+6 Pa
-150,0E+6 Pa
Case A119 Envelope Min 2D
z
y

Fig.73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Highlighted
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Von Mises
Stresses are at MIDDLE of elem
Output axis global
200,0E+6 Pa
190,0E+6 Pa
180,0E+6 Pa
170,0E+6 Pa
160,0E+6 Pa
150,0E+6 Pa
140,0E+6 Pa
130,0E+6 Pa
120,0E+6 Pa
110,0E+6 Pa
100,0E+6 Pa
90,00E+6 Pa
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70,00E+6 Pa
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50,00E+6 Pa
40,00E+6 Pa
30,00E+6 Pa
20,00E+6 Pa
10,00E+6 Pa
0,0 Pa
Case A120 Envelope Max 2D
z
y

Fig.74


Element list "Lateral Deck"
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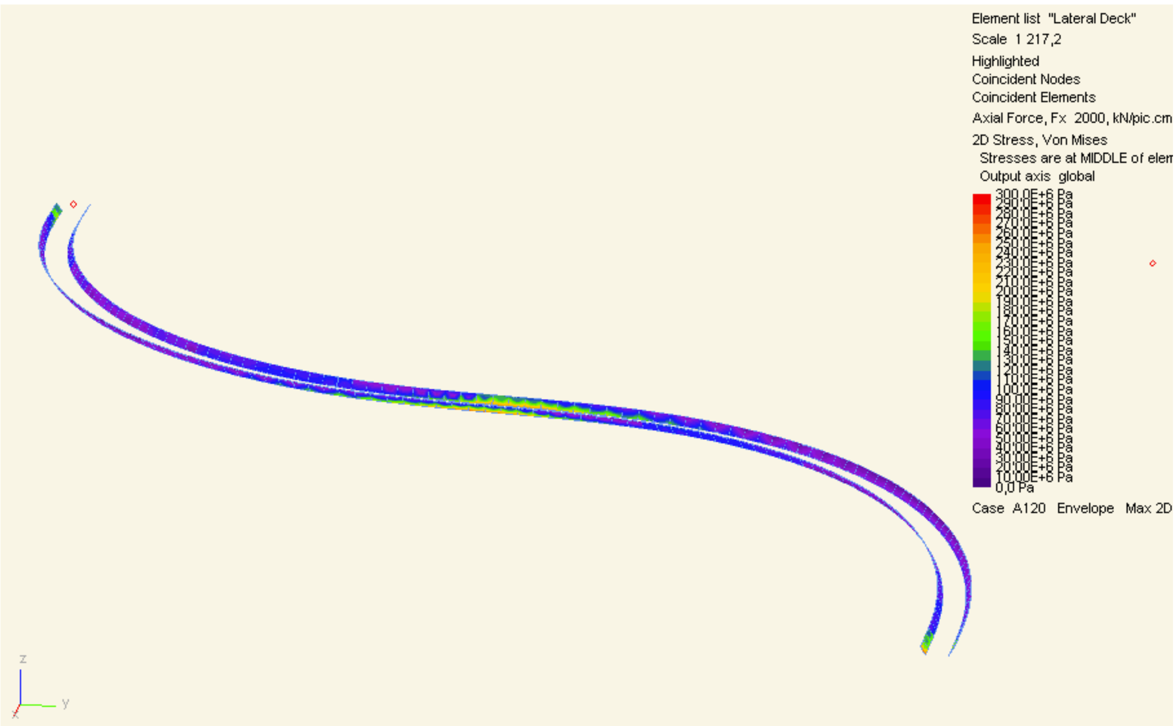
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Max
Stresses are at MIDDLE of elem
Output axis global
200,0E+6 Pa
190,2E+6 Pa
180,4E+6 Pa
170,5E+6 Pa
160,7E+6 Pa
150,9E+6 Pa
141,1E+6 Pa
131,3E+6 Pa
121,4E+6 Pa
111,6E+6 Pa
101,8E+6 Pa
91,96E+6 Pa
82,14E+6 Pa
72,32E+6 Pa
62,50E+6 Pa
52,68E+6 Pa
42,86E+6 Pa
33,04E+6 Pa
23,21E+6 Pa
13,39E+6 Pa
3,571E+6 Pa
-6,250E+6 Pa
-16,07E+6 Pa
-25,89E+6 Pa
-35,71E+6 Pa
-45,54E+6 Pa
-55,36E+6 Pa
-65,18E+6 Pa
-75,00E+6 Pa
Case A116 Envelope Max 2D
z
y

Fig.75


Element list "Lateral Deck"
Scale 1 217,2
Highlighted
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Min
Stresses are at MIDDLE of elem
Output axis global
100,0E+6 Pa
80,00E+6 Pa
60,00E+6 Pa
40,00E+6 Pa
20,00E+6 Pa
0,0 Pa
-20,00E+6 Pa
-40,00E+6 Pa
-60,00E+6 Pa
-80,00E+6 Pa
-100,0E+6 Pa
-120,0E+6 Pa
-140,0E+6 Pa
-160,0E+6 Pa
-180,0E+6 Pa
-200,0E+6 Pa
-220,0E+6 Pa
-240,0E+6 Pa
-260,0E+6 Pa
-280,0E+6 Pa
-300,0E+6 Pa
Case A119 Envelope Min 2D
z
y

Fig.76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Highlighted
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Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Von Mises
Stresses are at MIDDLE of elem
Output axis global
300,0E+6 Pa
290,0E+6 Pa
280,0E+6 Pa
270,0E+6 Pa
260,0E+6 Pa
250,0E+6 Pa
240,0E+6 Pa
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140,0E+6 Pa
130,0E+6 Pa
120,0E+6 Pa
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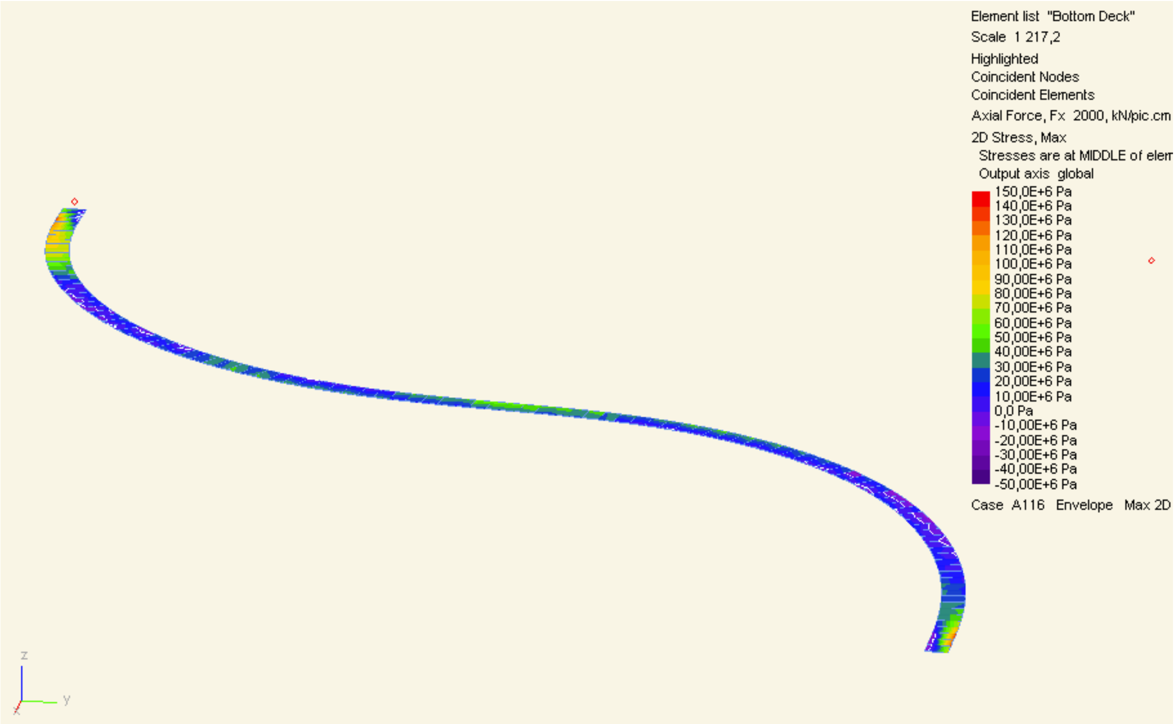
80,00E+6 Pa
70,00E+6 Pa
60,00E+6 Pa
50,00E+6 Pa
40,00E+6 Pa
30,00E+6 Pa
20,00E+6 Pa
10,00E+6 Pa
0,0 Pa
Case A120 Envelope Max 2D
z
y

Fig.77



Element list "Bottom Deck"
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Highlighted
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Max
Stresses are at MIDDLE of elem
Output axis global
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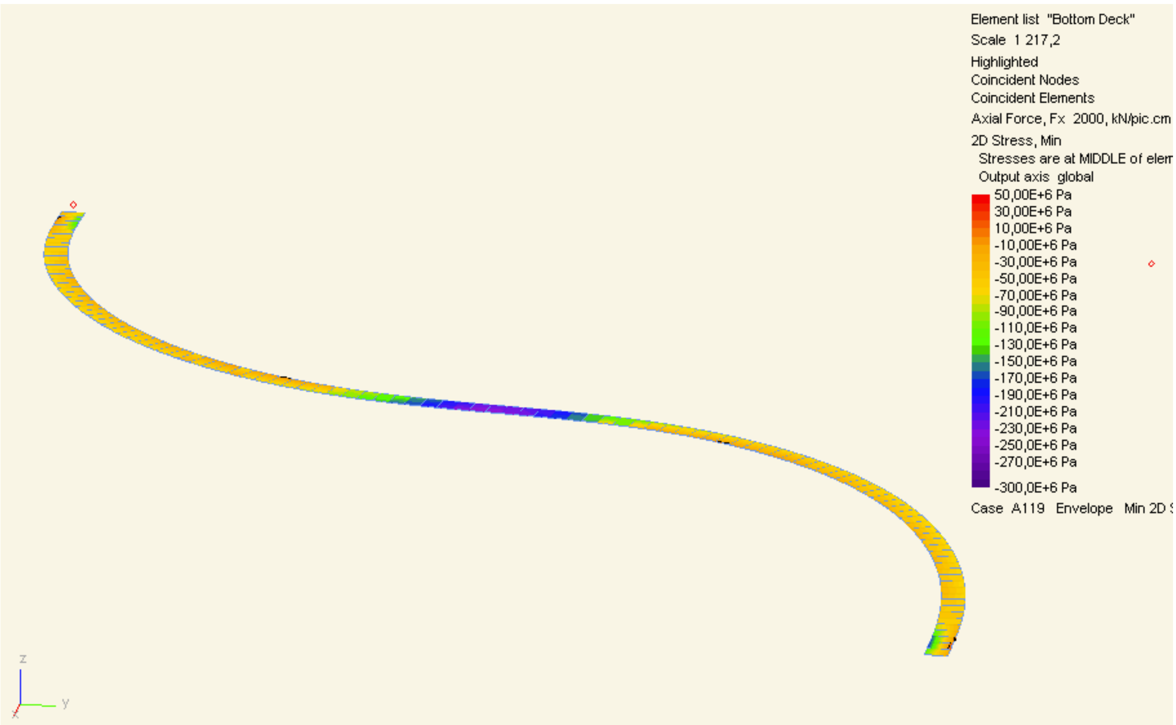
140,0E+6 Pa
130,0E+6 Pa
120,0E+6 Pa
110,0E+6 Pa
100,0E+6 Pa
90,00E+6 Pa
80,00E+6 Pa
70,00E+6 Pa
60,00E+6 Pa
50,00E+6 Pa
40,00E+6 Pa
30,00E+6 Pa
20,00E+6 Pa
10,00E+6 Pa
0,0 Pa
-10,00E+6 Pa
-20,00E+6 Pa
-30,00E+6 Pa
-40,00E+6 Pa
-50,00E+6 Pa
Case A116 Envelope Max 2D
z
y

Fig.78



Element list "Bottom Deck"
Scale 1 217,2
Highlighted
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Min
Stresses are at MIDDLE of elem
Output axis global
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30,00E+6 Pa
10,00E+6 Pa
-10,00E+6 Pa
-30,00E+6 Pa
-50,00E+6 Pa
-70,00E+6 Pa
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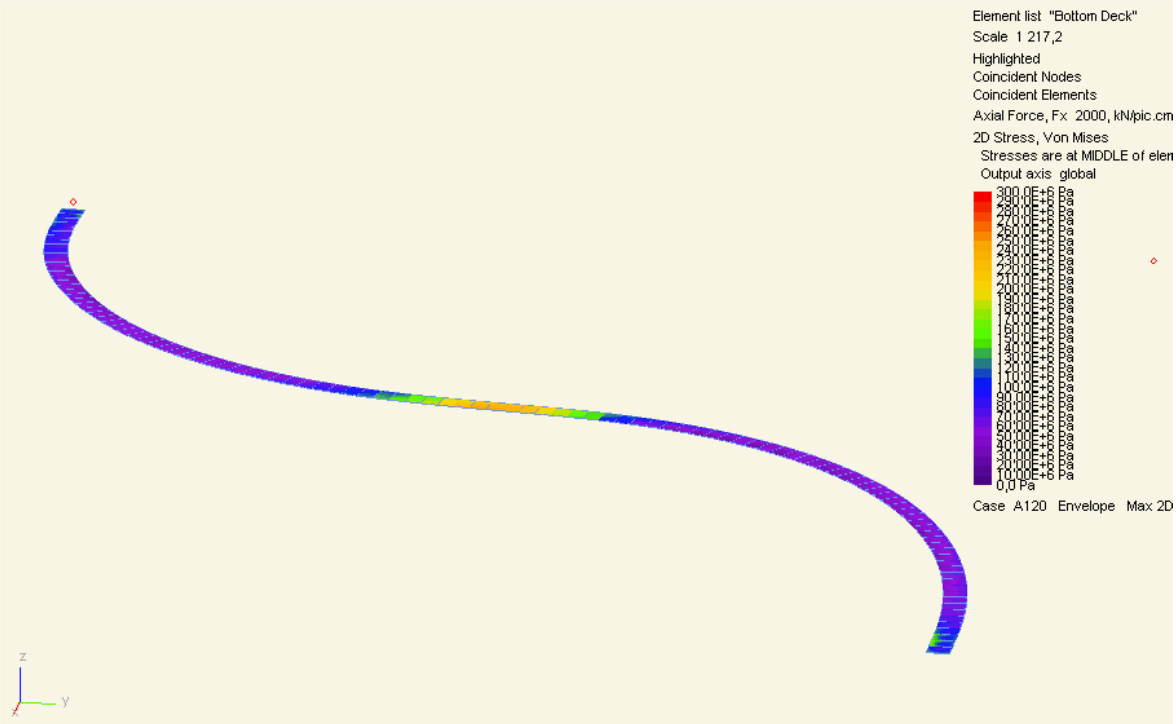
-190,0E+6 Pa
-210,0E+6 Pa
-230,0E+6 Pa
-250,0E+6 Pa
-270,0E+6 Pa
-300,0E+6 Pa
Case A119 Envelope Min 2D
z
y

Fig.79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Highlighted
Coincident Nodes
Coincident Elements
Axial Force, Fx 2000, kN/pic.cm
2D Stress, Von Mises
Stresses are at MIDDLE of elem
Output axis global
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290,0E+6 Pa
280,0E+6 Pa
270,0E+6 Pa
260,0E+6 Pa
250,0E+6 Pa
240,0E+6 Pa
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70,00E+6 Pa
60,00E+6 Pa
50,00E+6 Pa
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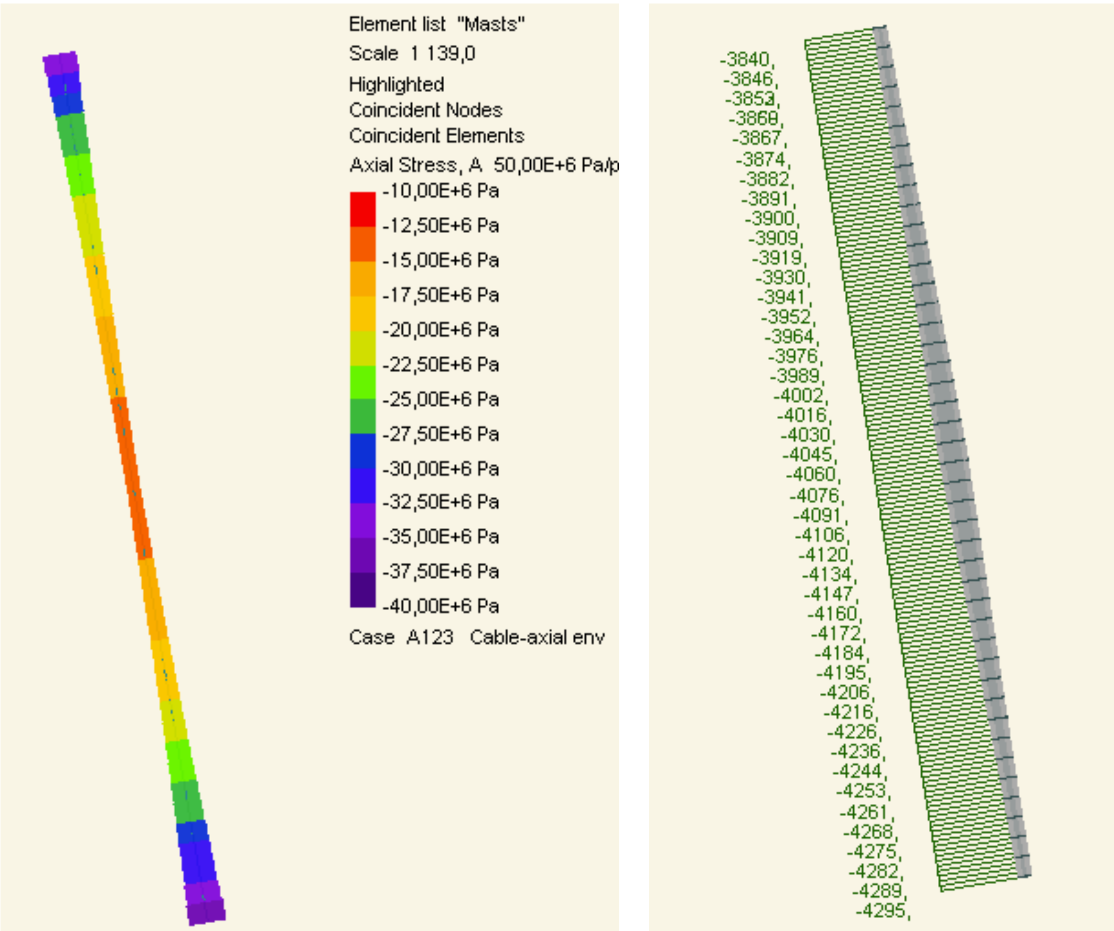
20,00E+6 Pa
10,00E+6 Pa
0,0 Pa
Case A120 Envelope Max 2D
z
y

Fig.80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Highlighted
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Coincident Elements
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-30,00E+6 Pa
-32,50E+6 Pa
-35,00E+6 Pa
-37,50E+6 Pa
-40,00E+6 Pa
Case A123 Cable-axial env
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-3900,
-3909,
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-3930,
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-4289,
-4295,

Fig.81

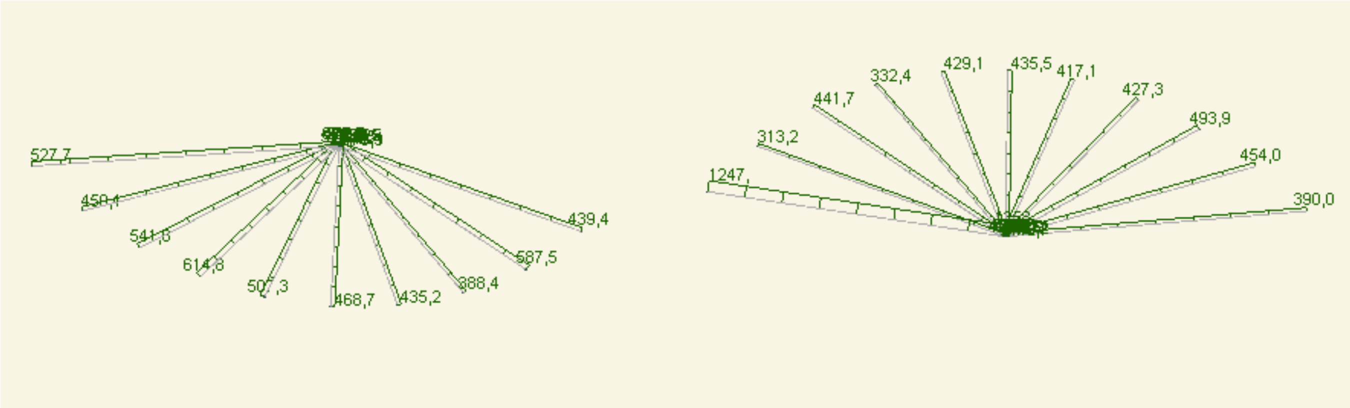


Fig.82

Elem	Case	Node	Max [Pa]	Min [Pa]	Angle to Max [°]	Max Shear [Pa]	Von Mises [Pa]	Average [Pa]
Maxima								
3022	A247	1571	*241,7E+6*	59,08E+6	83,09	91,31E+6	219,7E+6	100,3E+6
3026	A118	1573	163,8E+6	*109,9E+6* P9	81,37	26,93E+6	150,7E+6	91,23E+6
3198	A236	962	127,8E+6	2,127E+6	*90,00*	62,83E+6	126,9E+6	43,31E+6
1451	A119	1031	6,918E+6	-247,2E+6 P14	4,799	*127,1E+6*	250,8E+6	-80,10E+6
1423	A117	1031	-29,66E+6 P14	-282,9E+6	15,15	126,6E+6	*270,1E+6*	-104,2E+6
3022	A247	1571	241,7E+6	59,08E+6	83,09	91,31E+6	219,7E+6	*100,3E+6*
Minima								
3158	A117	1619	*-103,3E+6* P9	-115,2E+6	7,202	5,994E+6	110,3E+6	-72,84E+6
1450	A117	1040	-34,74E+6 P14	*-284,3E+6*	17,25	124,8E+6	269,1E+6	-106,3E+6
3014	A116	1569	133,4E+6 P9	-10,89E+6	*-89,99*	72,14E+6	139,4E+6	40,83E+6
3301	A327	1595	2,043E+6	2,018E+6	-4,048	*12120,*	19,26E+6	1,354E+6
1105	A475	912	235100,	50130,	-51,34	92490,	*484300,*	95080,
1423	A117	1022	-48,03E+6 P14	-273,4E+6	15,95	112,7E+6	253,8E+6	*-107,1E+6*

Tab.18 – deck panels

Elem	Case	Pos	Fx [kN]
Maxima			
3329	A122	1631	*1258,* P21
3318	A291	1630	185,7
3329	A285	1573	319,4
3309	A122	1630	450,5 P19
3309	A122	1630	450,5 P19
3309	A122	1630	450,5 P19
Minima			
3311	A123	1630	*-240,5E-9* P13
3318	A291	584	173,5
3329	A285	1631	324,0
3309	A122	1630	450,5 P19
3309	A122	1630	450,5 P19
3309	A122	1630	450,5 P19

Tab.19 – stay-cables

Elem	Case	Pos	Fx [kN]
Minima			
3400	A123	1635	*-4349,*
3401	A286	1630	-2076,
3358	A122	1631	-1703,
3381	A257	1661	-2618,
3379	A122	50,0%	-1918,
3379	A286	50,0%	-1971,

Tab.20 - mast

Elem	Case	Node	Max [Pa]	Min [Pa]	Angle to Max [°]	Max Shear [Pa]	Von Mises [Pa]	Average [Pa]
Maxima								
1433	A122	1031	*310,2* P14	0,01785	0,2835	0,07578	1,842	0,02119
2917	A247	1526	-4,158	*12,09*	11,04	0,4537	14,61	-3,322
3515	A248	1547	-138,2	-3,868	*138,8*	-0,3047	112,9	2,847
2971	A248	1543	141,7	9,375	66,76	*1,573*	45,97	4,331
3515	A248	1547	-138,2	-3,868	138,8	-0,3047	*112,9*	2,847
3444	A227	1	-191,3	-3,471	-134,2	-0,3167	-159,0	*4,444*
Minima								
3444	A123	3	*-191,6* P7	-3,402	-133,2	-0,3129	108,7	-2,447
1432	A259	1030	56,44	*-9,062*	-0,5446	0,1101	0,6322	-2,817
3444	A227	3	-191,3	-3,471	*-135,1*	-0,3167	110,3	-2,497
4	A227	4	120,1	1,932	4,388	*-0,8551*	-17,51	2,397
3515	A248	1543	-138,2	-3,868	137,9	-0,3047	*-163,8*	-4,889
3515	A248	1543	-138,2	-3,868	137,9	-0,3047	-163,8	*-4,889*

Tab.21 - diaphragms

5.3.2 Serviceability Limit State (SLS)

In order to verify the serviceability response of the structure, the characteristic, frequent and quasi-permanent load combinations were applied to the FE model and the most significant outputs of the analysis were reported below.

Since there is no code-based information such as limit displacements for footbridges and given that all the limit-deflection considerations related to other types of bridges are not relevant, choosing a comfort criterion to verify the compliance with the SLS seems to be a natural consequence.

Besides non-linear static analysis, p-delta and incremental load analyses were conducted in order to understand the structure's deformation behaviour under increasing loads and to assess the level of amplification of the service loads that produces unacceptable deck displacements. This was deemed necessary since the structure is statically unstable even though it is in equilibrium with each load case analysed. The results will be given in terms of load factors.

The following figures and tables' contents are listed (all stresses were obtained from a SLS combinations envelope):

- Fig.83 : Vertical displacements due to live loads distributed along the whole length [m];
- Fig.84 : Maximum vertical displacement [m];
- Fig.85 : Maximum vertical lifting [m];
- Fig.86 : Maximum horizontal displacement [m];
- Tab.22 : Numeric values;

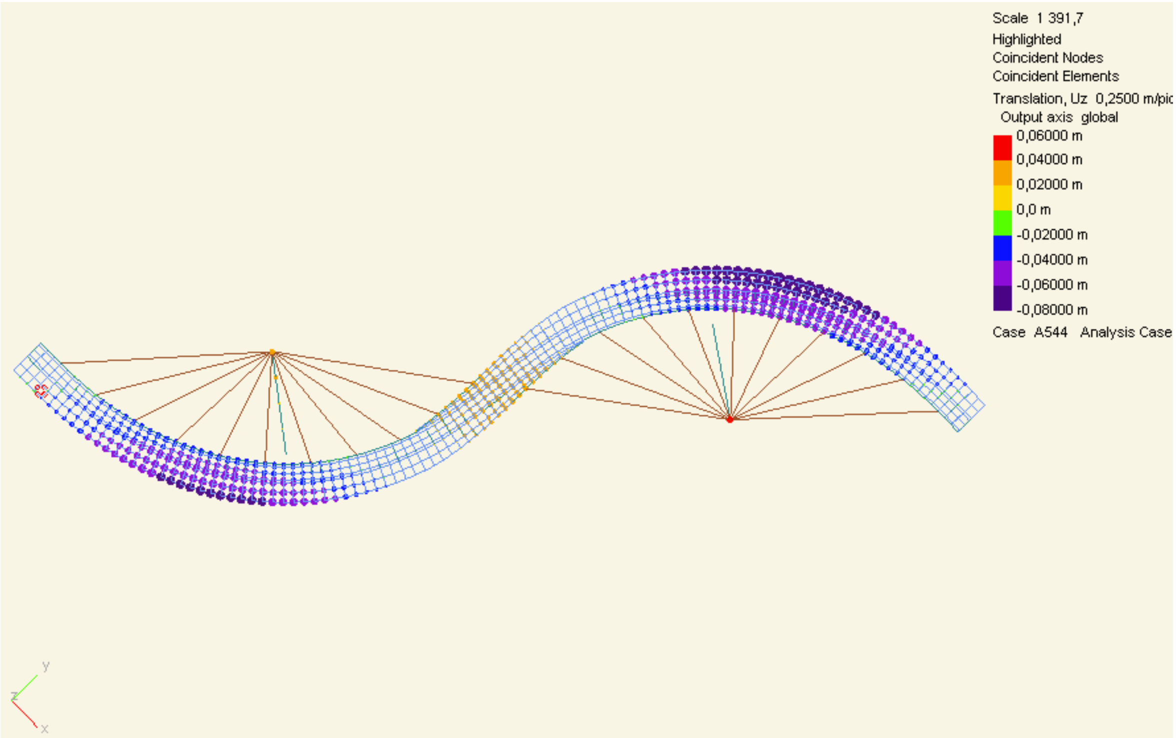


Fig.83

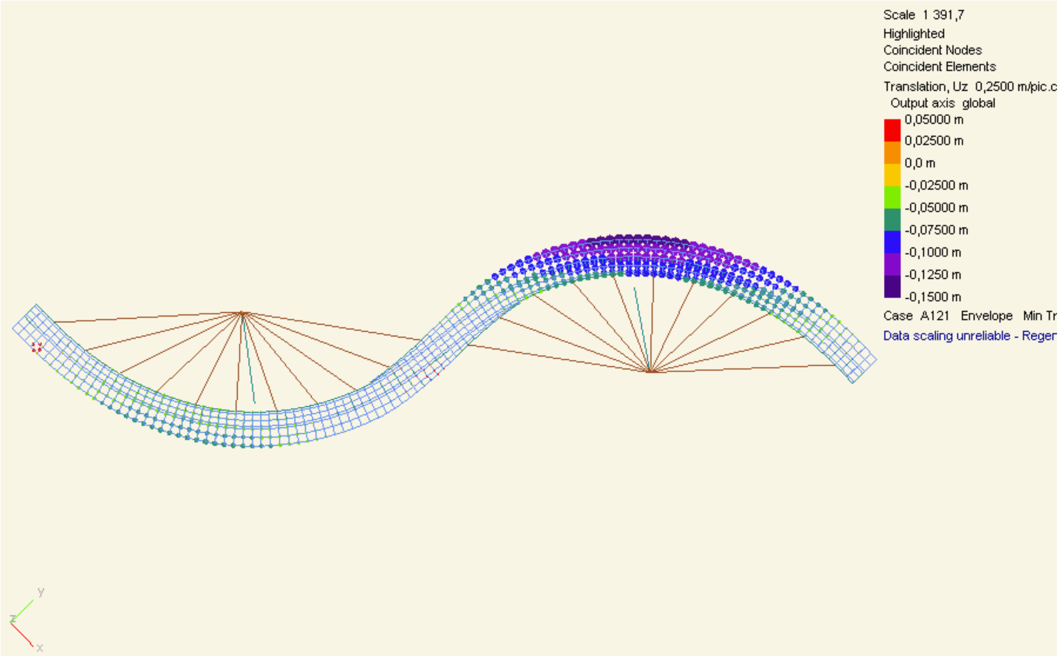


Fig.84 - Note: envelope nodal displacements are not centrally symmetric since asymmetric load distributions have been used to maximize responses according to the influence lines analysis

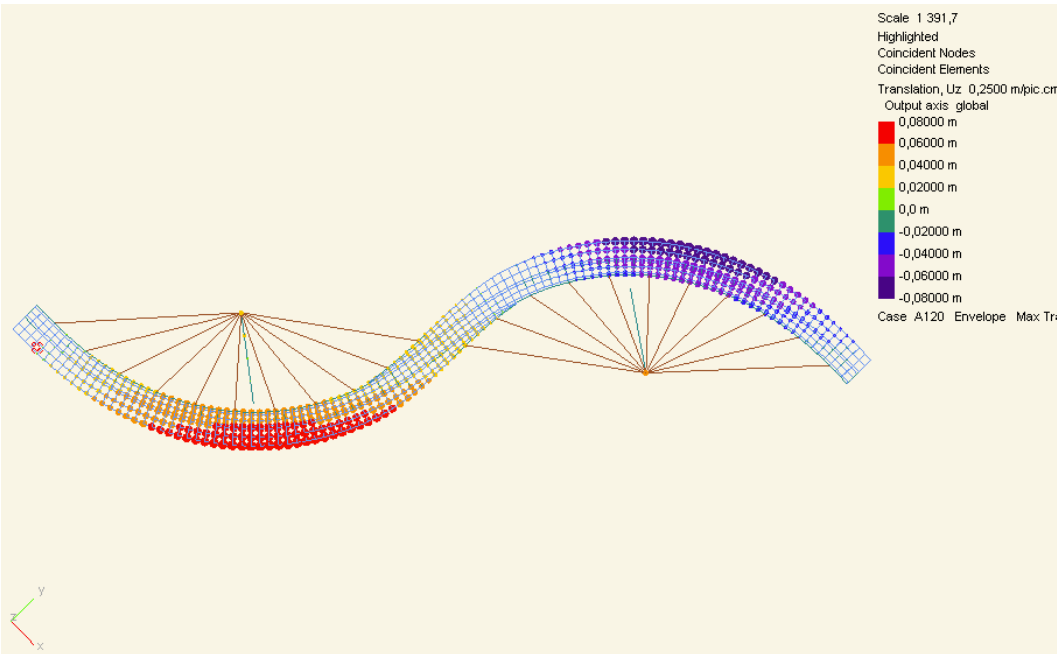


Fig.85

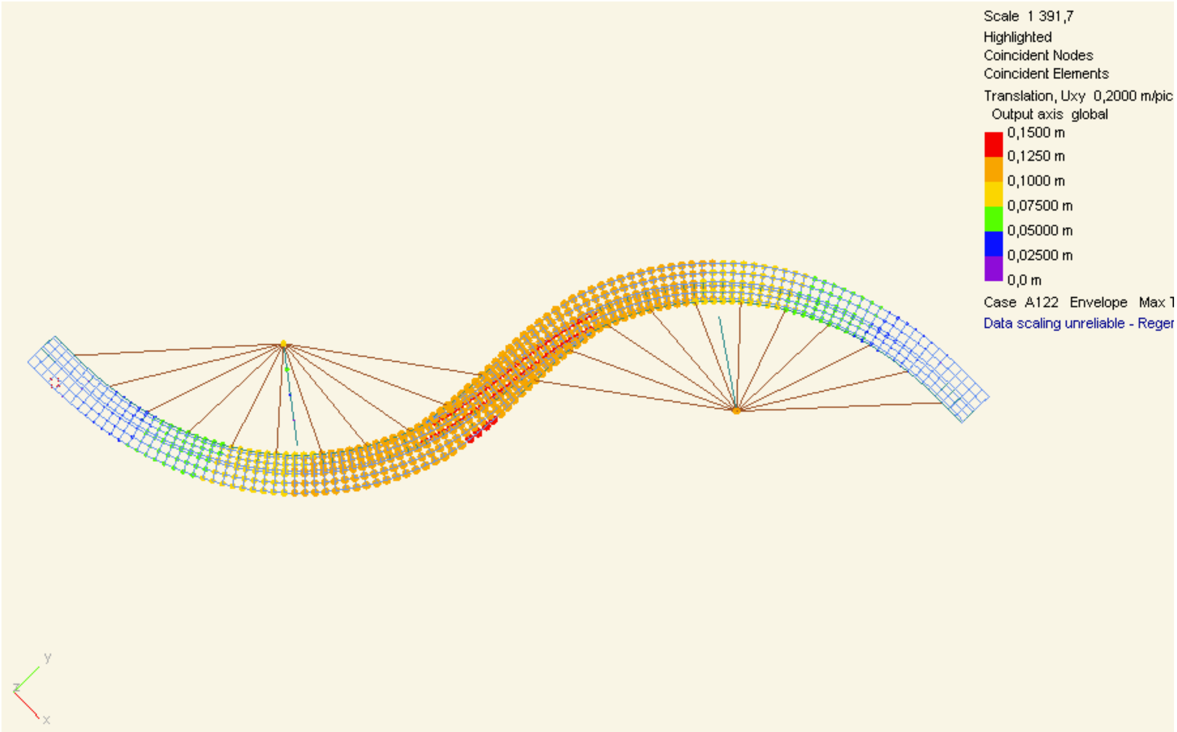


Fig.86

Displacement	Location (approximate)	
Maximum vertical (downwards)	Quarter-span section	-0.1312 m
Maximum vertical (upwards)	Quarter -span section	0.0789 m
Maximum horizontal	Midspan section	0.1294 m

Tab.22

In order to assess which value of the live loads factor would make deck nodes reach unacceptable displacement values (the limit value was taken arbitrarily as L/500=22cm, since footbridges' serviceability design is controlled by comfort criteria, as mentioned above), a series of static P-Delta analyses, each one corresponding to a 10% increment of the live load and to an initial stiffness equal to the stiffness reached at the previous increment (so that geometric stiffness effect would be accounted for), led to the following graph where the live loads factor is plotted against the vertical displacement of the critical node (Fig.87):

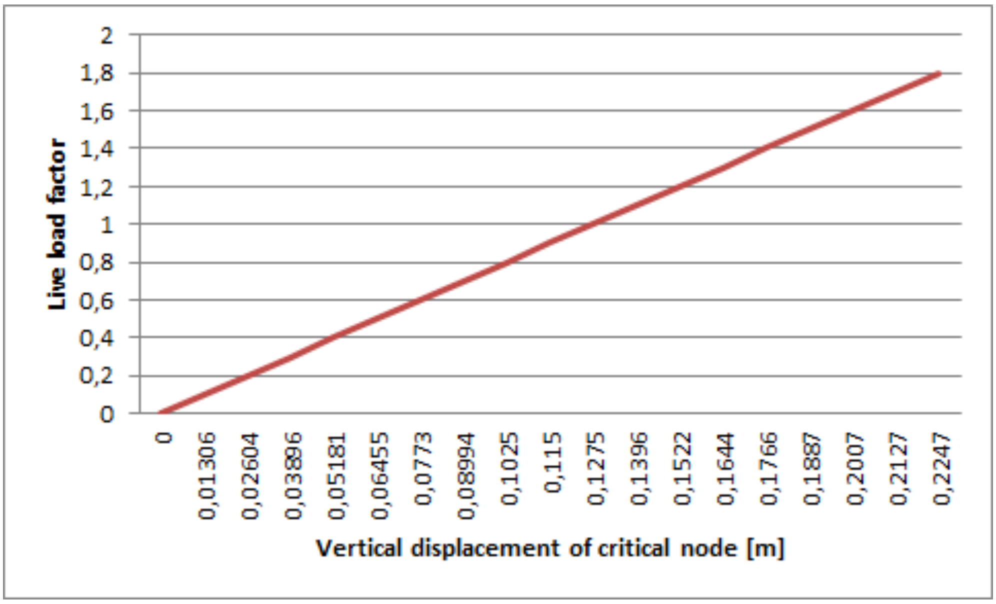


Fig.87

The graph depicts the deformative behaviour of the structure when subjected to live loading. The initial stiffness was set equal to that obtained from a non-linear static analysis under dead and prestress loads, this last configuration representing a no-traffic condition. It is clear that for the relevant range of displacements the structure behaves linearly under live loading, meaning that the choosen initial geometry and prestresses distribution allows the bridge to reach its live loads equilibrium position without experiencing large displacements.

We notice that a live load factor of approximately 1.8 corresponds to the target displacement, meaning that the service load can be increased by 80% until unacceptable displacements occur. This is deemed to be a sufficient safety margin.

The comfort criteria are based on the principle of limiting vertical and horizontal accelerations, thus a dynamic analysis is required to check the relevant serviceability limit states. This topic is treated in the following Chapter 6.

5.3.3 Seismic response

The Ultimate and Serviceability limit states related to seismic actions were checked using the relevant seismic load combination (5.2.5.3).

Since the structure has geometric and material non-linearities, a specific approach has been adopted and theoretically justified so that the seismic response of the bridge could be predicted. In fact, the existence of tension-only members would require a non-linear time history analysis, for which a time-history of the ground motion and significant hardware (and time) resources are needed. Since this would involve a whole new advanced study (i.e. a matter for another thesis), a simplified approximate approach was developed.

As explained in Chapter 6, all the dynamic properties of the structure were obtained by using a Modal P-Delta analysis, that is a modal analysis that takes into account that loading on the structure will affect its natural frequencies and mode shapes. It consists of two steps: the first is used to modify the stiffness matrix of the structure by applying a specific load case (i.e. the analysis case resulting from a non-linear static analysis where only permanent and prestress loads are applied) and by changing tension-only elements (*ties)* into linear element (*bars*), thus accounting for the geometric and material non-linearities; the second step is a standard modal analysis based on the modified stiffness matrix.

Response spectrum analyses were then run using the modal information to get the seismic response. Since Static P-Delta and Modal P-Delta linear results are based on a specific initial stiffness (i.e. the stiffeness obtained from dead loads and ties' prestresses) which varies depending on the p-delta load case, their combination, in general, should be avoided since superposition principle is not applicable. This is not, though, the case of our structure. In fact, response spectrum cases are combined with other Static P-Delta analysis cases (to get the seismic combination) which are based on a linearized model obtained by using a P-delta load case that corresponds to the one used for the Modal P-delta. This means that the cases can be combined, thus producing a total response.

The seismic action was modeled according to the italian code NTC 2008 given the site location. A total of four spectra were used in the analysis (response spectrum details are reported in the Annex):

- Horizontal and vertical component of the seismic action for the Life Safety limit state (SLV-Fig.88);
- Horizontal and vertical component of the seismic action for the Damage limit state (SLD-Fig.89).

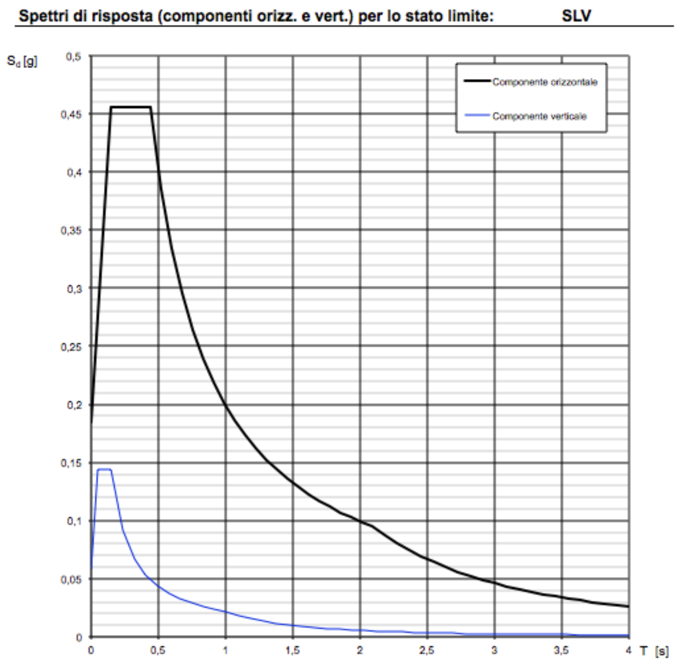


Fig.88

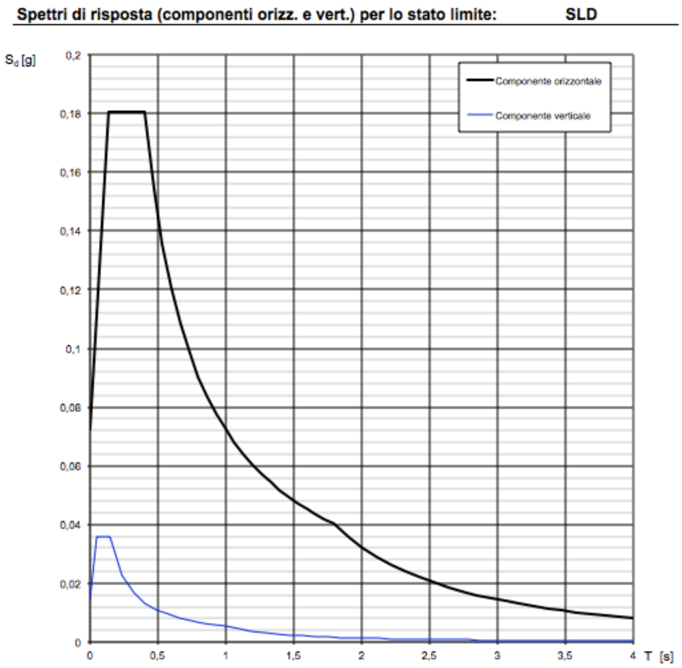


Fig.89

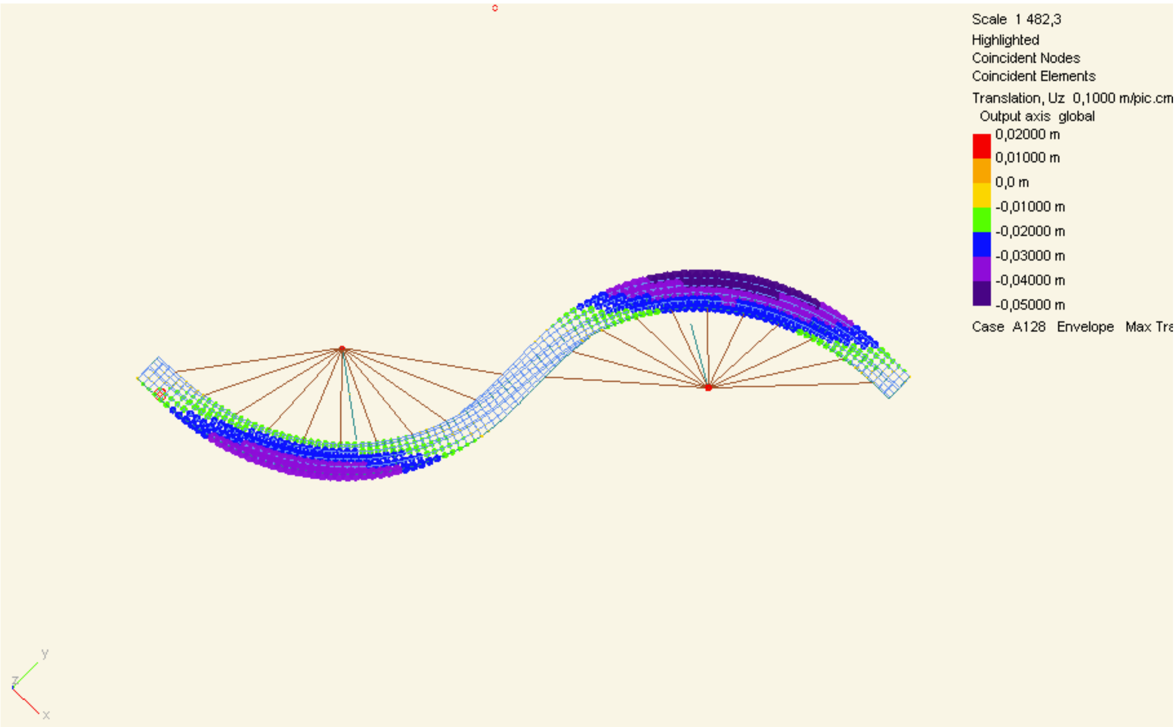
A number of 70 mode shapes was considered in order to achieve an acceptable percentage of effective modal mass (>85%). For each of the seismic limit states the following linear combination has been adopted to account for different potential earthquake space orientation:

$$1.00E_x + 0.30E_y + 0.30E_z$$

$$0.30E_x + 1.00E_y + 0.30E_z$$

$$0.30E_x + 0.30E_y + 1.00E_z$$

The following figures describe the seismic combinations envelope's displacements response in the vertical (Fig.90) and horizontal directions (Fig.91).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Coincident Elements
Translation, Uz 0,1000 m/pic.cm
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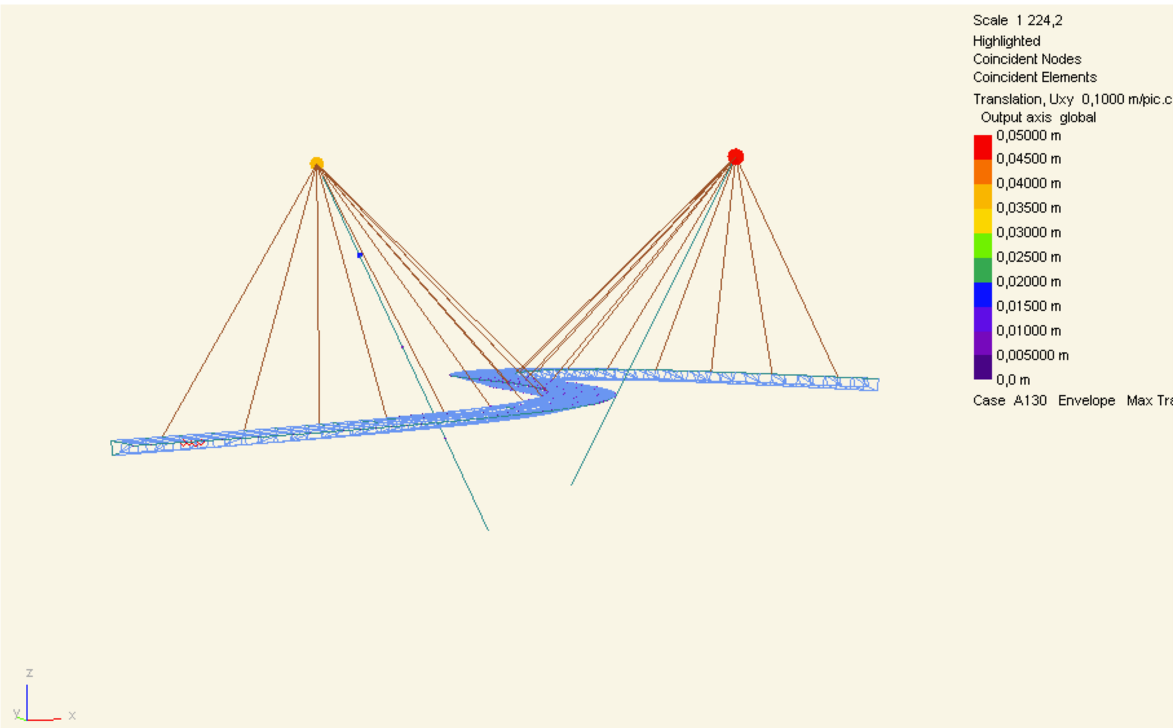
-0,03000 m
-0,04000 m
-0,05000 m
Case A128 Envelope Max Tra

Fig.90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Highlighted
Coincident Nodes
Coincident Elements
Translation, Uxy 0,1000 m/pic.c
Output axis global
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0,02500 m
0,02000 m
0,01500 m
0,01000 m
0,005000 m
0,0 m
Case A130 Envelope Max Tra

Fig.91

A specific investigation of the deck-mast relative displacement has been carried out in order to verify if any contact between the relevant nodes was possible. The relative displacement components are listed below:

Relative Nodal Displacements

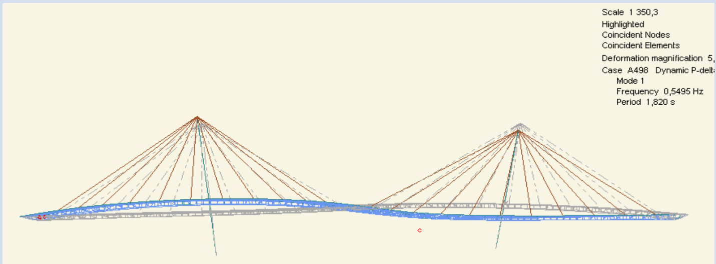
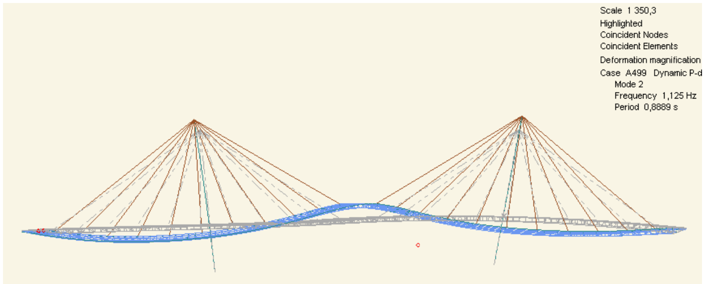
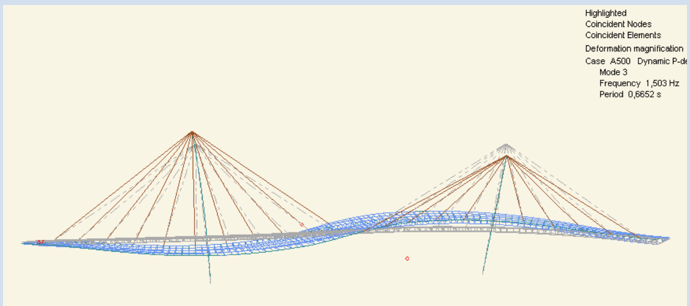
Name	Node 1	Node 2	Axis	Ux [m]	Uy [m]	Uz [m]
Deck-mast	827	1729	Global	350,9E-6	0,001149	592,7E-6

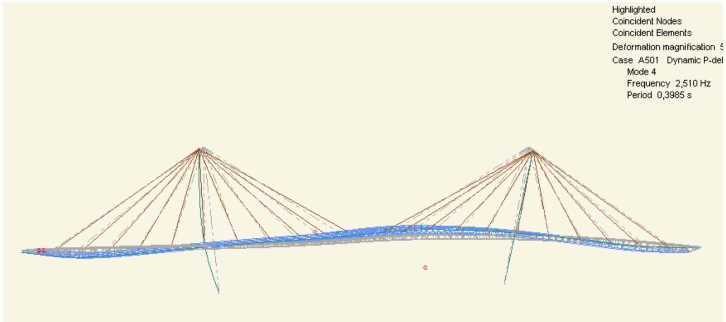
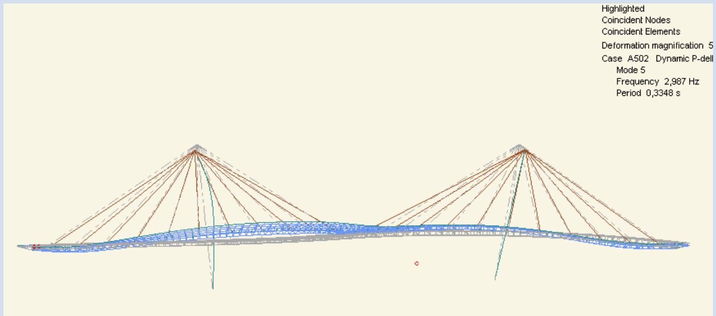
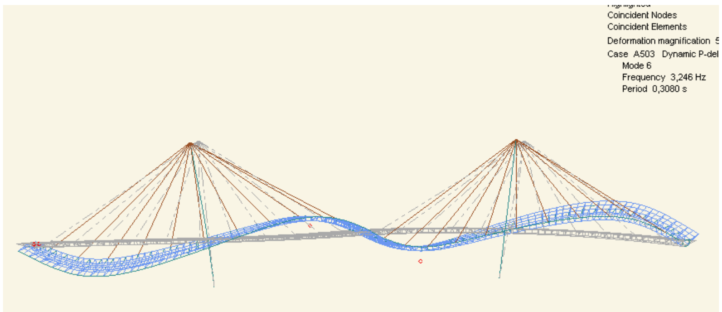
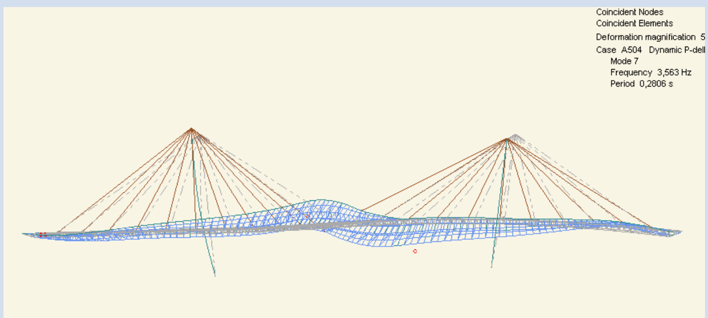
The resulting displacement is, then, deemed acceptable.

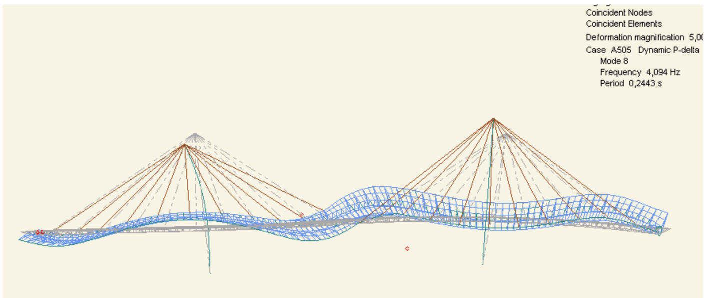
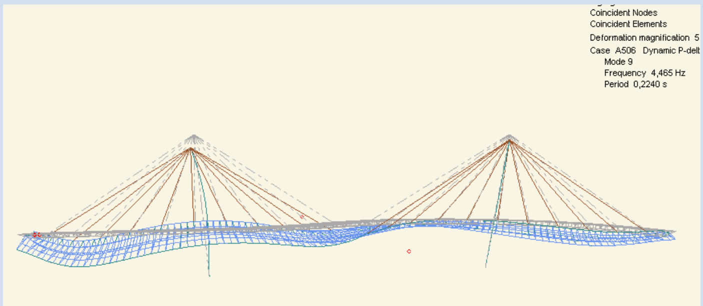
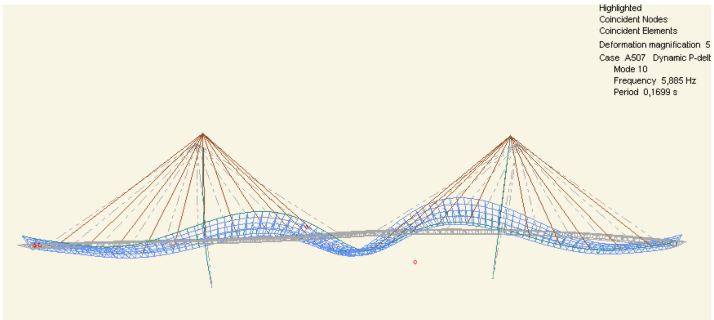
6. Bridge dynamics and vibration analysis

6.2 Mode shapes and eigenfrequencies

A summary of the predicted eigenfrequencies for the first ten mode shapes is given in the table below:

<table>
<tr><th>Description</th><th>Natural frequency [Hz]</th><th>Modal mass [kg]</th><th></th></tr>
<tr><td>Lateral</td><td>0.5495</td><td>171300</td><td>
</td></tr>
<tr><td>Vertical</td><td>1.1250</td><td>45820</td><td>
</td></tr>
<tr><td>Torsional</td><td>1.5030</td><td>42490</td><td>
</td></tr>
</table>

Vertical	2.5100	20820	
Vertical/torsional	2.9870	16660	
Vertical	3.246	34030	
Torsional	3.563	69330	

Vertical-torsional	4.094	58070	
Vertical-mast sway	4.465	42240	
Torsional	5.885	44880	

Tab.23

6.2 Dynamic input

Vibrations are an issue of increasing importance in current footbridge design.
It is a phenomenon that occurs in all structures and is caused by an energy pulse acting on a structural member.

Slender decks with more effective construction materials result in lightweight structures and a high ratio of live load to dead load. This trend leads to structures that are more susceptible to vibrations when subjected to dynamic loads. In walkways and footbridges, the most common energy source arises from pedestrians crossing the bridge and wind blowing.
While wind action must be considered when designing for failure, pedestrian induced footfall forces due to walking or jogging might control the design at a serviceability level.
When humans walk, the weight of the person will be transferred to the ground at approximately even intervals, which will lead to a periodic force. This force is dependent on the walking speed and stride length, in other words the pacing rate or the step frequency. Figure 92 shows that when the pacing rate increases, the time in which one foot has contact with the ground decreases and at the same time the dynamic amplification increases. Normal walking usually has a pacing frequency varying between 1.6 and 2.4 Hz with 2.0 Hz often used as a mean value, while running varies between 2.0 and 3.5 Hz (Živanović, Pavić, & Reynolds, 2005). From Fig. 92, it can be seen that a walking frequency of 2.0 Hz gives a dynamic effect where around 40% of the self-weight is added to the static force. When running, the dynamic force can be over 150% of the self-weight, and hence running will often be governing. However, Sétra guidelines (2006) states that it sometimes should be allowed to exceed design limits for running due to the short duration of the crossing. The short duration is not long enough to cause resonance and will onlydisturb other pedestrians for a short period of time.

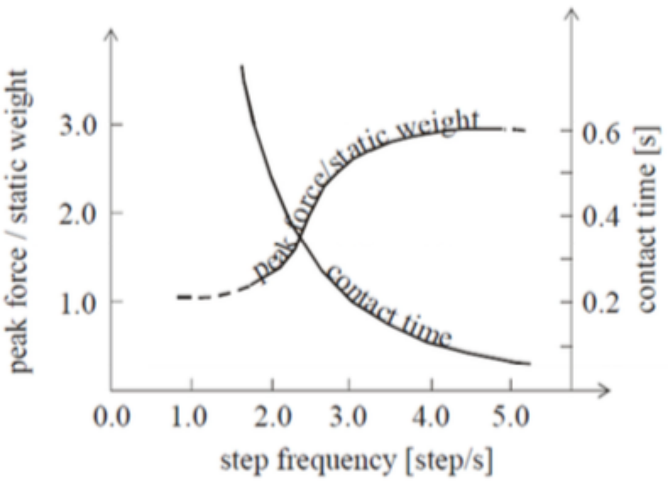


Fig. 92

When walking, dynamic forces arise in three directions: vertical, lateral and longitudinal. The vertical and longitudinal components have the same frequency, while the lateral component has half this frequency because every step is made with the same foot and hence in the same direction. The vertical direction is the most investigated due to the largest magnitude (Živanović, Pavić, & Reynolds, 2005). However, in the last decade the lateral force was more thoroughly studied and will be discussed in the following. The difference in periods and frequencies can be seen in Fig.93.

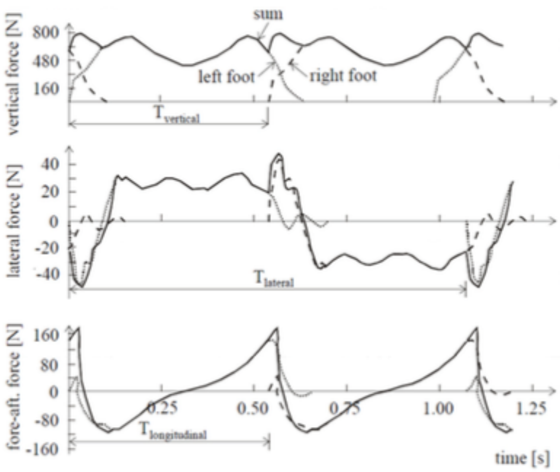


Fig. 93

In addition, it is possible to see how each step overlaps the previous one and that more than one harmonic is needed to describe these periodic forces. The total effect of a walking pedestrian can then, be described as the Fourier series:

$$F_p(t) = G + \sum_{i=1}^{n} G \cdot \alpha_i \cdot \sin\left(2\pi \cdot i \cdot f_p \cdot t - \varphi_i\right)$$

Where:

- G is the person's weight [N];
- α_i is the Fourier's coefficient of the ith harmonic (i.e. the dynamic load factor, DLF);
- f_p is the activity rate [Hz];
- φ_i is the phase shift of the ith harmonic;
- i is the order number of the harmonic;
- n is the total number of contributing harmonics.

Based on this decomposition, researchers have tried to quantify the DLFs. Blanchard et al. proposed a simple walking force model based on resonance due only to the first harmonic with the DLF equal to 0.257 and a pedestrian weight G = 700N, while Bachmann & Amman reported the first five harmonics for the vertical walking force and also harmonics for the lateral and longitudinal direction. By observing Fig.94, one might figure what are the dominant harmonics for the three directions and what DLF values are related to them.

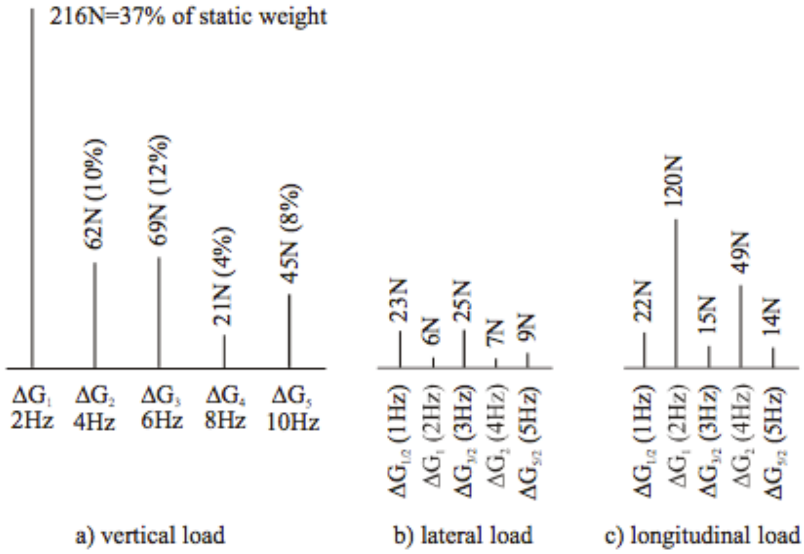


Fig.94

The values of DLFs that can be found in the literature are listed in the Table 24.

Young's coefficients (highlighted) are the resuls of the first attempt made by researchers to take into account the stochastic nature of human walking in day-to-day design and they outline the basic principles that are used by Arup Consulting Engineers when modeling walking forces and the corresponding structural responses.

Once the dynamic load from pedestrians is defined, it has to be applied to the appropriate spans of the bridge in order to maximize the dynamic effect, meaning that, given a mode shape, the load should have the same sign as the mode sags, according to Fig. 95

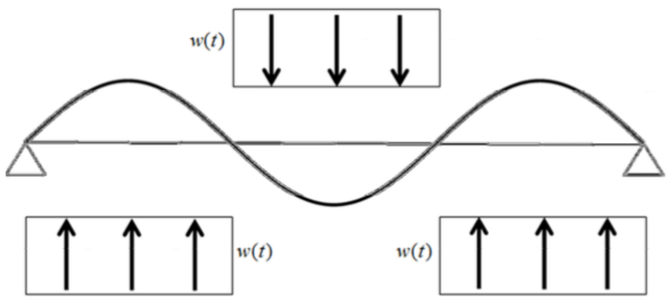


Fig. 95

Author(s)	DLFs for considered harmonics	Comment	Type of activity and its direction
Blanchard et al. [34]	$\alpha_1 = 0.257$	DLF is lessen for frequencies from 4 to 5 Hz	Walking – vertical
Bachmann & Ammann [14]	$\alpha_1 = 0.4 - 0.5$	Between 2.0 Hz and 2.4 Hz	Walking – vertical
	$\alpha_2 = \alpha_3 = 0.1$	At approximately 2.0 Hz	
Schulze (after Bacmann & Ammann, [14])	$\alpha_1 = 0.37$ $\alpha_2 = 0.10$ $\alpha_3 = 0.12$ $\alpha_4 = 0.04$ $\alpha_5 = 0.08$	At 2.0 Hz	Walking – vertical
	$\alpha_1 = 0.039$ $\alpha_2 = 0.01$ $\alpha_3 = 0.043$ $\alpha_4 = 0.012$ $\alpha_5 = 0.015$	At 2.0 Hz	Walking – lateral
	$\alpha_{1/2} = 0.037$ $\alpha_1 = 0.204$ $\alpha_{3/2} = 0.026$ $\alpha_2 = 0.083$ $\alpha_{5/2} = 0.024$	At 2.0 Hz	Walking – longitudinal
Rainer et al. [42]	$\alpha_1, \alpha_2, \alpha_3$ and α_4	DLFs are frequency dependent (Figure 10)	Walking, running, jumping – vertical
Bachmann et al. [48]	$\alpha_1 = 0.4/0.5$, $\alpha_2 = \alpha_3 = 0.1/-$	At 2.0/2.4 Hz	Walking – vertical
	$\alpha_1 = \alpha_3 = 0.1$	At 2.0 Hz	Walking – lateral
	$\alpha_{1/2} = 0.1$, $\alpha_1 = 0.2$ $\alpha_2 = 0.1$	At 2.0 Hz	Walking – longitudinal
	$\alpha_1 = 1.6$, $\alpha_2 = 0.7$, $\alpha_3 = 0.2$	At 2.0–3.0 Hz	Running – vertical
Kerr [36]	α_1 , $\alpha_2 = 0.07$ $\alpha_3 \approx 0.06$	α_1 is frequency dependent (Figure 11)	Walking – vertical
Young [56]	$\alpha_1 = 0.37(f - 0.95) \le 0.5$ $\alpha_2 = 0.054 + 0.0044f$ $\alpha_3 = 0.026 + 0.0050f$ $\alpha_4 = 0.010 + 0.0051f$	These are mean values for DLFs.	Walking – vertical
Bachmann et al. [48]	$\alpha_1 = 1.8/1.7$, $\alpha_2 = 1.3/1.1$ $\alpha_3 = 0.7/0.5$	Normal jump at 2.0/3.0 Hz	Jumping – vertical
	$\alpha_1 = 1.9/1.8$, $\alpha_2 = 1.6/1.3$ $\alpha_3 = 1.1/0.8$	High jump at 2.0/3.0 Hz	Jumping – vertical
	$\alpha_1 = 0.17/0.38$, $\alpha_2 = 0.10/0.12$ $\alpha_3 = 0.04/0.02$	At 1.6/2.4 Hz	Bouncing – vertical
	$\alpha_1 = 0.5$	At 0.6 Hz	Body swaying while standing – lateral
Yao et al. [52]	$\alpha_1 = 0.7$ $\alpha_2 = 0.25$	Free bouncing on a flexible platform with natural frequency of 2.0 Hz	Bouncing – vertical

Tab.24

For this project the software Oasys GSA was used also to carry out the human-induced vibration analysis and the expressions of the dynamic loads that the program automatically applies to the appropriate spans once a modal analysis has been run are based on Young's DLFs.

6.3 Vibration perception

The main receivers of vibrations on pedestrian bridges are walking people.
The reaction of human beings to vibrations is a very complex issue having in mind that humans are "the greatest variables with which anyone may deal with" . According to Lippert, not only different people react differently to the same vibration conditions, but also an individual exposed to the same vibrations on different days will likely react differently. This is known as the inter- and intra-subject variability of humans and their reactions to vibrations.
It is now widely accepted that the vibration tolerance for moving pedestrians on bridges is higher than for people in buildings, and that pedestrians can accept certain (initially unacceptable) level of vibrations when they accustom themselves to it.
Knowing that human sensitivity to vibrations is very high, it is clear that this issue is of paramount importance for footbridge vibration serviceability.
There are several specification defining limit acceleration levels to ensure pedestrian comfort.
For example, Eurocode sets the maximum acceleration in the vertical and horizontal direction to $0.7 m/s^2$ and $0.15 m/s^2$ respectively.
British Standard 5400 sets a limit vertical acceleration of $0.5\sqrt{f}$ (f = natural frequency), while it does not require a maximum horizontal acceleration, though stating that if the fundamental frequency of horizontal vibration is less than 1,5 Hz, the designer should consider the risk of lateral movements of unacceptable magnitude.
These approches are frequency independent. This means that threshlod of human perception of vibration is constant no matter what the pacing rate and the motion environment are.
Since human acceptance of vibration is very subjective, a level of vibration that causes one individual to complain might be unnoticed by another. Similarly, vibration that causes concern or distraction for an individual sitting in a quiet office could be quiet acceptable to the same person walking around a shopping center.
In order to account for these factors, a different approach was found by Irwin.
He constructed either the perception or maximum allowable magnitude curves for different types of structures and different type of vibrations. Among them, the limits for root-mean-square (RMS) accelerations for bridges are given, separately for everyday usage and storm conditions (Fig.96).

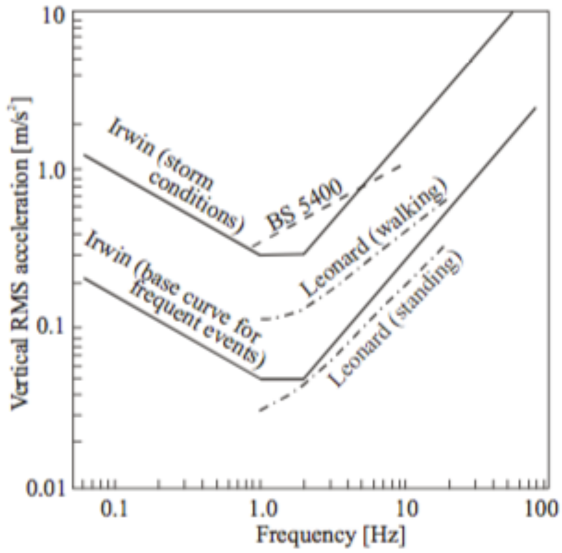


Fig.96

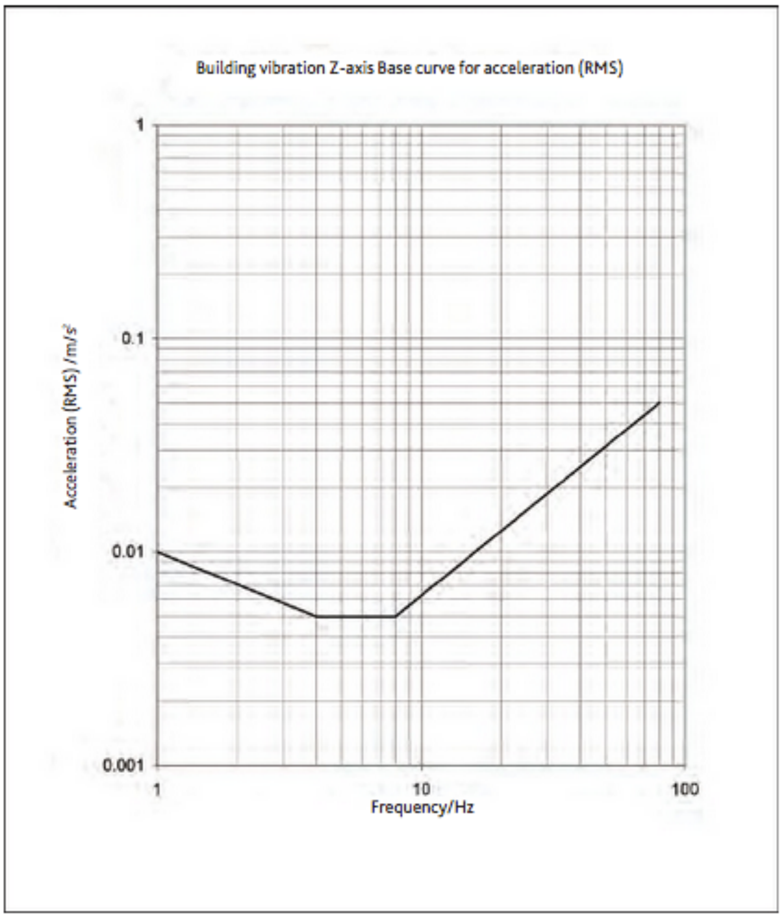


Fig.97 - Baseline for vertical acceleration

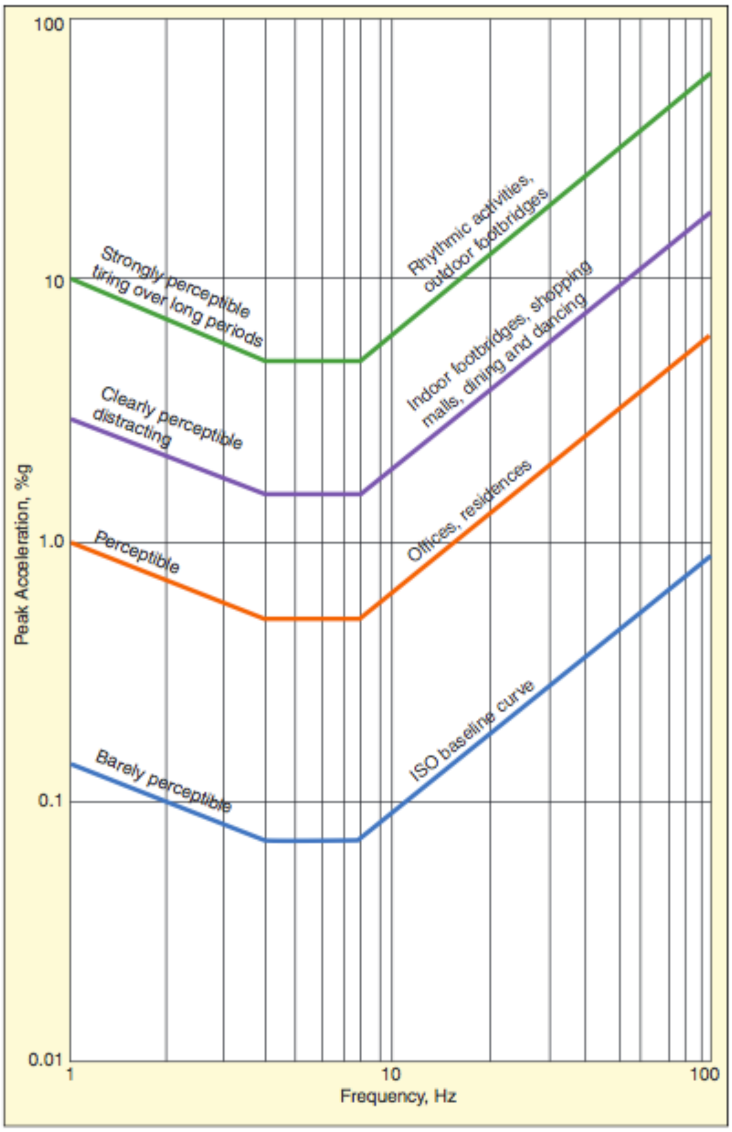


Fig. 98 - Peak acceleration baseline obtained by multiplying the RMS acceleration baseline by a factor of $\sqrt{2}$

The method outlined in his work defines a limit value of the root-mean-square acceleration

RMS = $\sqrt{\frac{\int_{t_1}^{t_2} \ddot{x}(t)^2 dt}{t_2 - t_1}}$ (where $\ddot{x}(t)$ is the acceleration time history, and t_1 and t_2 are the extremes of the time interval being considered) which varies vith frequency producing a base line. Irwin's work is, in fact, founded on the base curve principle, which means that a perceptibility RMS acceleration curve related to a specific environment can be obtained by multiplying the base curve by some factor.
The choice of RMS accelerations as the vibration perception descriptor is based primarily on the fact that it is relatively easy to measure accelerations and the corresponding RMS values, using both analog and digital methods

For the thesis project Young's approach has been used since it is the method implemented in the Oasys

GSA software.
The footfall analysis results consist of a set of values of the Response Factor (R) for a range of walking frequencies. That is, a multiplier on the level of vibration at the average threshold of human perception (in terms of RMS or peak acceleration) that has to be set against the maximum allowable vibration level obtained by multiplying the base curve by a factor of 64 (value for external footbridges suggested in "Willford, M.R. & Young, P. (2006) A Design Guide for Footfall Induced Vibration of Structures, The Concrete Centre, CCIP-016").
In details, the Response factors from footfall analysis are calculated using frequency weighting curves (FWC), i.e. the calculated RMS accelerations times the weighting factors from the chosen frequency weighting curve, then divided by 0.005 m/s^2 to get the response factors.
The frequency weighting curve can be standard or user defined, there are three standard frequency weighting curves (Wb, Wd and Wg) from BS6841 that can be used directly by footfall analysis.

Once calculated, R is compared with the aforementioned limit value and, if it is smaller than that, the footfall check will be satisfied, meaning that the structure will be subjected to an acceptable footfall induced vibration level.

6.4 Damping

The amount of damping present in a footbridge is very significant in the evaluation of the amplitude of oscillations induced by pedestrians. The attenuation of vibrations, i.e., the energy dissipation within the structure, depends both on the intrinsic damping of construction materials, which is of distributed nature, and on the local effect of bearings or other control devices. Additional damping is also provided by non-structural elements, like handrails and surfacing.
In general, the amount of damping depends on the level of vibrations, as higher amplitudes of vibration cause more friction between structural and non-structural elements and bearings.
Furthermore, it is now well-known that the presence of a stationary (standing or sitting) person changes the dynamic properties of a structure they occupy. And the most important effect is the increase in damping in the joint human-structure dynamic system compared with the damping of a bridge with no people on it.
The higher the number of people walking the greater would be the damping amplification effect.
Therefore it can be concluded that human bodies behave like damped dynamic systems attached to the main structural system (bridge).
This co-existence of various mechanisms of dissipation within the structure makes damping a complex phenomenon whose accurate characterisation can only rely on measurements taken once the footbridge has been constructed, including installation of handrails, surfacing and any type of furniture.

For the design project, a damping ratio of 2% has been choosen for all modes.

6.5 Lateral lock-in (SLE)

When walking, a person will shift the centre of gravity from left to right with around 2 cm (Fig. 99). This will induce a dynamic lateral force of a magnitude 10 times smaller than the vertical force. This lateral force is mainly of interest due to a behaviour that occurs in some bridges when subjected to a large crowd of pedestrians. If the bridge has a lateral natural frequency of approximately half the usual walking frequency, it becomes easier to walk in this frequency than any else when the vibration amplitude increases. Hence, this may cause even larger lateral vibrations, and some people then might feel uncomfortable, even though the bridge is structurally sound and safe to cross (Nakamura & Kawasaki, 2006).

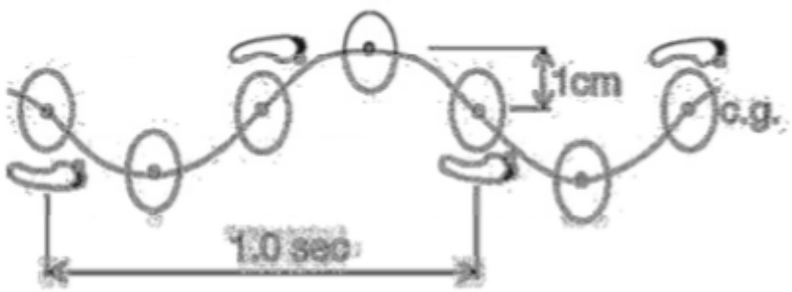


Fig.99

Two recent and famous cases of this phenomenon, called *Lock-in* or *Synchronous Lateral Excitation (SLE)*, are the Millennium Bridge in London and Pont de Solférino in Paris Both were closed shortly after opening and subjected to testing to determine the cause of the excessive and improve the bridges (Sétra, 2006). These two bridges have a very slim construction and special structural systems, but this problem is not restricted to slender bridges. According to Ingólfsson, Georgakis and Jönsson (2012), even more robust bridges exposed to unnatural huge crowds have experienced excessive lateral vibrations, among others the Brooklyn Bridge in New York City during a power blackout. The studies of the London Millennium Bridge by Dallard et al. (2001) shows that the load effect arises from synchronization of the lateral footfall of the crowd to the natural swaying of the bridge since it is more comfortable for the pedestrian. However, according to Nakamura (2004) when the velocity becomes large the pedestrians feel unsafe and this gives a maximum level of the vibrations. According to Ingólfsson, Georgakis, and Jönsson (2012) there is a discussion about the basic mechanism behin lateral forces and several hypothesis and load model exist, even thoug it may seem that the lock-in effect is the cause of excessive lateral vibrations.

According to Ingólfsson, Georgakis and Jönsson (2012), the most used method to assess suspectability of a bridge to excessive pedestrian-induced lateral vibrations is the theory formulated for the Millennium Bridge. The model is thoroughly explained by Dallard, et al.(2001) and gives a limiting

number of pedestrians before excessive vibrations occur.
This number can be defined as:

$$N_L = \frac{8\pi \cdot \xi \cdot M \cdot f}{k}$$

where:
- ξ is the structural damping ratio;
- M is the modal mass in the relevant modeshape;
- f is the natural frequency of the relevant modeshape;
- k is a constant (300 Ns/m approximately).

The investigations on Pont de Solférino agree with the investigations on the Millennium Bridge that there is a threshold for lock-in to occur. However, the threshold is instead expressed in terms of accelerations stating that the crowd behaviour is no longer random when the acceleration exceeds 0.10-0.15 m/s^2 (Sétra, 2006), and hence theories based on the randomness of pedestrians are no longer valid.

6.6 Footfall and Synchronous Lateral Excitation analysis

The footfall analysis has been conducted using Oasys GSA software.
In order to get the values of natural frequencies of the structure a Dynamic P-Delta analysis had to be run because of the geometric non-linearities characterizing the bridge.
As mentioned in a previous paragraph, the dynamic load are obtained from the general Fourier's expression by entering different values of DLFs (dynamic load factors) according to the method choosen (e.g. Arup, SCI, AISC or CCIP-016 method).
An appropriate frequency weighting curve (FWC) shall then be selected among the BS6472 curves (Wb, Wd and Wg) in order to get the response factors. That is, the weighting factors from a specific FWC are to be multiplied by the RMS acceleration and divided by 0.005m/s^2 in order to get the Response Factor (R).
The analysis settings are highlighted in the figure below (Fig.100).

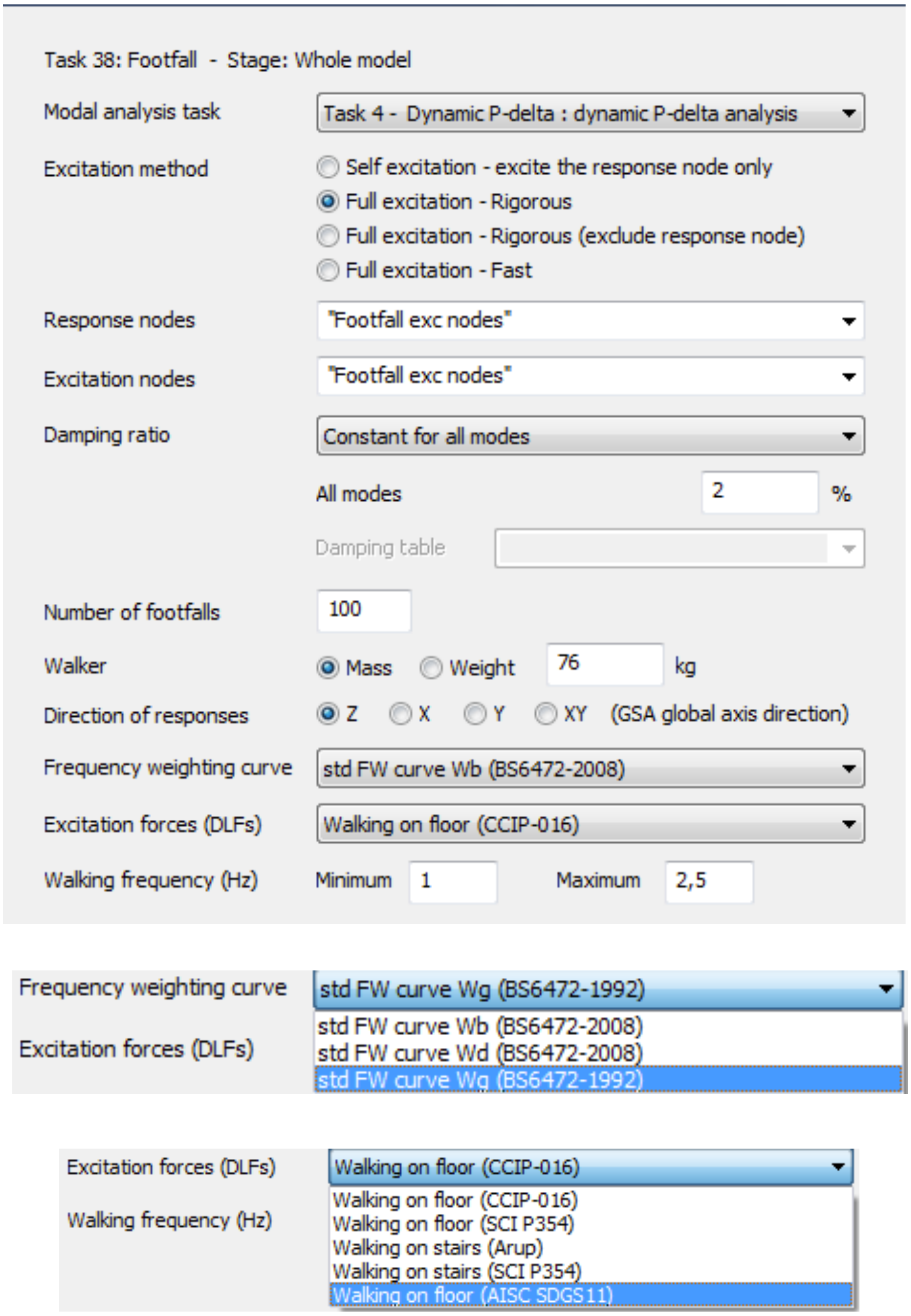


Fig.100

Each set of DLFs is related to a specific walking frequency range. For the project, BS6472 curves and the CCIP-016 dynamic load factors have been used, thus a walking frequency ranging from 1 to 2.5 Hz will be adopted. In addition a Wb frequency weighting curve is choosen according to what BS6472 suggests regarding the evaluation of the vertical vibration response.

An amplified default pedestrian mass of 300 kg (representing a group of four pedestrians) has been used since it seems to be appropriate considering an average walker weight of 75kg.

As a result we found that the overall maximum response factor R for the vertical direction is equal to 28.88 and it occurs in a region close to the quarter span section (Fig.101).

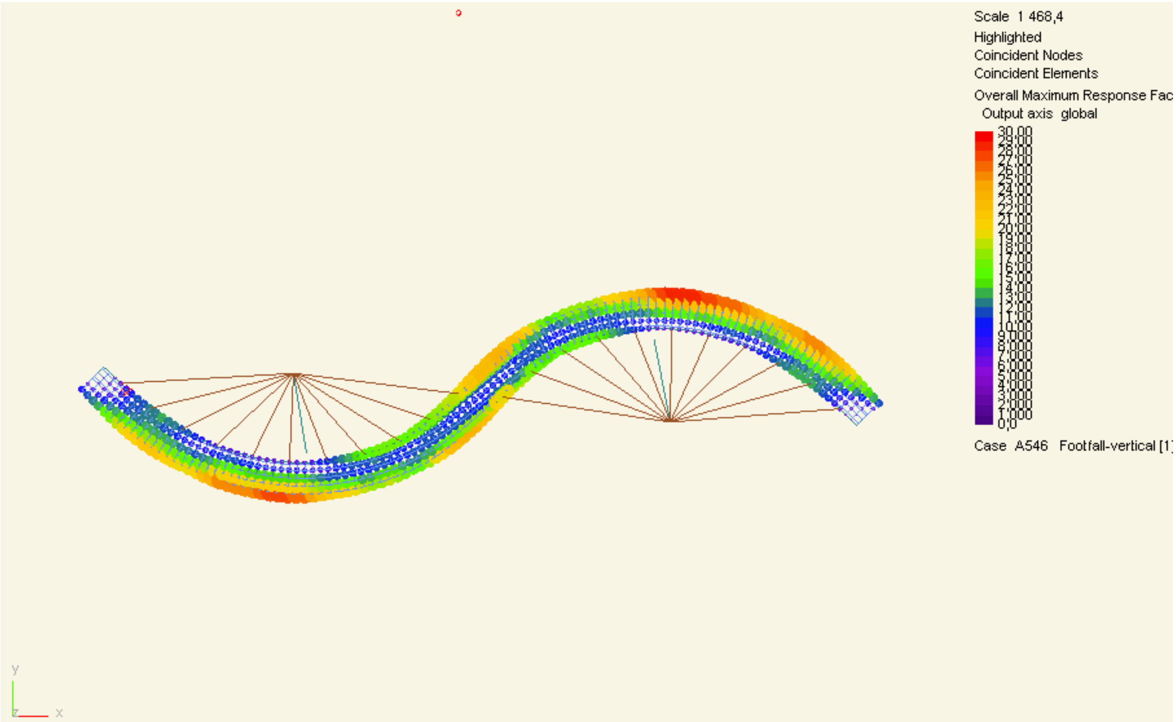


Fig.101

Node	Case	Maximum Resonant Response Factor	Maximum Transient Response Factor	Overall Maximum Response Factor
1306	A522	1,318	0,2842	1,318
	A523	2,365	0,3247	2,365
	A545	2,708	0,4160	2,708
	A546	28,88	3,814	28,88
Maxima				
1306	A546	*28,88*	3,814	28,88
1306	A546	28,88	*3,814*	28,88
1306	A546	28,88	3,814	*28,88*
Minima				
1306	A522	*1,318*	0,2842	1,318
1306	A522	1,318	*0,2842*	1,318
1306	A522	1,318	0,2842	*1,318*

Tab.25

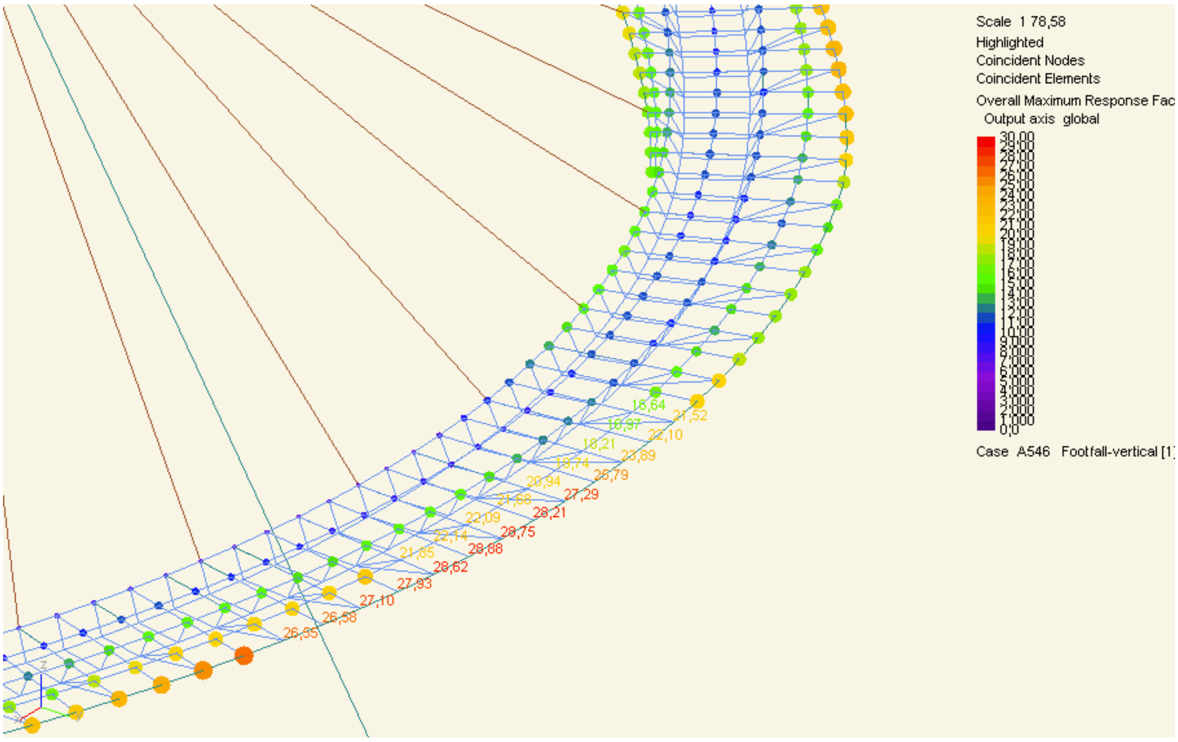


Fig.102

An alternative way to look at the results is plotting the Response Factor envelope for the first 4 harmonics of the excitation force versus the walking frequency (notice that the plot is cut at a walking frequency of 2.5 Hz since that is the maximum value allowed by the selected frequency range.

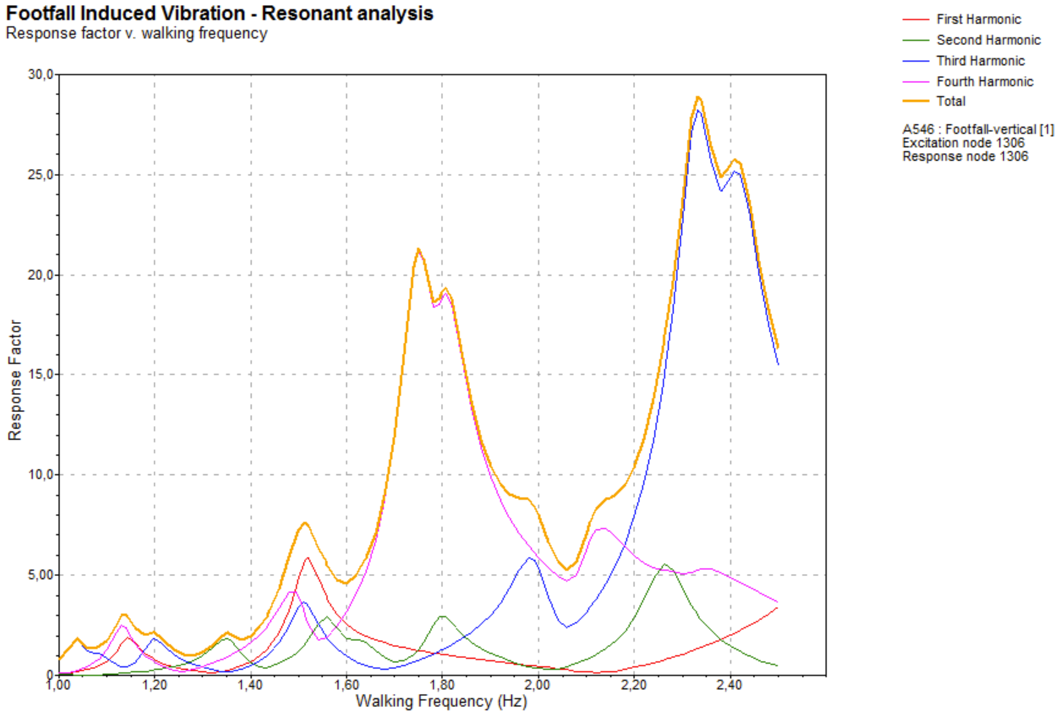
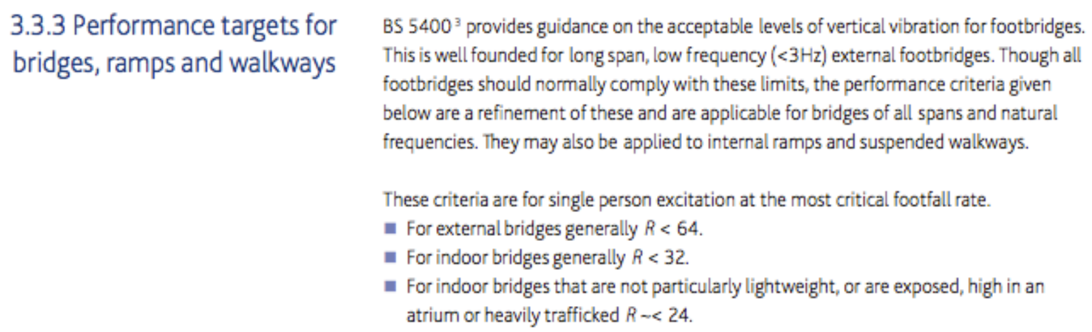


Fig.103

The Concrete Center guideline (" A Design Guide for Foottfall Induced Vibrations of Strucures") states that for external bridges the limit value for R be 64 (Fig.104).



3.3.3 Performance targets for bridges, ramps and walkways

BS 5400[3] provides guidance on the acceptable levels of vertical vibration for footbridges. This is well founded for long span, low frequency (<3Hz) external footbridges. Though all footbridges should normally comply with these limits, the performance criteria given below are a refinement of these and are applicable for bridges of all spans and natural frequencies. They may also be applied to internal ramps and suspended walkways.

These criteria are for single person excitation at the most critical footfall rate.

- For external bridges generally $R < 64$.
- For indoor bridges generally $R < 32$.
- For indoor bridges that are not particularly lightweight, or are exposed, high in an atrium or heavily trafficked $R \sim< 24$.

Fig.104

Thus, the footbridge will not experience disturbing pedestrian-induced vibrations in the vertical direction. To verify the compliance with the Eurocode prescription regarding the maximum vertical acceleration allowed in a footbridge to ensure pedestrian comfort, the dynamic response of the structure to the excitation forces (same as for footfall) has been evaluated and the resulting nodal acceleration values are displayed in Fig.105.

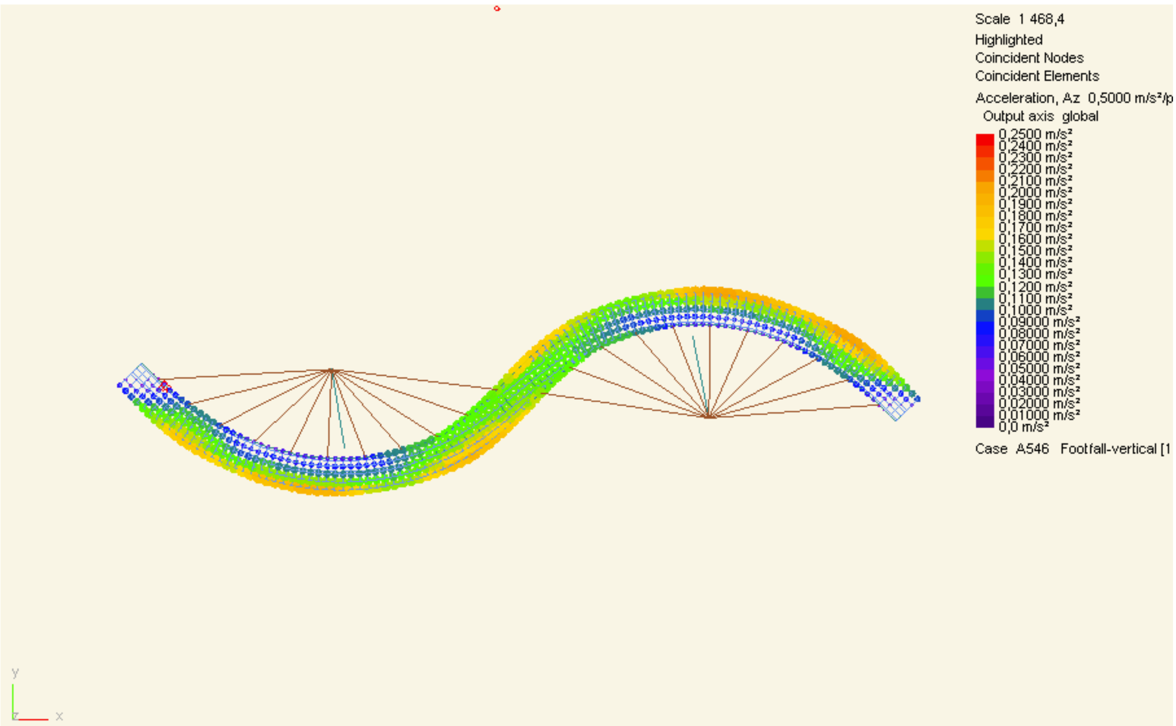


Fig.105

Node	Case	Ax [m/s²]	Ay [m/s²]	Az [m/s²]	|A| [m/s²]
1450	A546	0,0	0,0	0,2061	0,2061
Maxima					
1450	A546	*0,0*	0,0	0,2061	0,2061
1450	A546	0,0	*0,0*	0,2061	0,2061
1450	A546	0,0	0,0	*0,2061*	0,2061
1450	A546	0,0	0,0	0,2061	*0,2061*
Minima					
1450	A546	*0,0*	0,0	0,2061	0,2061
1450	A546	0,0	*0,0*	0,2061	0,2061
1450	A546	0,0	0,0	*0,2061*	0,2061
1450	A546	0,0	0,0	0,2061	*0,2061*

Tab.26

Eurocode (EN1990-A2) states that the maximum acceleration of any part of the deck, in the vertical direction, should not exceed 0.7 m/s^2, thus even this last compliance is satisfied since the maximum vertical acceleration experienced by the bridge is equal to 0.2061 m/s^2.

The same analysis has been used to evaluate the vibration response in the horizontal direction. The appropriate BS6472 W_d curve has been adopted since we are focused on the horizontal behaviour. The overall maximum response factor R for the horizontal plane is equal to 4.849 and it occurs at the midspan region (Fig.106).

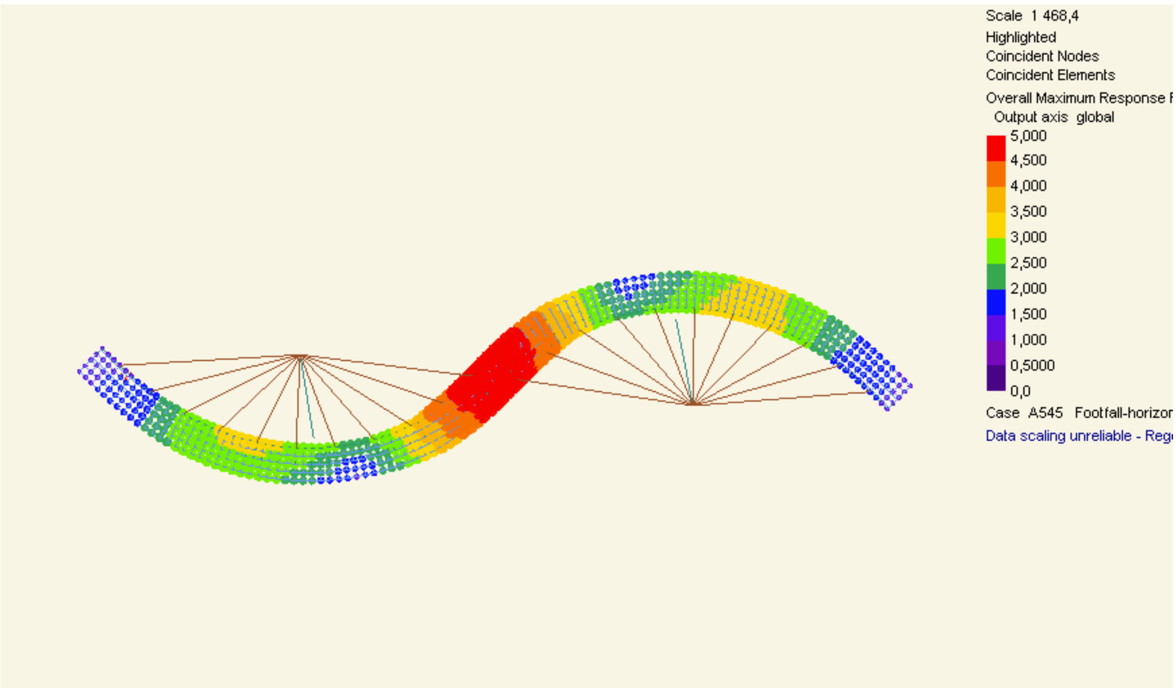


Fig.106

Node	Case	Maximum Resonant Response Factor	Maximum Transient Response Factor	Overall Maximum Response Factor
1608	A545	4,849	0,8189	4,849

Maxima

1608	A545	*4,849*	0,8189	4,849
1608	A545	4,849	*0,8189*	4,849
1608	A545	4,849	0,8189	*4,849*

Minima

1608	A545	*4,849*	0,8189	4,849
1608	A545	4,849	*0,8189*	4,849
1608	A545	4,849	0,8189	*4,849*

Tab.27

As mentioned for the vertical vibration analysis, the footbridge Response Factor for the horizontal vibration is smaller that the limit value set by the Concrete Centre footfall guideline, thus the structure will not experience disturbing pedestrian-induced vibrations in the horizontal direction as well.

To verify the compliance with the Eurocode prescription regarding the maximum horizontal acceleration allowed in a footbridge to ensure pedestrian comfort, the dynamic response of the structure to the excitation forces (same as for footfall in the horizontal direction) has been evaluated and the resulting nodal acceleration values are displayed in Fig.107,108.

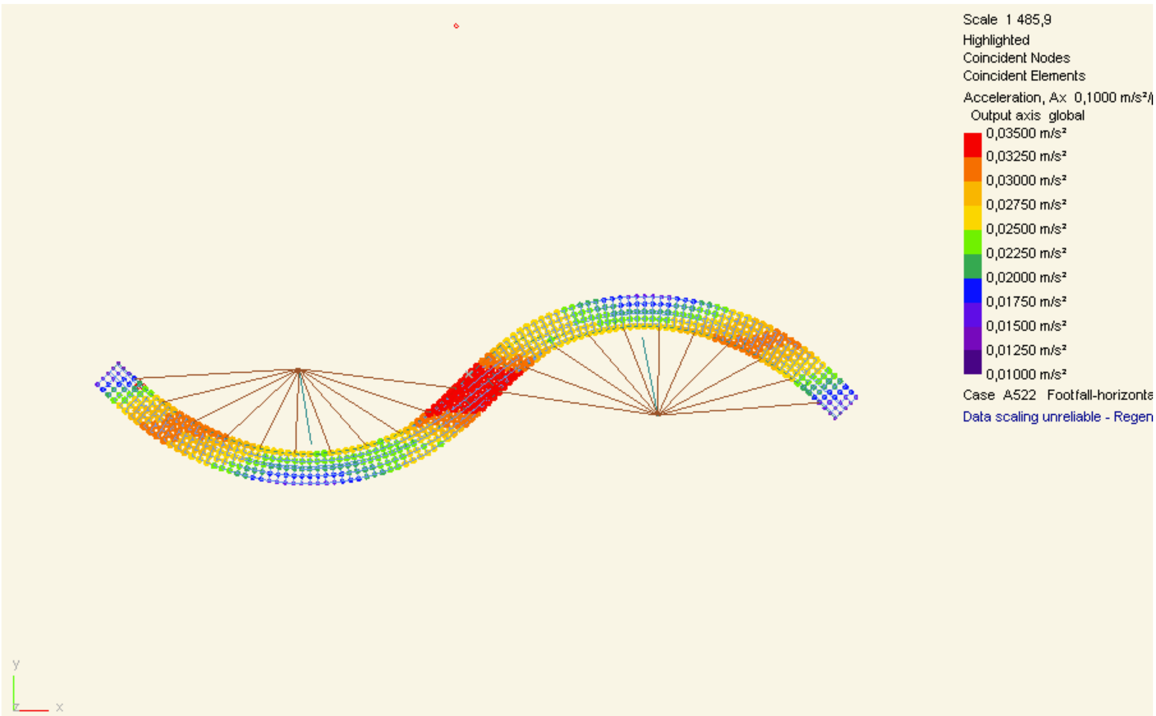


Fig.107

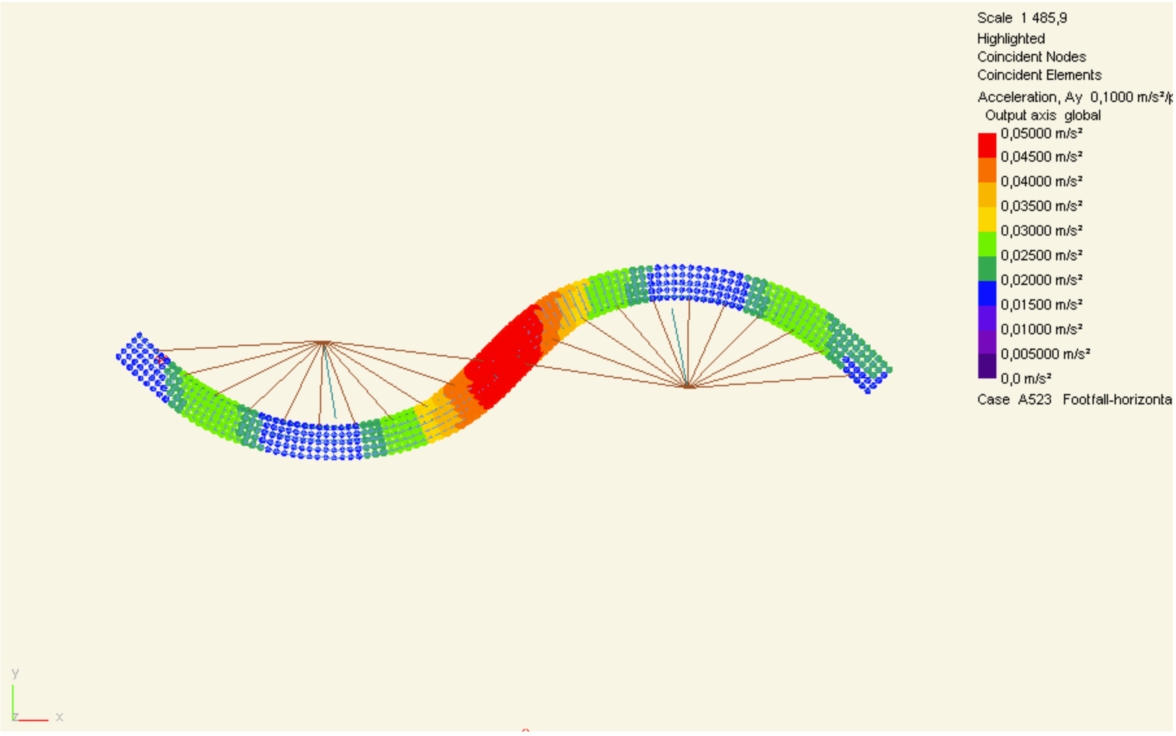


Fig.108

Node	Case	Ax [m/s²]	Ay [m/s²]	Az [m/s²]	|A| [m/s²]
Maxima					
1608	A545	*0,04262*	0,04262	0,0	0,06027
1608	A545	0,04262	*0,04262*	0,0	0,06027
1	A545	0,01188	0,01188	*0,0*	0,01680
1608	A545	0,04262	0,04262	0,0	*0,06027*
Minima					
566	A545	*0,01133*	0,01133	0,0	0,01602
566	A545	0,01133	*0,01133*	0,0	0,01602
1	A545	0,01188	0,01188	*0,0*	0,01680
566	A545	0,01133	0,01133	0,0	*0,01602*

Tab.27

Eurocode states that the maximum acceleration of any part of the deck, in the horizontal direction, should not exceed 0.15 m/s^2, thus even this last compliance is satisfied since the maximum horizontal acceleration experienced by the bridge is equal to 0.06027 m/s^2. In addition, a synchronous lateral excitation analysis was carried out since the structure appears to be potentially inclined to be susceptible to the lateral lock-in phenomenon (the first mode is, in fact, a horizontal sway with a frequency of 0.5495 Hz, less than what it is believed to be the SLE threshold (1.5Hz). The Sétra/AFGC guidelines and the Eurocode EN1990-A2 have been adopted to set a criterion for maximum horizontal acceleration and an appropriate horizontal dynamic load model. The load shall be taken as:

$$f(t) = N_{eq} \cdot F_0 \cdot \sin(2\pi f_s t)$$

where:

-N_{eq} is the number of pedestrians (uniformly distributed and walking in phase with the same frequency as the footbridge) that produces tha same effect as random pedestrians;

-F_0 is taken as 35N;

-f_s is the frequency of the first horizontal mode (0.5495Hz).

For our case $N_{eq} = \begin{cases} 10.8\sqrt{N\xi} \ , \ \rho < 1 \frac{ped}{m^2} \\ 1.85\sqrt{N} \ , \ \rho \geq 1 \frac{ped}{m^2} \end{cases} = 39 \, pedestrians$, being ρ (density of pedestrian crossing the bridge) equal to 1 ped/m^2 (depicting a very dense traffic condition where freedom of movement is restricted and walking is obstructed, i.e. Traffic Class 4 according to HIVOSS guidelines) and thus N = 1*440m^2 = 440 pedestrians.

The dynamic point load has been spread all over the deck surface with a resulting amplitude of 3.1N/m^2. The forcing load per unit surface is then: $f(t) = 3.1 \frac{N}{m^2} \cdot \sin(2\pi \cdot 0.5495 \cdot t)$. The results are shown in the following figures.

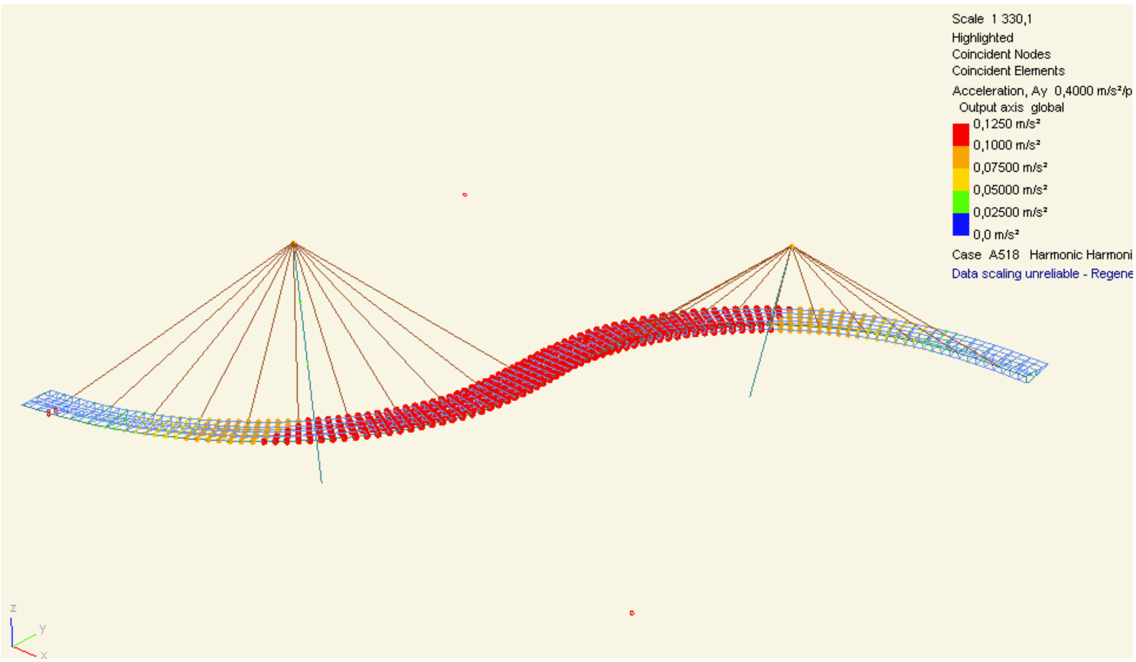


Fig.109 - Acceleration in y-direction

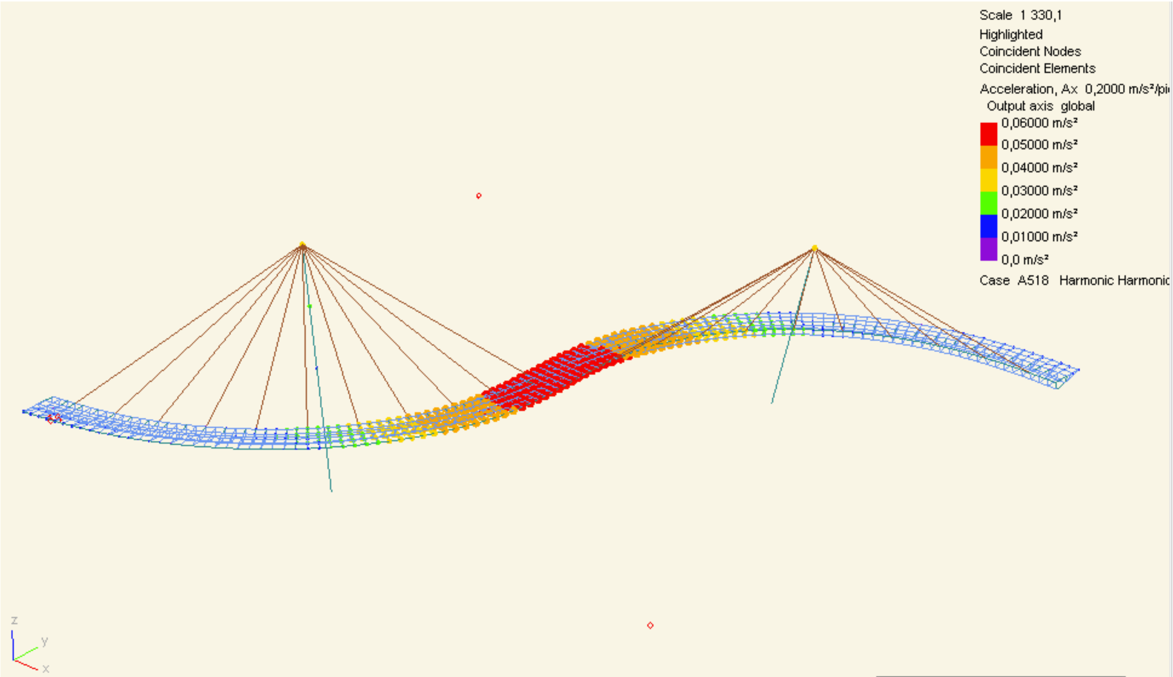


Fig.110 - Acceleration in x-direction

Maxima

1049	A518	*0,05492*	0,1239
995	A518	0,05332	*0,1243*
1270	A518	0,02087	0,1126
911	A518	0,04095	0,1233
1631	A518	0,03332	0,06267
1631	A518	0,03332	0,06267
1631	A518	0,03332	0,06267
1631	A518	0,03332	0,06267

Tab.28

Combining the components together we obtain a resultant maximum horizontal acceleration of 0.135254 m/s² and a RMS acceleration of 0.100772 m/s².

The Eurocode dynamic pedestrian load consists of a set of two concentrated vertical and horizontal loads (modeling a small group of people) and two distributed vertical and horizontal loads (representing a stream of pedestrians). Thus, for SLE assessment the following harmonic loads were used and the maximum response was recorded:

$$F_H = 70 \cdot k_H(f_H) \cdot \sin(2\pi f_H t) \quad [\mathrm{N}]$$

$$F_{s,H} = 4 \cdot k_H(f_H) \cdot \sin(2\pi f_H t) \quad [\frac{\mathrm{N}}{\mathrm{m}^2}]$$

where f_H is the natural frequency of the bridge closest to 1 Hz and $k_H(f_H)$ is a suitable coefficient depending on the frequency according to Fig.111:

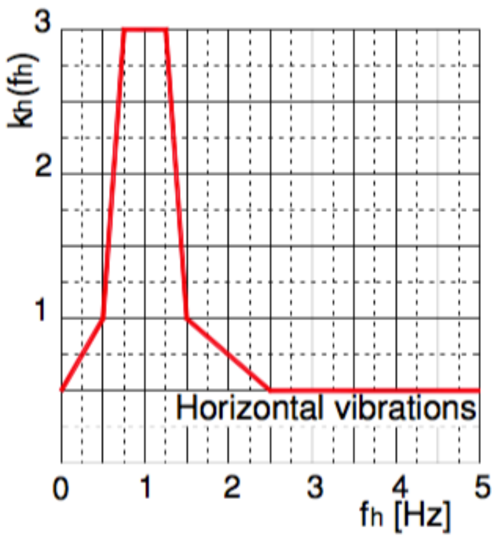


Fig.111

In our case we have:

$$f_H = 0{,}5495\ Hz$$

$$k_H = 2 \ \text{(safe-sided)}$$

The outputs are the nodal accelerations in the horizontal plane (Fig.112 and Fig.113):

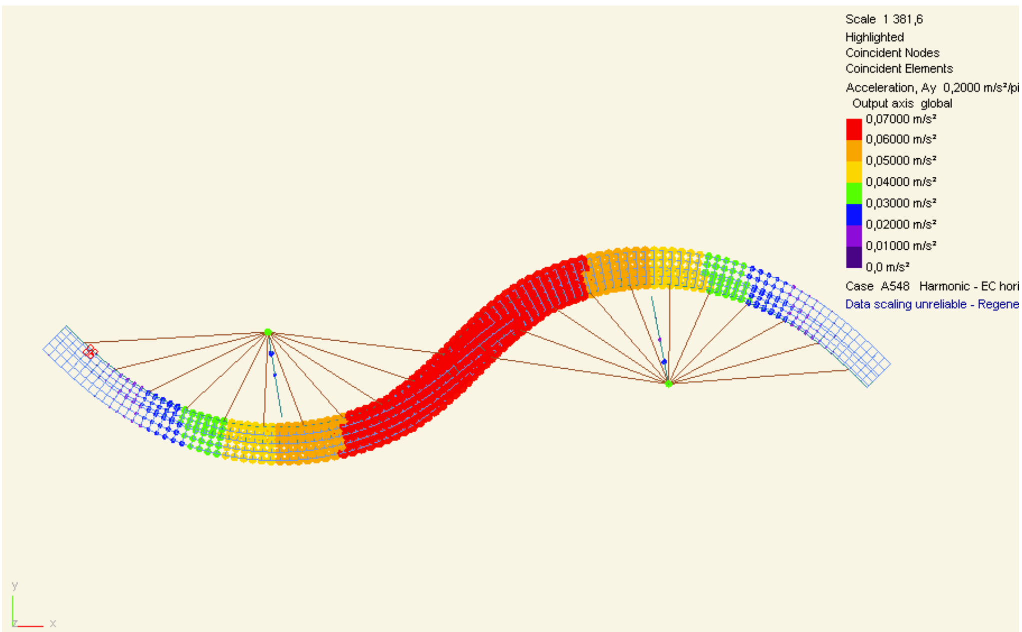


Fig.112 - concentrated horizontal load

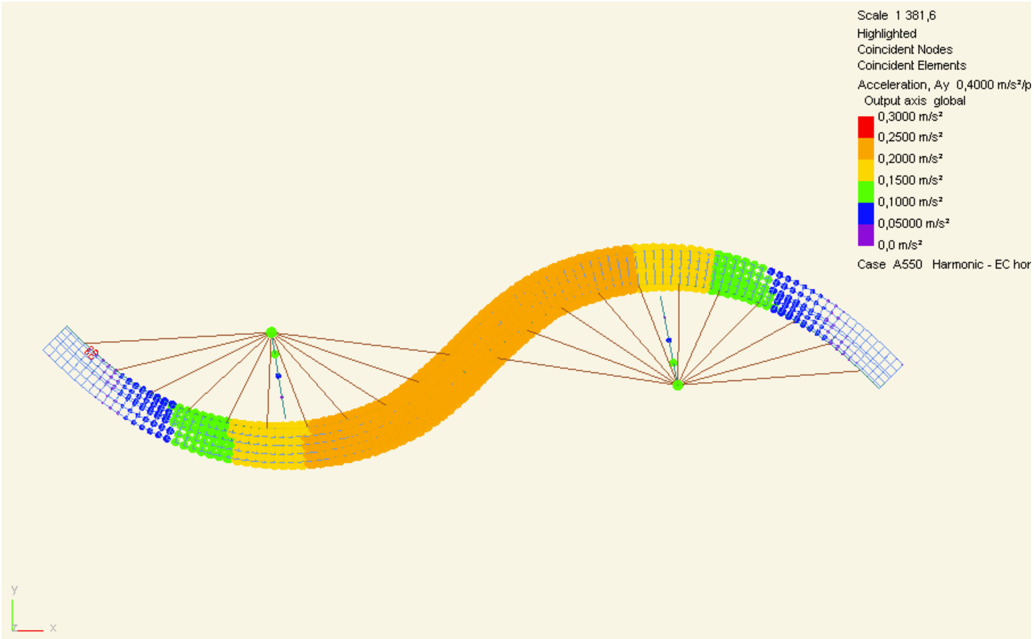


Fig.113 – distributed horizontal load

Maximum nodal accelerations are given in Tab.29:

Dynamic loading	Critical nodal acceleration [m/s²]
Horizontal concentrated (F_H)	0.0674
Horizontal distributed ($F_{s,H}$)	0.2477

Tab.29

By comparing the result obtained from the two pedestrian dynamic loading with the limit value for horizontal acceleration set by Sétra (0.1m/s²) and Eurocode (0.15m/s²), it appears that the nodal horizontal acceleration produced by Sétra loading is acceptable, while the second one is not. However, having in mind that the design damping ratio adopted is only an approximate value of the real damping (and so are the acceleration results based on that) and that the excitation forces were applied to the deck as if the bridge would have been loaded for its whole length with a stream of pedestrian walking at the same pace (i.e. a conservative choice), we can state that, with good chance, the structure will not experience unacceptable synchronous lateral oscillations.

Even if SLE was deemed to be likely to occur, several possible measures could be choosen to increase structural damping, such as viscous dampers, tuned mass dampers (TMD), pendulum dampers, tuned liquid column dampers (TLCD) or tuned liquid dampers (TLD), that would perfectly fit inside the box-girder cross section. An evaluation of the optimal distribution of the dampers along the span, though, lies outside the goal of this thesis.

7. Verification of members

The verification of structural members was conducted by using the limit states semi-probabilistic method and adopting the following recognized standards:

19. UNI EN 1536:2010.
20. UNI EN 1537:2013.
21. UNI EN 1990:2006.
22. UNI EN 1991:2004.
23. UNI EN 1992:2005.
24. UNI EN 1993:2005.
25. UNI EN 1997:2005.
26. UNI EN 1090-1:2012.
27. UNI EN 1090-2:2011.
28. British National Annexes to Eurocodes .
29. BS 5400.
30. BS 6841.
31. Sètra/AFGC footfall guideline.

7.1 Cable system

The stay-cables consist of a 85mm locked-coil strand. According to Bridon specifications the following loads represents the design loads that shall be used in the verification:

Product Code / Strand Diameter	Minimum Breaking Load	Design Load GR,d = MBL / 1,5 / 1,1	Nominal Metallic Cross Section	Nominal Axial Stiff-ness	Nominal Metallic Mass
d	MBL	GR,d	A	EA	Mass
mm	kN	kN	mm2	MN	kg/m
LC 20	368	223	254	42	2.04
LC 25	574	348	398	66	3.20
LC 30	858	520	594	98	4.77
LC 35	1170	709	808	133	6.49
LC 40	1580	958	1090	180	8.76
LC 45	2000	1212	1390	229	11.1
LC 50	2470	1497	1710	282	13.7
LC 55	3020	1830	2090	345	16.8
LC 60	3590	2176	2490	411	20.0
LC 65	4220	2558	2920	482	23.5
LC 70	4890	2964	3390	559	27.2
LC 75	5620	3406	3890	642	31.3
LC 80	6390	3873	4420	729	35.5
LC 85	7220	4376	5000	824	40.1

Tab.30

The cable-to–mast joint is made out of a one-piece melt steel shaped as a double curvature surface to which cables' sockets cylindrical extensions are anchored to. The special joint is conceived as the mast's cap so that the two elements act as a whole, being CJP welded together.

The geometry of the cantilevered arms was choosen so that they would fall onto the cables' surface, creating an elegant and efficient channel for stress-flow (from suspension to the load bearing systems).

The analyis of the special joint was conducted using a finite element sofware (ANSYS v.14.0).

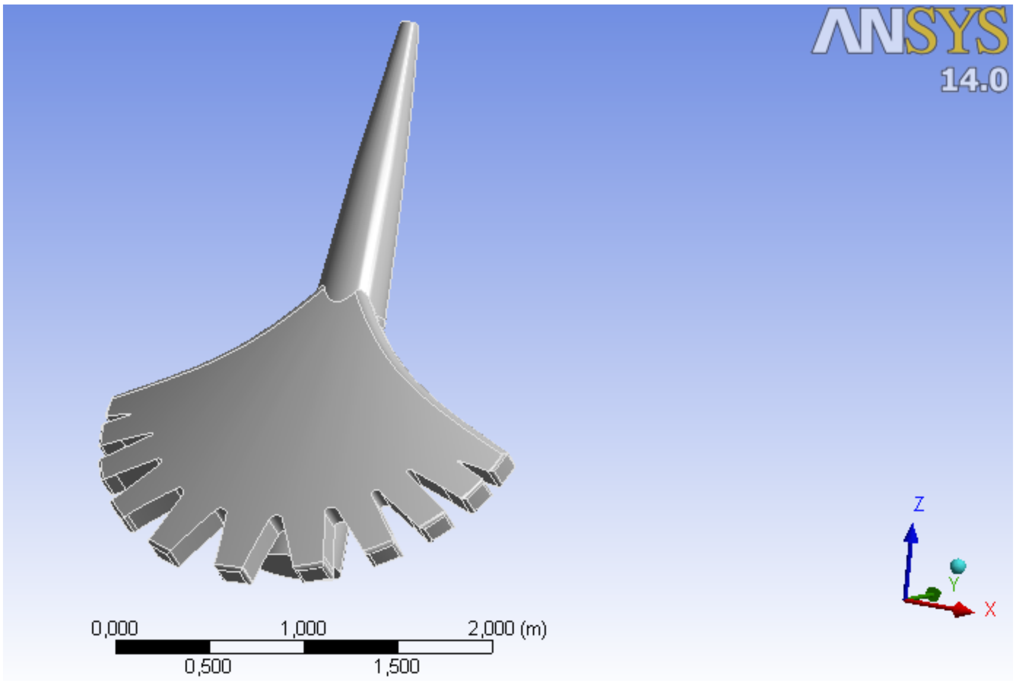


Fig.114

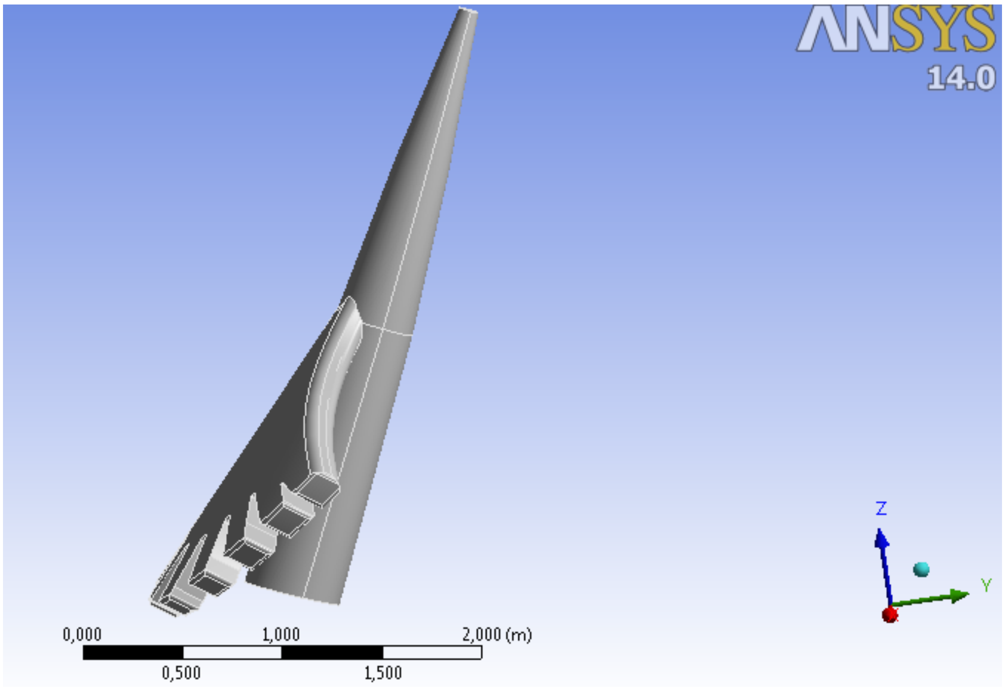


Fig.115

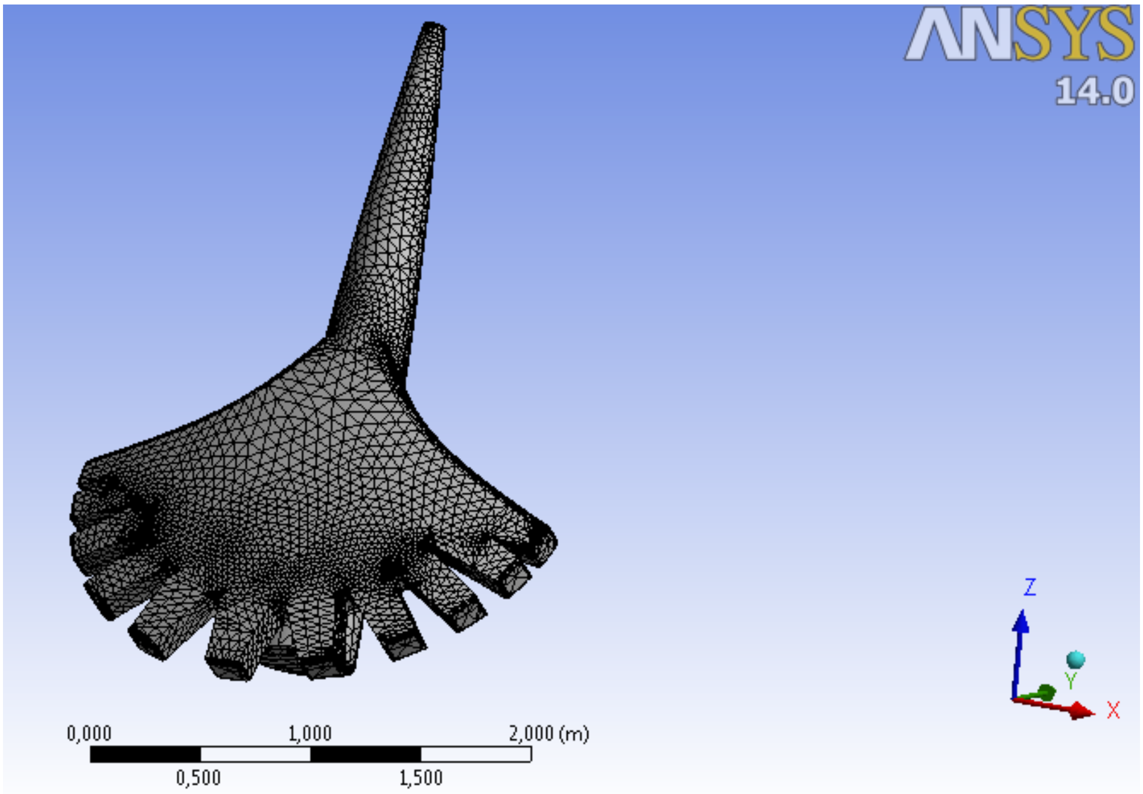


Fig.116

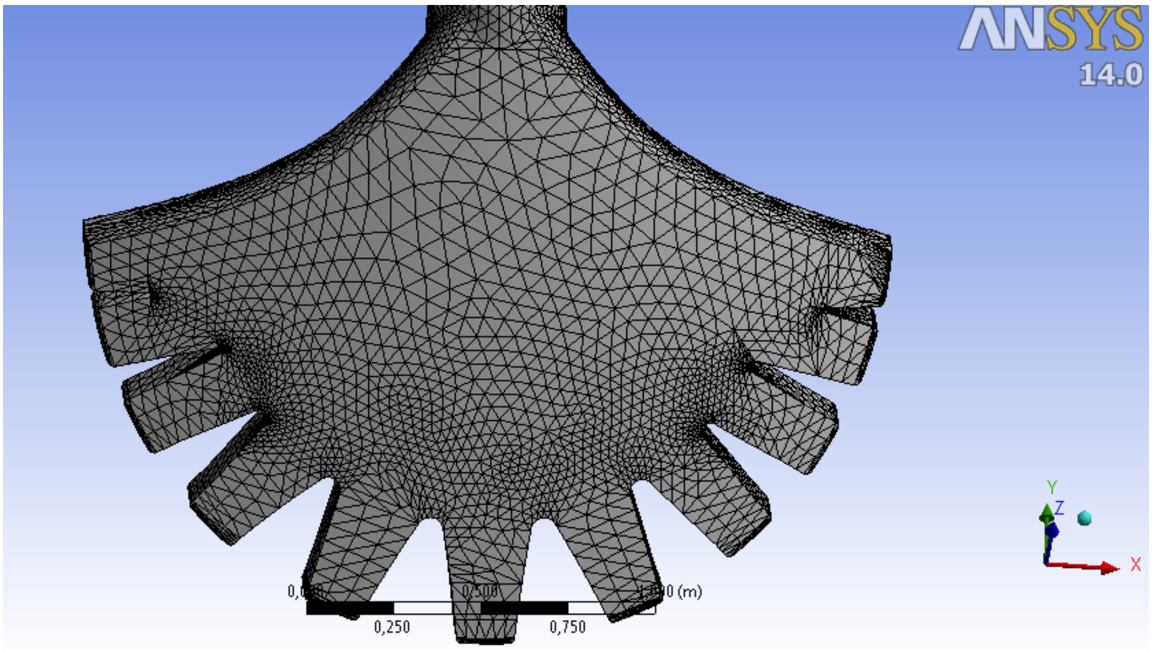


Fig.117

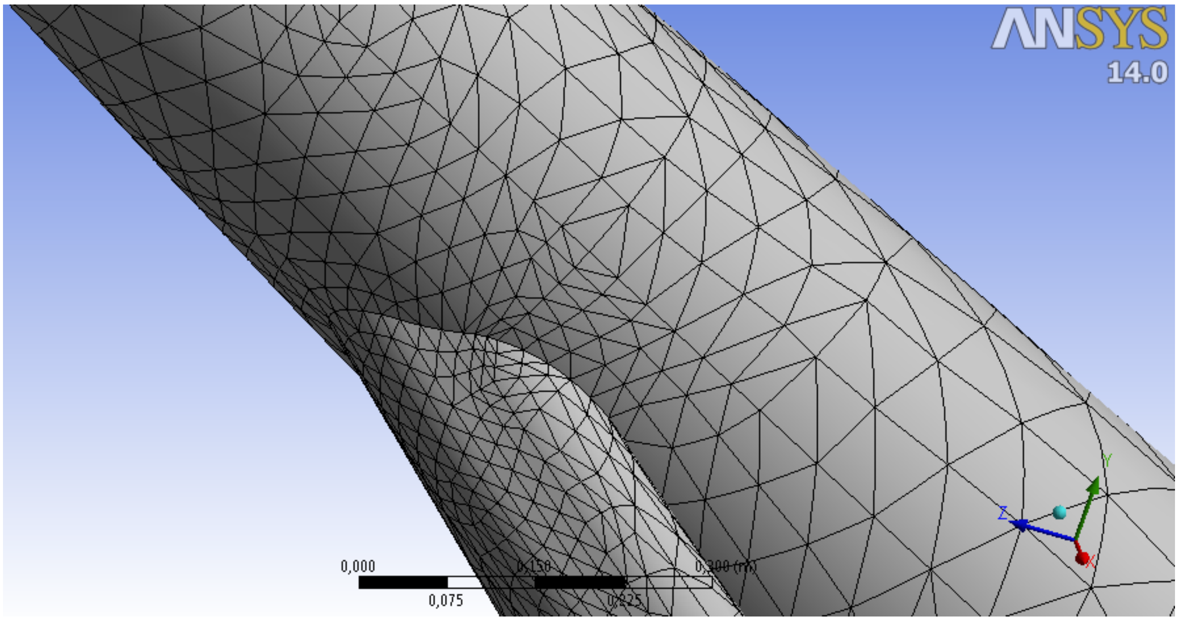


Fig.118

The mesh parameters have been choosen to get accurate results particularly in those regions where geometric complexity leads to stress concentrations.

The stress results and verification of the special joint are reported in the Annex 1.G2.

The cables are pinned to triangle gusset plates with variable tilt angles depending on the location on the deck. The gusset is attached to the frame diaphragm, whose function, besides increasing the torsional stiffness of the deck, is to transfer the tension load from the cables to the spine box-girder. This detail was also analysed with ANSYS.

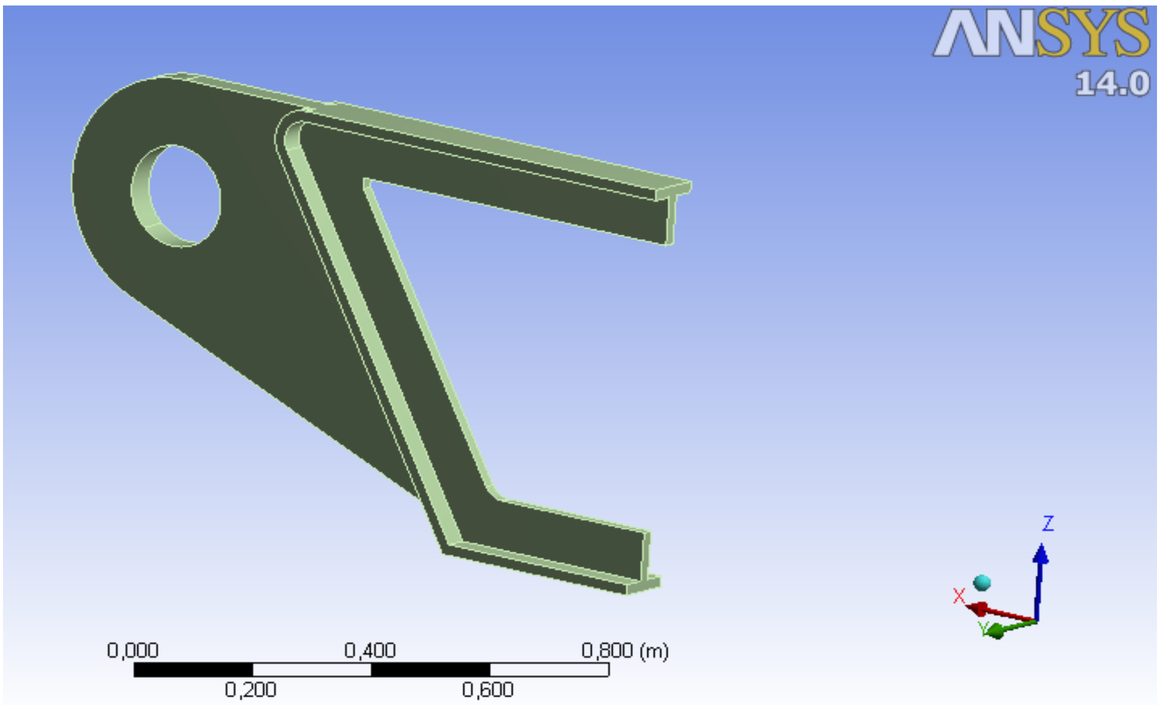


Fig.119

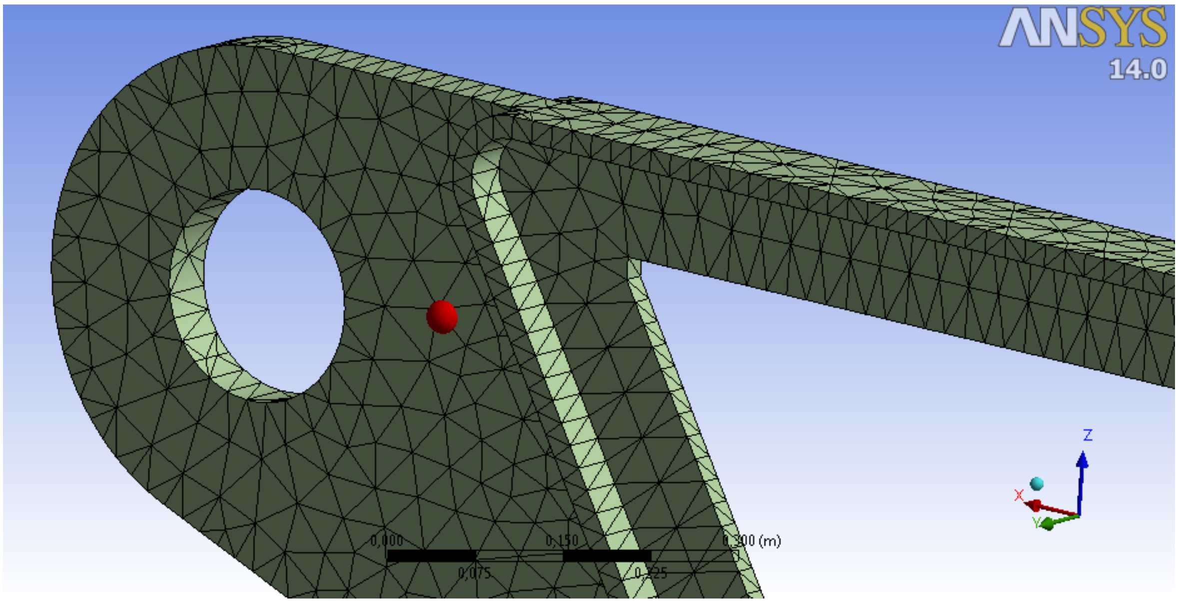


Fig.120

The verification of this second detail can be consulted in Annex 1.G1.

Cables' sockets are of two types. An adjustable fork socket was used at the deck-cable joint, while a standard fork socket connects the stays to the shell-shaped special piece through adapter bars. These last ones are fasten to circular pipes that are provided with a internal spiral groove, so that tensile stresses can flow with safety.

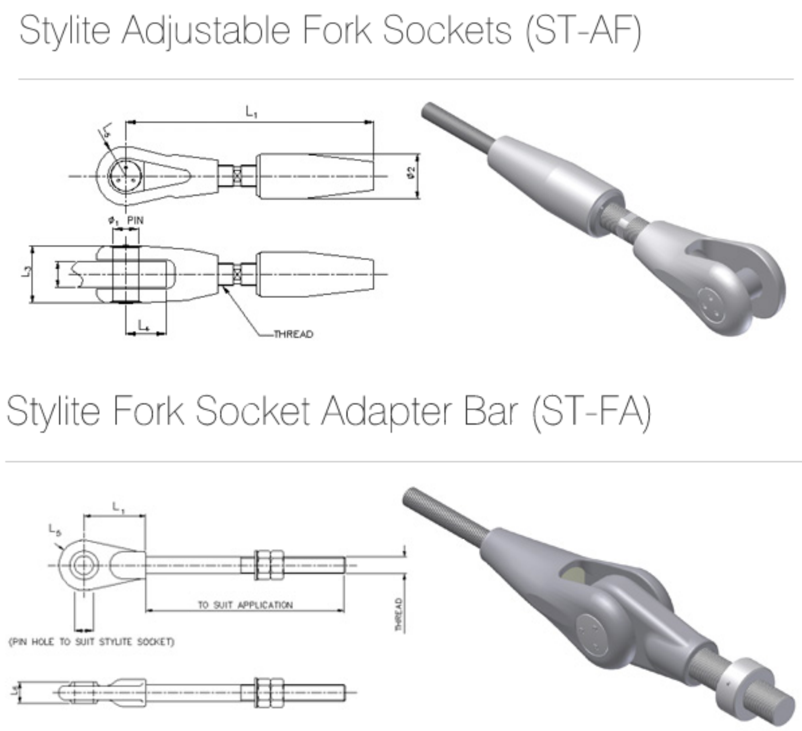


Fig.121

Product Code / Strand Diameter	L1		ADJT +/-	L3	L4	L5	L6		ø1	ø2	Thread Size	Weight
	Max	Min					Max	Min	Pin			Assy
mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	Metric	kg
ST-AF 25	631	531	50	121	73	58	57	55	45	87	M42 x 4.5	20
ST-AF 30	701	601	50	154	86	70	71	68	55	108	M52 x 5.0	36
ST-AF 35	815	695	60	175	100	82.5	85	79	65	120	M60 x 5.5	49
ST-AF 40	865	745	60	187	120	95	90	82	75	135	M68 x 6.0	69
ST-AF 45	894	774	60	210	124	100	95	90	80	145	M76 x 6.0	87
ST-AF 50	974	834	70	221	144	115	100	93	90	155	M80 x 6.0	112
ST-AF 55	1030	890	70	233	155	115	110	100	100	170	M90 x 6.0	131
ST-AF 60	1088	948	70	250	168	125	120	110	110	185	M95 x 6.0	163
ST-AF 65	1161	1021	70	267	187	137	129	115	120	205	M105 x 6.0	210
ST-AF 70	1247	1087	80	285	202	148	137	125	130	220	M105 x 6.0	260
ST-AF 75	1316	1156	80	307	211	154	147	135	135	235	M115 x 6.0	309
ST-AF 80	1416	1236	90	322	226	166	157	145	145	245	M120 x 6.0	365
ST-AF 85	1480	1300	90	348	240	177	167	155	155	265	M125 x 6.0	447

Tab.30

Stylite Fork Sockets (ST-F)

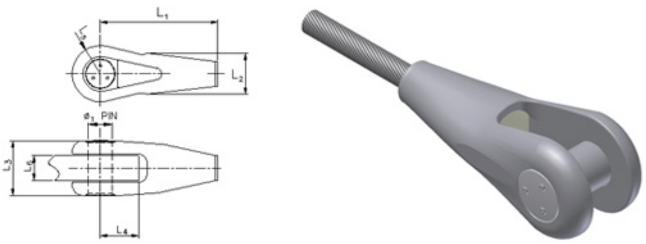
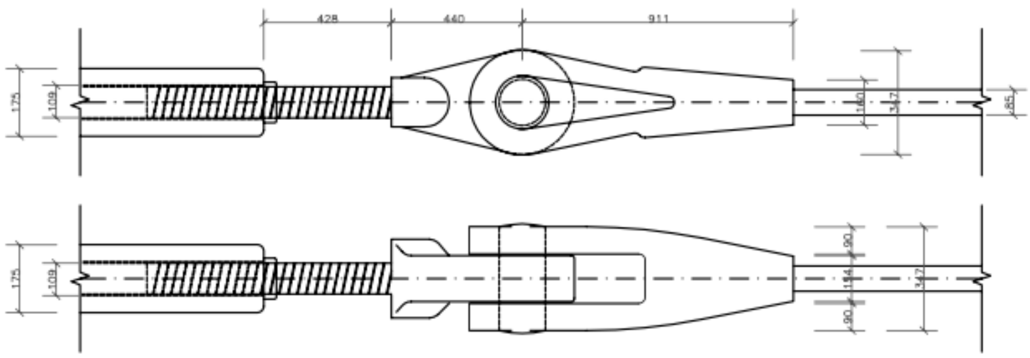


Fig.122

Product Code / Strand Diameter	L1	L2	L3	L4	L5	L6		ø1	Weight	
						Max	Min	Pin	Socket	Pin+Caps
mm	mm	mm	mm	mm	mm	mm	mm	mm	kg	kg
ST-F 25	275	88	121	73	58	57	55	45	8	1.7
ST-F 30	330	105	154	86	70	71	68	55	15.5	2.9
ST-F 35	385	120	175	100	82.5	85	79	65	22.5	4.7
ST-F 40	410	130	187	120	95	90	82	75	31.5	6.6
ST-F 45	420	145	210	124	100	95	90	80	43	8.4
ST-F 50	440	155	221	144	115	100	93	90	56	11
ST-F 55	477	168	233	155	115	110	100	100	60	14
ST-F 60	520	180	250	168	125	120	110	110	74	18
ST-F 65	565	195	267	187	137	129	115	120	94	24
ST-F 70	610	210	285	202	148	137	125	130	117	30
ST-F 75	645	225	307	211	154	147	135	135	141	34
ST-F 80	690	240	322	226	166	157	145	145	167	42
ST-F 85	735	255	348	240	177	167	155	155	209	51

Tab.31



Fig,123

7.2 Box-girder

The bridge spine consists of a trapezoidal asymmetric box-girder (panels' thickness between 16 and 20mm), stiffened with closed-section longitudinal ribs whose dimensions vary as a function of the span. The box is torsionally stiffened by a T-section frame diaphragm that runs along the external perimeter, allowing for a sufficient clear depth utilized for maintenance, lighting wires/pipes installation, etc.

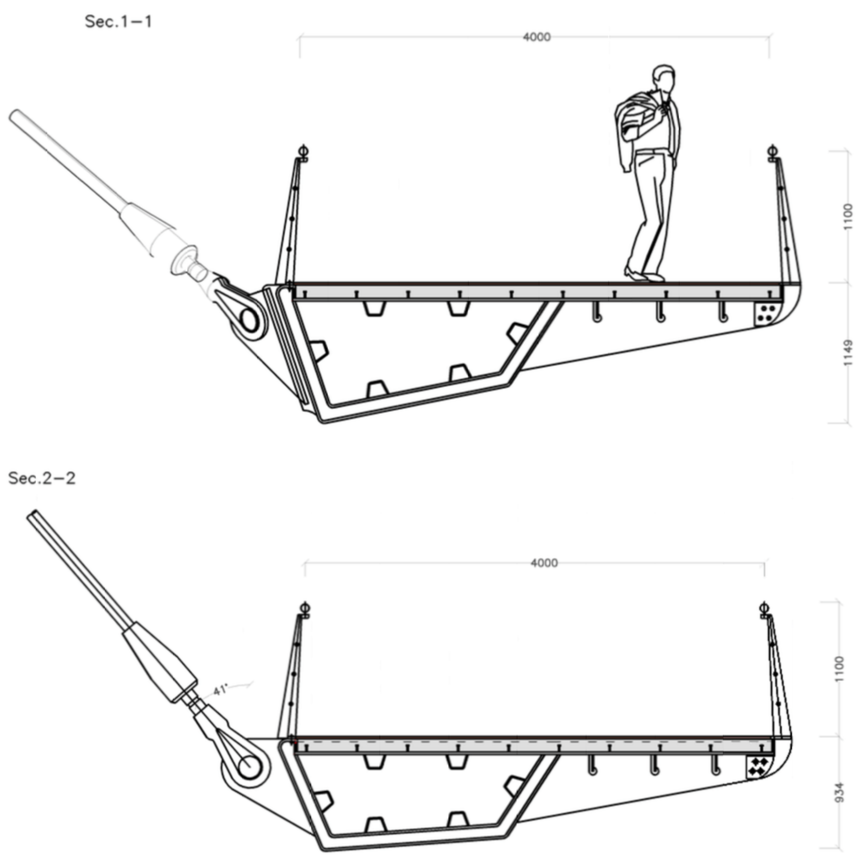
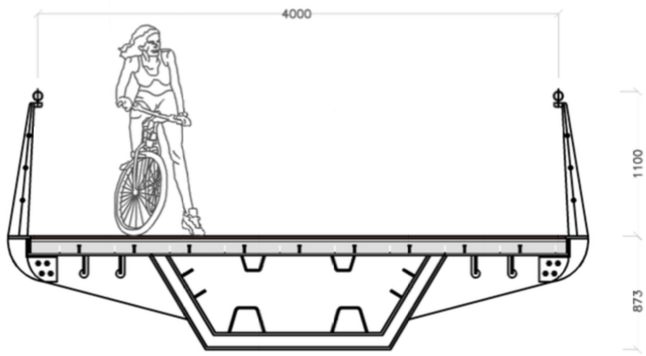


Fig.124

Sec.3–3



All box-girder panels were verified according to the relevant code and the results are reported in Annex 1A.

7.3 Cantilever plates

20mm-thick triangle plates cantilevers out from the box section a every 2m, with dimensions varying with the longitudinal abscissa. In addition to the standard verifications (reported in Annex 1.E), a flexural-torsional buckling analysis has been conducted via GSA modal buckling tool, where a finite element model with cut-outs has been built in order to assess the compliance with the relative limit state. Shear buckling is prevented by using sufficiently thick plates.

7.4 Cantilevered deck panels

Switching from one side of the deck to the other, an 18mm steel plate connected to a 120mm thick RC slab, is supported by the undelying cantilever plates. Buckling is prevented by using both open-section longitudinal stiffeners and the RC slab (the details on how the cooperation between the two element has been evaluated are treated in a following paragraph). Verifications of such elements are reported in Annex 1.B.

7.5 Masts

Two 40mm-thick CHS masts with a diameter varying from 400mm (at supports) to 800mm (at midspan), support the cables system. At the top node they are anchored to the stays via a special one-piece solid steel joint (7.1), while at the base node a pinned connection is realised with a ball-joint, as shown in Fig.125. Stresses in all the elements that masts are composed of were evaluated using a FE software (ANSYS v. 14.0).

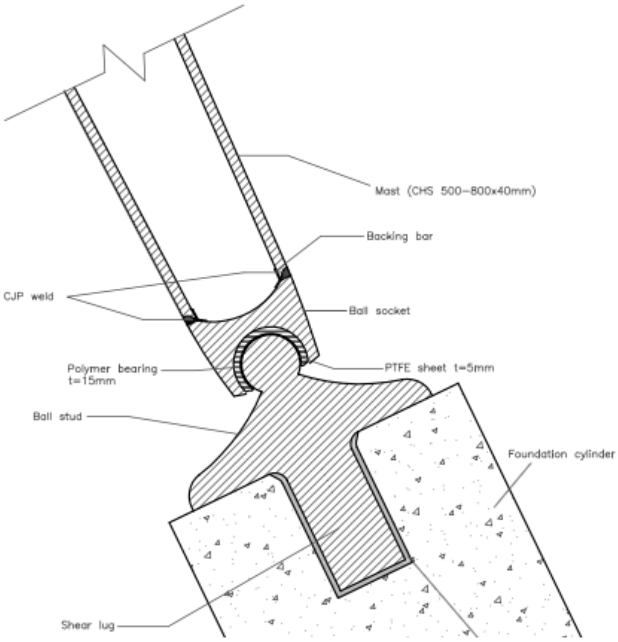


Fig.125

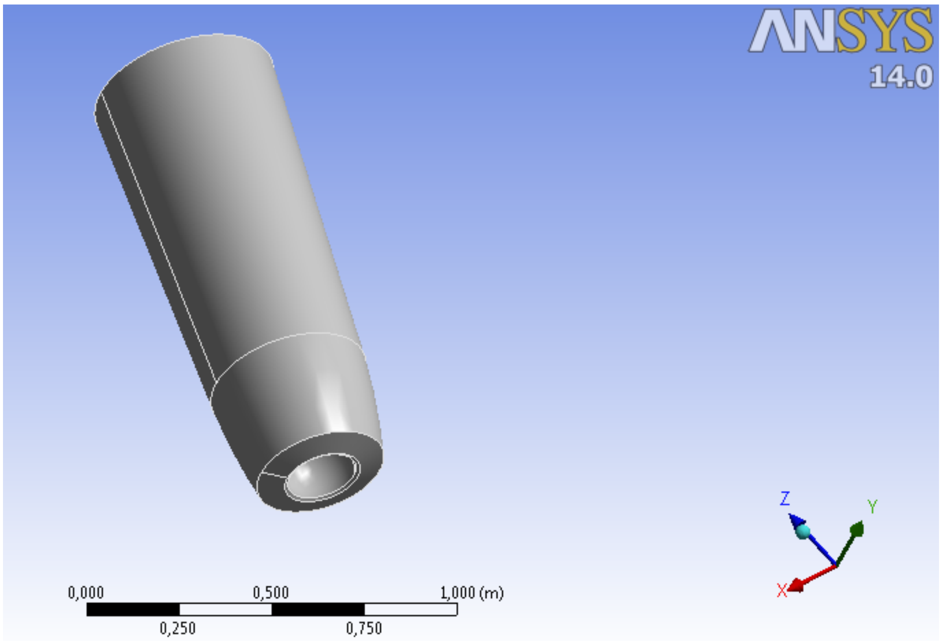


Fig.126

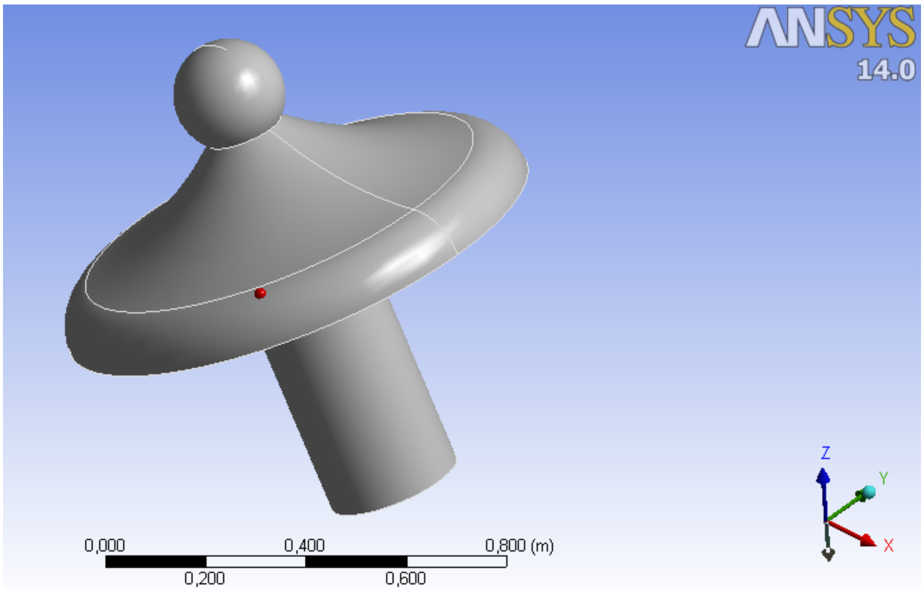


Fig.127

Since the choosen scheme leads to a formally ill-conditioned structure, buckling issues were treated with different approaches. A first step was used to assess the behaviour of an equivalent constant-diameter mast using the classic structural analysis methods. A second step consisted of a simplified FE model, where a single mast is restrained by the stay-cables which are pinned at their base. This way the model is more stable (bottom nodes of tie elements are pinned instead of being restrained by the deck), so that an equilibrium position could be found in the proximity of the initial configuration and, even if non-linear issues could still not be neglected, a faster calculation procedure and a preliminary check of the buckling behaviour were possible. Moreover, the initially found buckled geometry (i.e. the final geometry of a first non-linear load-increment analysis where the mast's axial load is increased until a transerve limit displacement occurs along the mast, Fig.128,129) was scaled and used as a geometric imperfection in the second analysis. This is due to the fact that a modal buckling analysis was not possible and, thus, no buckling modes shapes were available. A second load-increment analysis was, then, run on the new geometry. The output of this latter analysis is a Buckling P-Delta curve (Fig.130), where the mast's load factor (i.e. the load factor of a unit axial force of 10kN applied on top of the mast) is plotted against the transversal displacement of the critical mast's node (i.e. the midspan node).

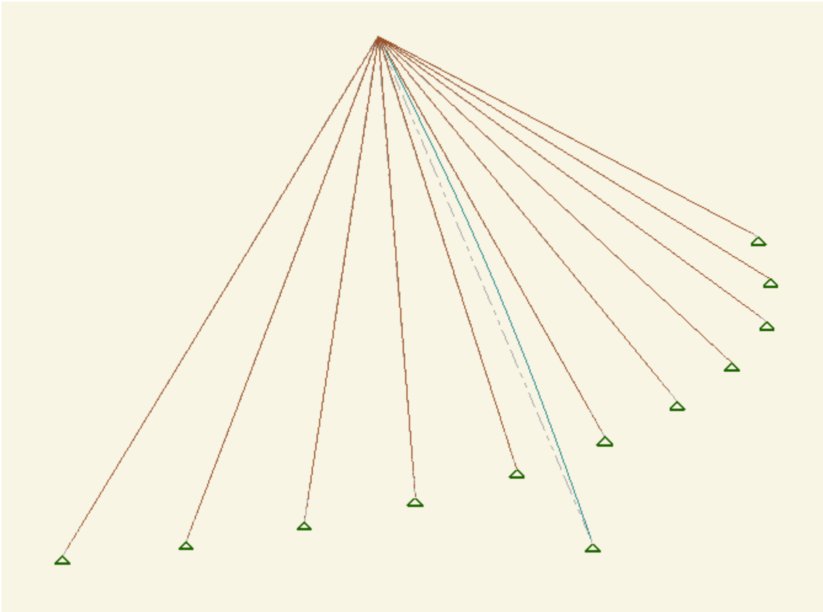


Fig.128

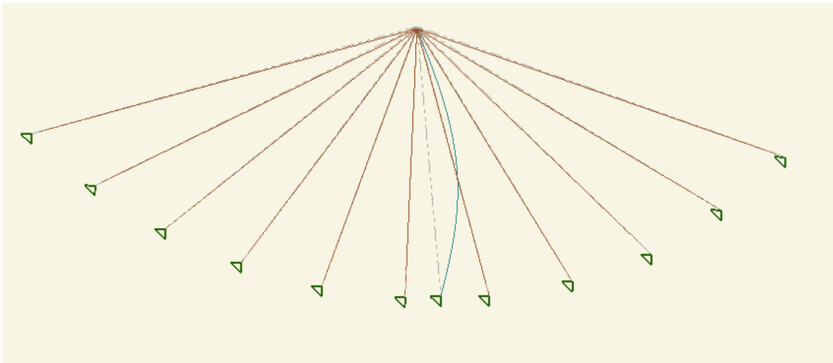


Fig.129

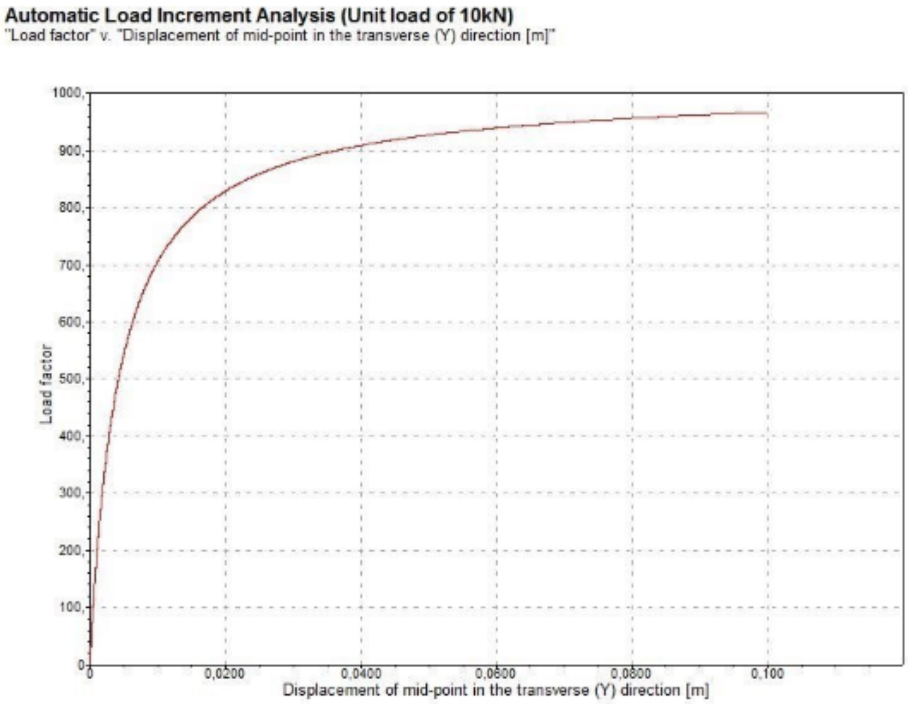


Fig.130

Being on the safe side, we should set a Load Factor limit up to 800 so that the corresponding displacement would be less than 20mm. Even by doing so we get a buckling load that is approximately 2.7 times the actual axial design load.

A third step consisted of a Buckling P-delta analysis run on the actual model, so that mast's effective restraint condition could be evaluated. The resulting curve is plotted below:

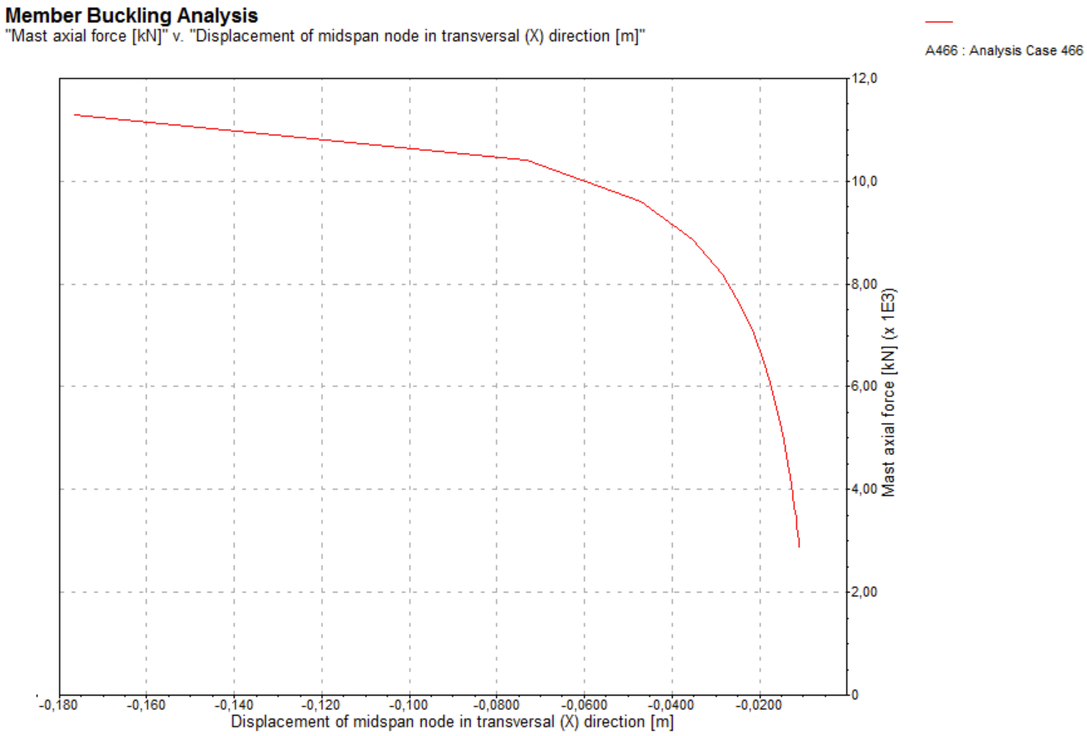


Fig.131

By limiting the mast's midspan node displacement to 20mm, a buckling load of approximately 6250kN is obtained, less than what resulted from the previously analysed simplified model (as expected).

A fourth step was then used to assess the possibility of a snap-through buckling picturing a failure scenario where mast displacements increase until unacceptable deflections occur at the deck nodes. The displacement limit was set equal to the one used for serviceability checks (Par. 5.3.2). Thus, a live load factor of 1.8 was used to calculate the corresponding axial force in the mast (2556kN), that was then compared to the buckling load derived from the third step.

The choosen wall thickness satisfies the axial stiffness and local buckling requirements. It also reflects the will to increase masts's selfweight in order to counterbalance wind overturning forces (as largly explained in Par. 7.9). Verifications related to these elements are reported in Annex 1.F.

7.6 Mast foundation

The mast's base lies on a deep foundation consisting of a 1.5m-deep pile cap and four 52cm-diameter drilled shafts. The shafts are inclined by 15° from the vertical direction to reduce their shear and moment demand. Compression flows from the mast to the pile cap through a concrete cylinder conveniently reinforced. Geotechnical and structural verifications are reported in Annex 1.I.

7.7 Abutment

The bridge abutments were designed to integrate with the river banks' environment. Since the embankment walls are 1.80m high, a minimum height of 3m was considered for the abutment elevation. A six-drilled-shafts-foundation (pile's diameter of 60cm) has been design to carry the vertical and horizontal loads coming from the superstructure. A 640x290x100cm pile cap distributes the loads among the shafts. Pedestrian/cycle access is garanteed by stairs and elevators, which are built-in with the abutment's elevation structures, reaching a full height of 4.65m.

7.8 Bearings

The only fixed node within the structure is at the abutment sections. This allows for the use of pinned masts, statisfying the architectural requirements. Since the only external torsional restraint is provided by the couple generated by the bearings (two for each abutment), the tension must be transfered to the foundations through a conveniently designed anti-lifting bearing, able to resist the design negative load (uplift force). This bearing would alse have to carry horizontal loads, becoming a fixed bearing. Additionally a multi-direction confined elastomeric disc bearing will be carrying the vertical loads on the opposite side of the abutment's stem wall. The geotechnical and structural verifications of the abutment structures are reported in Annex 1.L.

7.8 Fatigue resistance

In view of the fact that pedestrian loads may cause failure of the critical parts of the structure, a fatigue check was deemed necessary and was performed. For the fatigue check due to pedestrian loading, the static and dynamic components of the loads were considered. Static fatigue loads derive from the main load model used for assessing static resistance, where the load values are simply reduced to the frequent ones (multiplied by 0.4). The dynamic fatigue model consist of the loads used in the dynamic analyses illustrated in Chapter 6. Only modes 2 and 3 were consider, since other modes do not give a relevant contribution to the verification because of their low value of stress range in the elements and their high eigenfrequencies (modes with a natural frequency that falls outside the footfall pacing range). The two component cannot be added since this would result in a physical non-sense. In fact, this would involve the use of superposition principle to responses that were obtained from non-linear static and harmonic analyses. In addition stress ranges were the output of two different envelopes so that their combination was even further meaningless. An envelope of the results was recorder instead.

For each of the modeshapes the forcing load is in resonance and in phase with it, acting in the relevant directions, as shown in Fig.132 and 133.

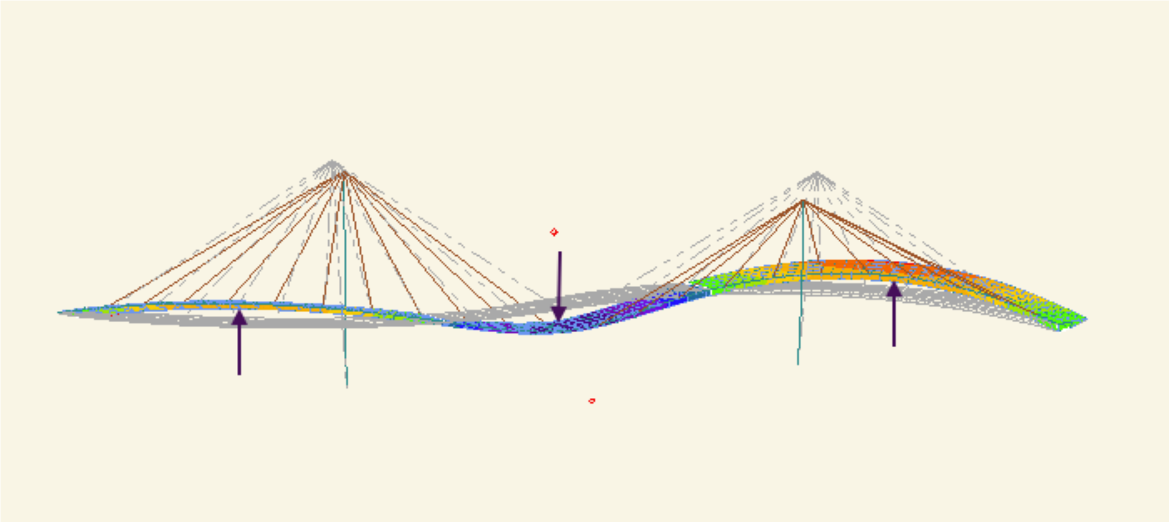


Fig.132

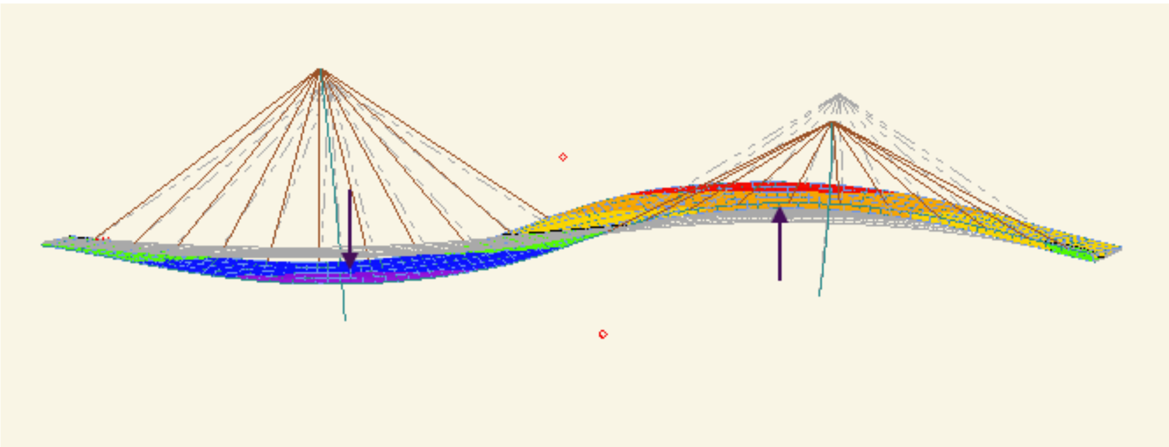


Fig.133

The boundless fatigue life verification was conducted according to UNI EN 1993-1-9, checking if the design maximum stress-range experienced by the structural elements of the deck and the cables system ($\Delta\sigma_{max,d}$) would reach the constant amplitude fatigue limit $\Delta\sigma_D$, being:

$$\Delta\sigma_{max,d} = \gamma_{Mf} \cdot \Delta\sigma_{max}$$

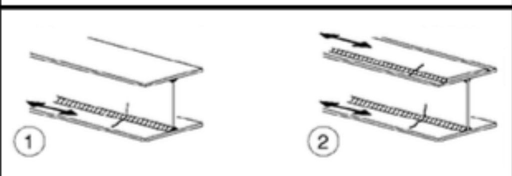
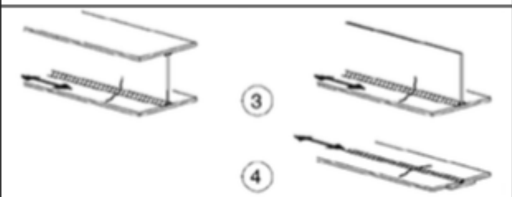
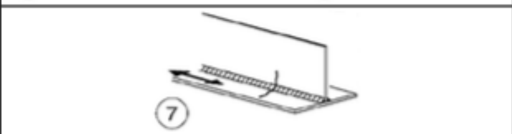
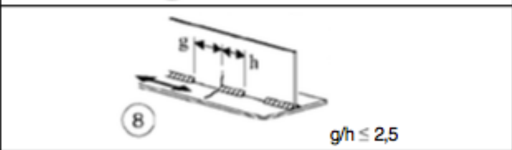
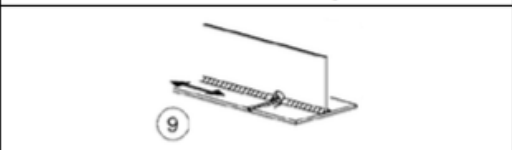
$$\Delta\sigma_D = 0.737 \cdot \Delta\sigma_c$$

where:

- γ_{Mf} can be obtained from Table ** (Table 3.1 UNI EN 1993-1-9)

Assessment method	Consequence of failure	
	Low consequence	High consequence
Damager tolerant	1.00	1.15
Safe life	1.15	1.35

- the reference value $\Delta\sigma_c$ can be selected from the appropriate code charts based on the structural detail and weld typology as well as the manifacturing operations.

Detail category	Constructional detail	Description	Requirements
125		<u>Continuous longitudinal welds:</u> 1) Automatic butt welds carried out from both sides. 2) Automatic fillet welds. Cover plate ends to be checked using detail 6) or 7) in Table 8.5.	<u>Details 1) and 2):</u> No stop/start position is permitted except when the repair is performed by a specialist and inspection is carried out to verify the proper execution of the repair.
112		3) Automatic fillet or butt weld carried out from both sides but containing stop/start positions. 4) Automatic butt welds made from one side only, with a continuous backing bar, but without stop/start positions.	4) When this detail contains stop/start positions category 100 to be used.
100		5) Manual fillet or butt weld. 6) Manual or automatic butt welds carried out from one side only, particularly for box girders	5), 6) A very good fit between the flange and web plates is essential. The web edge to be prepared such that the root face is adequate for the achievement of regular root penetration without break-out.
100		7) Repaired automatic or manual fillet or butt welds for categories 1) to 6).	7) Improvement by grinding performed by specialist to remove all visible signs and adequate verification can restore the original category.
80		8) Intermittent longitudinal fillet welds.	8) $\Delta\sigma$ based on direct stress in flange.
71		9) Longitudinal butt weld, fillet weld or intermittent weld with a cope hole height not greater than 60 mm. For cope holes with a height > 60 mm see detail 1) in Table 8.4	9) $\Delta\sigma$ based on direct stress in flange.

Tab.32

Group		Tension components	Detail category $\Delta\sigma_c$ [N/mm²]
A	1	Prestressing bars	105
B	2	Fully locked coil rope with metal or resin socketing	150
	3	Spiral strands with metal or resin socketing	150
C	4	Parallel wire strands with epoxy socketing	160
	5	Bundle of parallel strands	160
	6	Bundle of parallel wires	160

Tab.33

For what concerns box-girder and cantilever plates' details a value of $\Delta\sigma_c = 100$ MPa ($\Delta\sigma_D = 73$ MPa) was choosen, while for frame diaphragms $\Delta\sigma_c = 36$ MPa ($\Delta\sigma_D = 26$ MPa). Fully locked coil stay-cables' (with metal sockets) fatigue strength is represented by a $\Delta\sigma_c$ of 150 MPa ($\Delta\sigma_D = 110$ MPa).

A partial safety factor of 1.15 was choosen to amplify the design stress-range.

Static loading (load frequent combination)

Deck elements				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D = 73$ MPa
Top-deck plate	Orthotropic (t=16)+closed ribs	53.90	61.98	Verified
Lateral deck plate	Orthotropic (t=16)+closed ribs	56.50	64.97	Verified
Bottom-deck plate	Orthotropic (t=16)+closed ribs	62.00	71.30	Verified
Cantilever plate	t=20	59.40	68.31	Verified
Overhung plate	Orthotropic (t=16)+closed ribs	56.47	64.94	Verified
Stay-cables				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D = 110$ MPa
Stay-cable	Locked coil 85	95.62	109.93	Verified
Diaphragms				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D = 26$ MPa
Frame diaphragm	T-section (t=18)	13.20	15.20	Verified

MODE 2 – Dynamic loading

Deck elements				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D$ = 73 MPa
Top-deck plate	Orthotropic (t=16)+closed ribs	2.58	2.97	Verified
Lateral deck plate	Orthotropic (t=16)+closed ribs	4.33	4.98	Verified
Bottom-deck plate	Orthotropic (t=16)+closed ribs	4.11	4.73	Verified
Cantilever plate	Orthotropic (t=16)+closed ribs	7.94	9.13	Verified
Overhung plate	Orthotropic (t=16)+closed ribs	4.55	5.23	Verified
Stay-cables				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D$ = 110 MPa
Stay-cable	Locked coil 85	2.79	3.21	Verified
Diaphragms				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D$ = 26 MPa
Frame diaphragm	T-section (t=18)	5.35	6.15	Verified

MODE 3 – Dynamic loading

Deck elements				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D$ = 52 MPa
Top-deck plate	Orthotropic (t=16)+closed ribs	1.44	1.67	Verified
Lateral deck plate	Orthotropic (t=16)+closed ribs	1.52	1.75	Verified
Bottom-deck plate	Orthotropic (t=16)+closed ribs	1.56	1.79	Verified
Cantilever plate	Orthotropic (t=16)+closed ribs	3.20	3.68	Verified
Overhung plate	Orthotropic (t=16)+closed ribs	2.87	3.30	Verified

Stay-cables				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D = 111$ MPa
Stay-cable	Locked coil 85	2.21	2.54	Verified
Diaphragms				
Element	Cross section	$\Delta\sigma_{max}$ [MPa]	$\Delta\sigma_{max,d}$ [MPa]	$\Delta\sigma_D = 41$ MPa
Frame diaphragm	T-section (t=18)	3.55	4.08	Verified

7.9 Global equilibrium under transverse wind action

A special study has been carried out in order to assess the possibility of a rigid-body equilibrium loss. This condition is associated with specific horizontal wind loadings acting transversally with respect of the walking path. A load-increment analysis has been used, where, at each step, only 10% of the load is applied. The result of the analysis is a horizontal wind load factor that produces unacceptable vertical nodal displacement at the deck level.

The first loading condition is represented by a transverse uniform wind load acting on all of the superstructure's elements, deck, masts and cables (wind acting on balustades has been neglected, since a more advanced wind analysis would have been required) with the same sign, as shown in Fig.134.

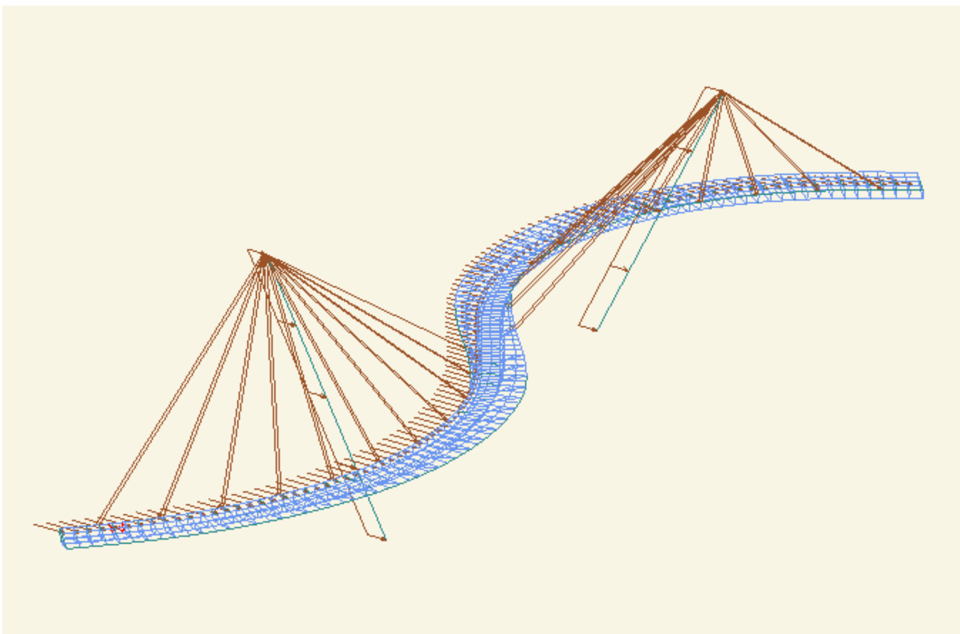


Fig.134

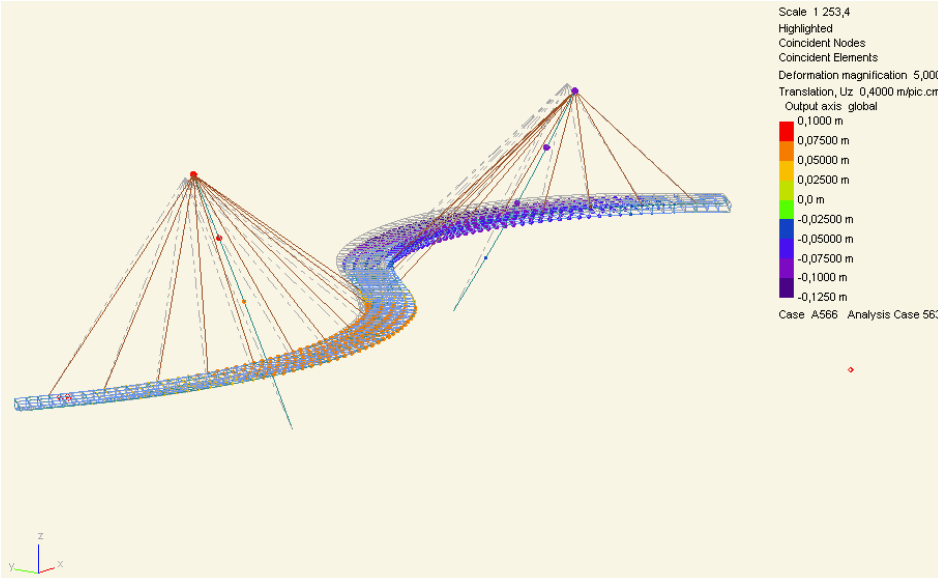


Fig.135

Surprisingly, the maximum vertical displacement occurs approximately at the quarter-span section that, under this particular loading, is supposed to be restrained by cables. This result is justified by observing that when such wind loads act on the bridge, the mast rotation causes a vertical translation that produces vertical displacement in the deck nodes. The maximum vertical displacement is equal to 0.09543m, corresponding to a wind load factor of 3 (judged sufficiently safe).

The second loading condition is represented by a centrally-symmetric transverse uniform wind load, as shown in Fig.136.

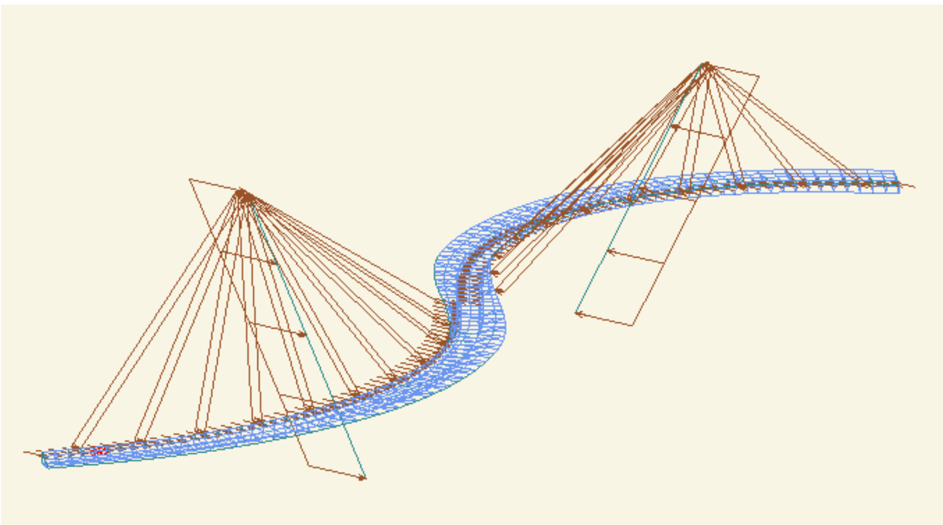


Fig.136

This particular loading was selected so that the mast top horizontal movement would be maximize.

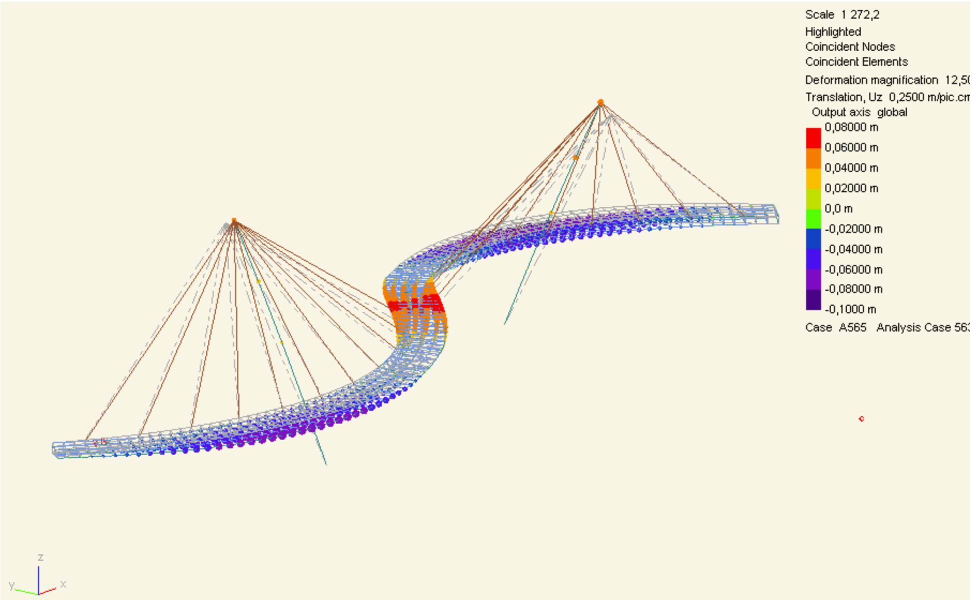


Fig.137

The maximum vertical displacement experienced by the bridge is equal to 0.08318m, corresponding to a wind load factor of 3 (as above).

This results ruled out any possibility of equilibrium loss due to forces acting in a critical direction with respect of restraining condition.

7.10 Top box panel steel-RC plate

The top deck panel consists of a longitudinally stiffened steel plate. Closed-section ribs belong to the width that correspond to the box-girder top panel, while open-section stiffeners were used for the cantilevered width. A C 35 concrete slab is fixed to the steel plate via shear studs, making the global panel act as a composite plate.

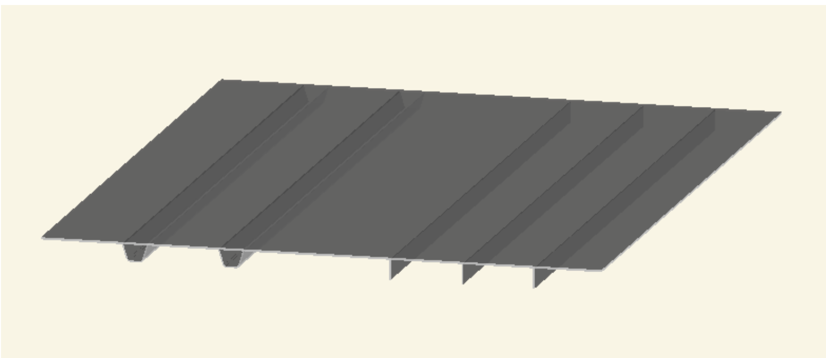


Fig.138

In order to understand the buckling behaviour of such a stiffened plate and to be able to design the shear connectors appropriately, a FE model (Fig.139) was built and pin restraints were defined at each stud location (Fig.140).

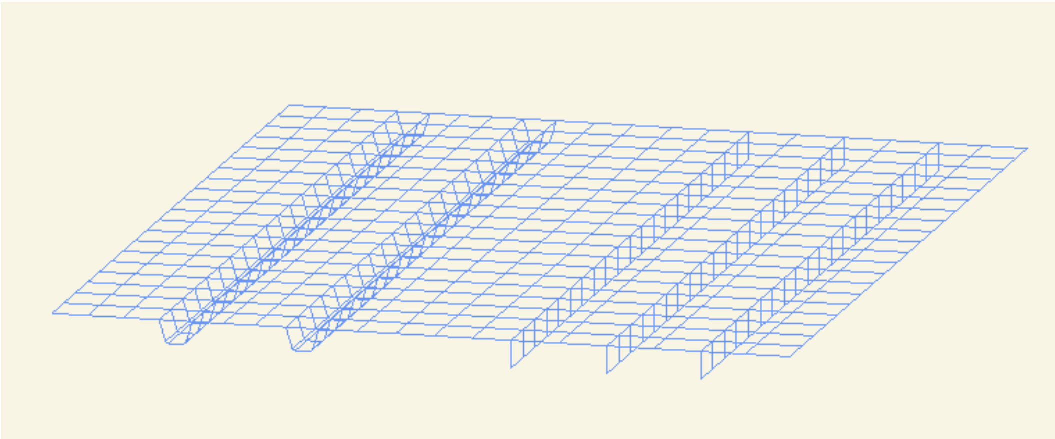


Fig.139

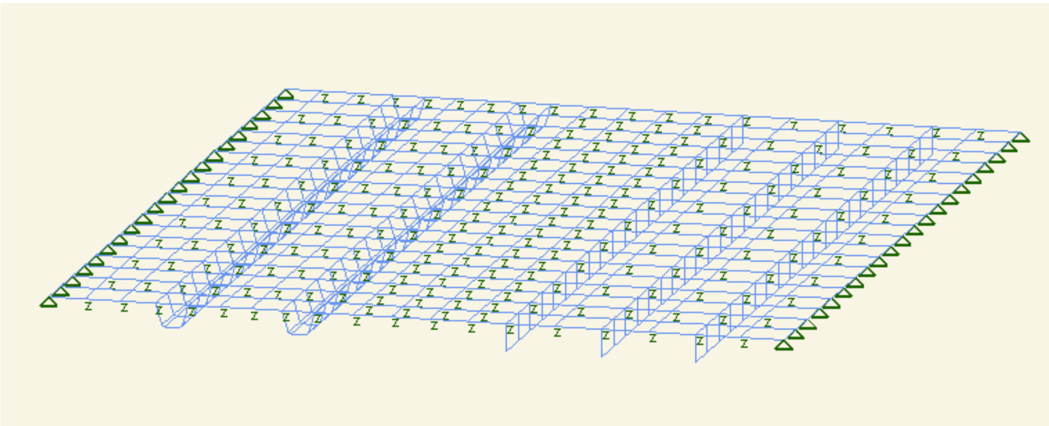


Fig.140

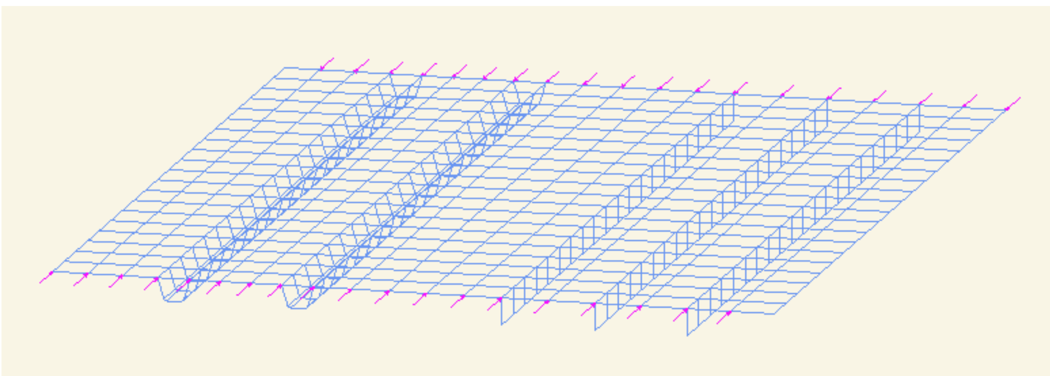


Fig.141

The panel was loaded with an edge distributed stress of 355MPa (equal to the strength of the steel used) and a modal buckling analysis was run. In order to estabilish the maximum vertical reaction among the studs (i.e. the maximum tension force that would produce a separation of the concrete and the steel layers), the first buckling mode shape geometry was adopted as geometric imperfection and the new structure is loaded again. A design load of 10.25 kN controls the design of the studs since a minimum stud-concrete adherence resistance of such an amplitude is required.

Being N_{Rd} the adherence resistance for a single stud to be compared with the tension force:

$$N_{Rd} = f_{bd} \cdot \pi \cdot \phi \cdot L \,, \qquad \text{where:}$$

- $f_{bd} = \frac{f_{bk}}{\gamma_c} = \frac{2.25 \cdot \eta \cdot f_{ctk}^{\frac{2}{3}}}{1.5 \cdot \gamma_c} = \frac{2.25 \cdot 1 \cdot 0.3 \cdot 35^{\frac{2}{3}} \cdot 0.7}{1.5 \cdot 1.5} = 2.246 \ \frac{N}{mm^2}$;
- ϕ is the stud's diameter;
- L is the stud's length.

Nelson 19mm-diameter, 8mm long, improved-adherence studs (Fig.142) were used, spaced 200mm in the transverse and 300mm in the longitudinal direction.

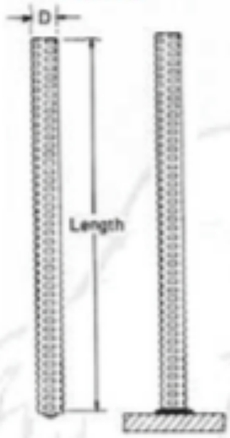
The verification is satisfied since : $\frac{N_{Ed}}{N_{Rd}} = \frac{10.25}{10.72} = 0.96 < 1.$

NELSON STUD WELDING

SPECIFICATION: D2L Deformed Bar Anchors (DBA)

✔ Check Standard Stock

D2L



Nelson deformed bar anchors deliver full tension capacity when embedded according to code requirements and provide specified shear strength when embedded at proper edge distances and spacing between bars. Nelson deformed bar anchors are used for deep embedment anchors in such applications as precast columns, tee and beam connections, seismic shear walls and securing steel plates to concrete structures. Nelson deformed bars meet requirements of the following codes and are also USNRC approved:

AWS D1.1 Structural Welding Code – Steel
ASTM A – 496 Steel Wire, Deformed, For Concrete Reinforcement
ACI-318 Building Code Requirements for Structural Concrete
Precast/Prestresssed Concrete Institute Design Handbook
Canadian Standards Association
International Building Code Section 19

See ICBO Evaluation Report ER-5217

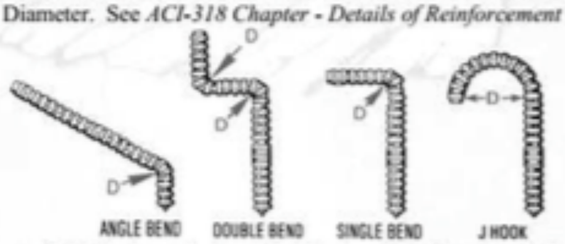
For similar function studs, see Nelson B4L Reinforcing Standoff Support studs, H4L Headed Concrete Anchors, J2L "J" Bolt studs, and S3L Shear Connectors.

When ordering, specify Type, Diameter, Before Weld Length, Material, Quantity, and Part Number
***Example:* D2L 1/2 x 18-1/8"; Mild Steel; 5000 pieces #101064765**

Stud Diameter	Burn Off	Recommended Standard Accessories			
		Chuck	Grip	Ferrule	Foot
3/8 10 mm	0.125 3 mm	500001011	501003009	100101099	502002001*
1/2 13 mm	0.125 3 mm	500001014	501003010	10010114	502002002*
5/8 16 mm	0.187 4 mm	500001016	501003014	10010187	502002002*
3/4 19 mm	0.187 4 mm	500001018	501003019	10010152	502002009

* Feet 502002001 and 502002002 are used with Nelson's heavy duty gun.
Feet 502002045 and 502002046 is used with Nelson's standard duty gun.

Nelson deformed bar anchors are also available on special order bent to the following shapes. The minimum bend diameter on the standard hooks shown is 6 x Bar Diameter and on 135° tie hooks 4 x Bar Diameter except for 3/4" bar which should be 6 x Bar Diameter. See *ACI-318 Chapter - Details of Reinforcement* for additional requirements.



MATERIALS: Studs are available in Low Carbon Mild Steel and Stainless Steel. For specific grade information and physical and chemical properties, and conforming standards, please see General Material Specifications. Certified Material Test Reports (CMTR) and Certificates of Compliance (COC) are available and must be requested at time of order.

For special ferrules and grips used in welding at an angle to plate, welding to angles, and welding to a vertical base plate, see the Special Applications section of the Ferrule Specifications.

FLUX: All Nelson deformed bar anchors have a solid flux load.

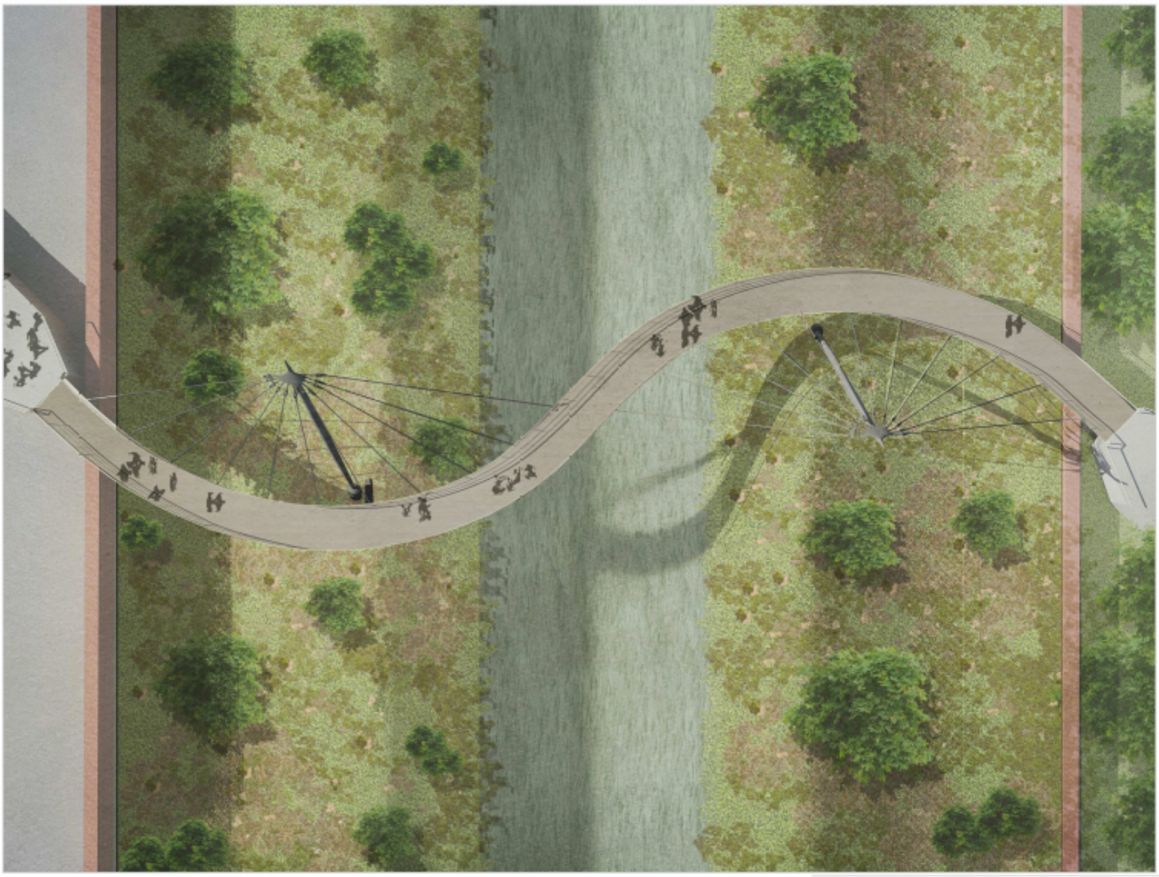
Fig,142

8. Conclusions

A new cable-stayed footbridge over river Arno was designed, focusing on the critical aspects of geometric and structural optimization, structural dynamics and vibration control. An architecturally bold design was choosen in order to fulfill both aesthetic and recreational requirements, in a continuous dialogue with the needs of the linked municipalities.

General analysis and design of the structural members were conducted according to the European, British and Italian building codes, while for what concerns specific aspects of design, as the pedestrian-induced vibrations, relevant guidelines were adopted. Different finite elements models were created in order to assess the global and local behaviours of the structure, with the help of GSA Oasys (v.8.7) and ANSYS (v.14) softwares. Analytical tools were developed to research solutions for the optimization process, scientifically supporting the architectural choices and leading to an efficiency-based design.

This project is a humble attempt to make the two approaches of structural design coexist. Architectural inspiration deeply influences the bridge's nature. Structural analysis and optimization guarantee that a solution to the problem exist and it is optimal. Both aspects affect each other, celebrating an holistic approach that has driven the whole design process, from the global layout to the smallest details.



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My sincere thanks also go to Alberto Carlucci, whose technical expertise helped this design project become a feasible reality, and to Marco Malloggi, who helped me develop an architectural inspiration that I now see as a fundamental basis of any challenging structural project.

I would like to thank my parents and my family for supporting me during these years of study in such a way that I always felt I could reach my goals without any pressure and with all the encouragement that I needed. I thank them for being always willing to grant me great opportunities, letting me become, day by day, more passionate about what I was doing.

Last but not least, I thank all of my italian and international friends for the great time we spent together during these years. My deepest gratitute especially goes to those who are, were and will always be part of my Pisa "family", a crowd of incredibly-awesome people, that in all these years turned out to be such a strong and beautiful company.

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A. Box panels verification

***Box top panel** (stiffened by trapezoidal stiffeners)*

Cross-section (1) - Anchorage of the sixth stay-cable (from abutments)

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1+\nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 88.08\ MPa$ $\sigma_{Ed.cmin} := 3.68\ MPa$

Tension: $\sigma_{Ed.t} := 100.50\ MPa$

Shear max: $\tau_{Edmax} := 43.28\ MPa$

Shear min: $\tau_{Edmin} := 4.50\ MPa$

Von Mises: $\sigma_{VonMises} := 90\ MPa$

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 2000\ mm$

Plate thickness: $t_{pl} := 16\ mm$

STIFFENERS

Number of stiffeners: $n_{st} := 4$

Thickness: $t_{st} := 11\ mm$

Major width: $B_{st} := 185\ mm$

Minor width: $b_{st} := 108\ mm$

Depth: $d_{st} := 150\ mm$

Inclined width: $a := \sqrt{{d_{st}}^2 + \left(\frac{B_{st} - b_{st}}{2}\right)^2} = 154.862\ mm$

Angle: $\theta := \mathrm{acos}\left(\frac{d_{st}}{a}\right) = 0.251\ rad$

Center of gravity (stiffened plate):

$$y_G := \frac{n_{st} \cdot b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2}\right) + 2 \cdot n_{st} \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2}\right)}{n_{st} \cdot (b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}) + a_2 \cdot t_{pl}} = 36.836\ mm$$

Second moment of area (plate):

$$I_{pl} := \left(a_2 \cdot \frac{{t_{pl}}^3}{12} + a_2 \cdot t_{pl} \cdot {y_G}^2\right) \cdot \frac{1}{(1-\nu^2)} = \left(4.847 \cdot 10^7\right)\ mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{{t_{st}}^3}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G\right)^2 + 2 \cdot \left(a \cdot \frac{{t_{st}}^3}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2} - y_G\right)^2\right) = \left(2.362 \cdot 10^7\right)\ mm^4$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(1.429 \cdot 10^{8}\right)\ mm^{4}$$

Properties for equivalent thickness
(see Point C) :

$$I_{TD} := I_{tot} = \left(1.429 \cdot 10^{8}\right)\ mm^{4}$$

$$A_{TD} := n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}\right) + a_2 \cdot t_{pl} = \left(5.038 \cdot 10^{4}\right)\ mm^{2}$$

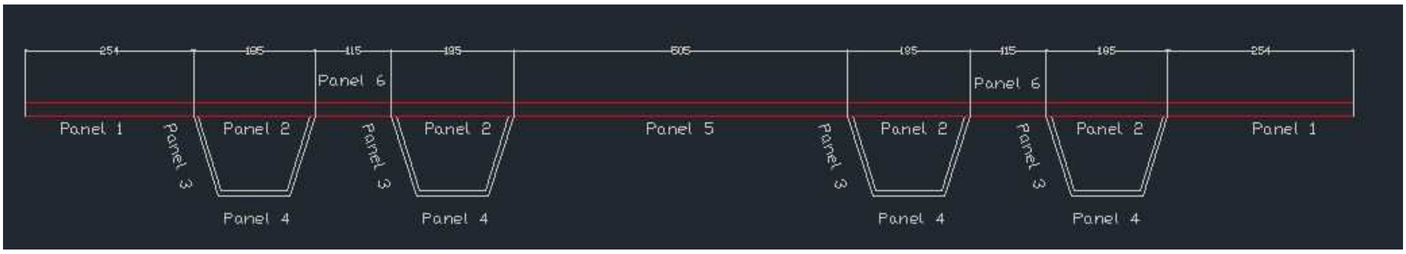
$h := 500\ mm$ *(total depth of box section)*

$$y_{G.TD} := h - \left(y_G + \frac{t_{pl}}{2}\right) = 455.164\ mm$$

(distance b/w G of box top panel and bottom panel external surface)

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels



Panel 5 (INTERNAL ELEMENT)

Panel width: $b_{pl5} := 505\ mm$

Panel thickness: $t_{pl5} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 5} := 4$

Slenderness:

$$\lambda_{p5} := \frac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$$

Reduction factor:
('internal compression element')

$$\rho_5 := \begin{cases} 1 & \text{if } \lambda_{p5} \le 0.673 \\ \dfrac{\lambda_{p5} - 0.055 \cdot (3 + \psi)}{\lambda_{p5}^{2}} & \text{else} \end{cases} = 0.993$$

Effective panel width: $b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 501.234\ mm$

Effective panel area: $A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(8.02 \cdot 10^{3}\right)\ mm^{2}$

Panel 6 (INTERNAL ELEMENT)

Panel width: $b_{pl6} := 115\ mm$

Panel thickness: $t_{pl6} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 6} := 4$

Slenderness: $\lambda_{p6} := \dfrac{b_{pl6}}{t_{pl6} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma6}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_6 := \begin{cases} 1 & \text{if } \lambda_{p6} \le 0.673 \\ \dfrac{\lambda_{p6} - 0.055 \cdot (3 + \psi)}{\lambda_{p6}^{\ 2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl6} := \rho_6 \cdot b_{pl6} = 115\ mm$

Effective panel area: $A_{eff.pl6} := b_{eff.pl6} \cdot t_{pl6} = \left(1.84 \cdot 10^3\right)\ mm^2$

Panel 1 (INTERNAL ELEMENT)

Panel width: $b_{pl1} := 254\ mm$

Panel thickness: $t_{pl1} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma1} := 4$

Slenderness: $\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma1}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_1 := \begin{cases} 1 & \text{if } \lambda_{p1} \le 0.673 \\ \dfrac{\lambda_{p1} - 0.055 \cdot (3 + \psi)}{\lambda_{p1}^{\ 2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 254\ mm$

Effective panel area: $A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(4.064 \cdot 10^3\right)\ mm^2$

Panel 2 (INTERNAL ELEMENT)

Panel width: $b_{pl2} := B_{st} = 185\ mm$

Panel thickness: $t_{pl2} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma2} := 4$

Slenderness: $\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \begin{cases} 1 & \text{if } \lambda_{p2} \le 0.673 \\ \dfrac{\lambda_{p2} - 0.055 \cdot (3 + \psi)}{\lambda_{p2}^{\ 2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 185\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(2.96 \cdot 10^3\right)\ mm^2$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := a = 154.862\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 3} := 4$

Slenderness:
$$\lambda_{p3} := \frac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 3}}\right)}$$

Reduction factor:
('internal compression element')
$$\rho_3 := \begin{cases} 1 & \text{if } \lambda_{p3} \le 0.673 \\ \dfrac{\lambda_{p3} - 0.055 \cdot (3+\psi)}{\lambda_{p3}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 154.862\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(1.703 \cdot 10^3\right)\ mm^2$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := b_{st} = 108\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness:
$$\lambda_{p4} := \frac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$$

Reduction factor:
('internal compression element')
$$\rho_4 := \begin{cases} 1 & \text{if } \lambda_{p4} \le 0.673 \\ \dfrac{\lambda_{p4} - 0.055 \cdot (3+\psi)}{\lambda_{p4}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 108\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.188 \cdot 10^3\right)\ mm^2$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(1.838 \cdot 10^4\right)\ mm^2$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.838 \cdot 10^4\right)\ mm^2$

Total effective section under compression: $A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + 2 \cdot A_{eff.pl6} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(4.598 \cdot 10^4\right)\ mm^2$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + 2 \cdot b_{pl6} \cdot t_{pl6} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(4.604 \cdot 10^{4}\right)\ mm^{2}$$

Area ratio: $$\delta := \frac{A_{sl}}{t_{pl} \cdot a_2} = 0.574$$

Plate aspect ratio: $$\alpha := \frac{a_1}{a_2} = 1$$

Inertial ratio: $$\gamma := \frac{I_{tot}}{a_2 \cdot \dfrac{t_{pl}^{3}}{12\left(1-\nu^2\right)}} = 190.541$$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} = 122.932$$

$$k_{\sigma.p} := \begin{array}{l} \text{if } \alpha \le \sqrt[4]{\gamma} \\ \quad \dfrac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} \\ \text{else} \\ \quad \dfrac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1)\cdot(1+\delta)} \end{array} = 122.932$$

Euler plate buckling stress: $$\sigma_E := \pi^2 \cdot E \cdot \frac{t_{pl}^{2}}{12 \cdot \left(1-\nu^2\right) \cdot a_2^{2}} = 12.147\ MPa$$

Critical elastic plate buckling stress: $$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = \left(1.493 \cdot 10^{3}\right)\ MPa$$

Slenderness of stiffened plate: $$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c} = 0.999$$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.487$$

$$\rho := \begin{array}{l} \text{if } \lambda_{p\_} \le 0.673 \\ \quad 1 \\ \text{else} \\ \quad \dfrac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{\lambda_{p\_}^{2}} \end{array} = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener 1 (b/w panel 5 and panel 6)

$$b_{I.inf1} := \frac{3-\psi}{5-\psi} \cdot b_{pl6} = 57.5\ mm \qquad b_{I.sup1} := \frac{2}{(5-\psi)} \cdot b_{pl5} = 252.5\ mm$$

$$b_{II.inf1} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm \qquad b_{II.sup1} := \frac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$$

$$A_{sl1} := b_{I.inf1} \cdot t_{pl6} + b_{I.sup1} \cdot t_{pl5} + b_{II.inf1} \cdot t_{pl2} + b_{II.sup1} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(1.251 \cdot 10^{4}\right)\ mm^{2}$$

Angle: $$\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.251\ rad$$

Center of gravity (column):

$$y_{G.c1} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl1}} = 37.071\ mm$$

Second moment of area (column):

$$I_{sl1'} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^{2} + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^{2} + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^{2} + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^{2}\right)$$

$$I_{sl1''} := b_{I.inf1} \cdot \frac{t_{pl1}^{3}}{12} + b_{I.sup1} \cdot \frac{t_{pl6}^{3}}{12} + b_{II.inf1} \cdot \frac{t_{pl2}^{3}}{12} + b_{II.sup1} \cdot \frac{t_{pl2}^{3}}{12} + b_{I.inf1} \cdot t_{pl1} \cdot y_{G.c1}^{2} + b_{I.sup1} \cdot t_{pl6} \cdot y_{G.c1}^{2} + b_{II.inf1} \cdot t_{pl2} \cdot y_{G.c1}^{2} + b_{II.sup1} \cdot t_{pl2} \cdot y_{G.c1}^{2}$$

$$I_{sl1} := I_{sl1'} + I_{sl1''}$$

Elastic critical column buckling stress: $$\sigma_{cr.sl1} := \pi^{2} \cdot E \cdot \frac{I_{sl1}}{A_{sl1} \cdot a_{1}^{2}} = \left(1.43 \cdot 10^{3}\right)\ MPa$$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $$\sigma_{cr.c1} := \sigma_{cr.sl1} = \left(1.43 \cdot 10^{3}\right)\ MPa$$

Column-stiffener 2 (b/w panel 1 and panel 6)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 127\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl6} = 57.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl6} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(1.051 \cdot 10^{4}\right)\ mm^{2}$$

Angle: $$\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.251\ rad$$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl2}} = 44.156\ mm$$

Second moment of area (column):

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^{2} + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^{2} + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^{2} + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^{2}\right)$$

$$I_{sl2''} := b_{I.inf2} \cdot \frac{t_{pl1}^{3}}{12} + b_{I.sup2} \cdot \frac{t_{pl6}^{3}}{12} + b_{II.inf2} \cdot \frac{t_{pl2}^{3}}{12} + b_{II.sup2} \cdot \frac{t_{pl2}^{3}}{12} + b_{I.inf2} \cdot t_{pl1} \cdot y_{G.c2}^{2} + b_{I.sup2} \cdot t_{pl6} \cdot y_{G.c2}^{2} + b_{II.inf2} \cdot t_{pl2} \cdot y_{G.c2}^{2} + b_{II.sup2} \cdot t_{pl2} \cdot y_{G.c2}^{2}$$

$$I_{sl2} := I_{sl2'} + I_{sl2''}$$

Elastic critical column buckling stress: $$\sigma_{cr.sl2} := \pi^{2} \cdot E \cdot \frac{I_{sl2}}{A_{sl2} \cdot a_{1}^{2}} = \left(1.733 \cdot 10^{3}\right)\ MPa$$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1-------->

$$\sigma_{cr.c2} := \sigma_{cr.sl2} = \left(1.733 \cdot 10^{3}\right)\ MPa$$

Slenderness of column *(column with the least critical stress is considered, i.e. column 1)*:

$$\sigma_{cr.c} := min\left(\sigma_{cr.c1}, \sigma_{cr.c2}\right) = \left(1.43 \cdot 10^{3}\right)\ MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := \frac{b_{eff.pl6}}{2} \cdot t_{pl6} + b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.248 \cdot 10^{4}\right)\ mm^{2}$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 0.998$$

Slenderness of column:

$$\lambda_c := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.c}}} = 0.498$$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 100.969\ mm$$

Distance:

$$e_1 := y_{Gst} - y_{G.c1} = 63.897\ mm$$

Distance between plate and column centroids:

$$e_2 := y_{G.c1} = 37.071\ mm$$

$$e := \max\left(e_1, e_2\right) = 63.897\ mm$$

$$\alpha := 0.34$$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 52.53\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.449$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + \lambda_c^{2}\right) = 0.691$$

Reduction factor:

$$\chi_c := \frac{1}{\phi + \sqrt{\phi^{2} - \lambda_c^{2}}} = 0.855$$

Interaction between plate and column buckling behaviours

$$\xi_{\_} := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = 0.044$$

$$\xi := \begin{array}{l} \text{if } \xi_{\_} \le 0 \\ \quad \| \ 0 \\ \text{else if } \xi_{\_} \ge 1 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \xi_{\_} \end{array} = 0.044$$

Final reduction factor: $\rho_c := (\rho - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c = 0.867$

Effective cross-sectional area: $A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \dfrac{t_{pl1}}{2} = \left(4.395 \cdot 10^4\right) mm^2$

Axial stress resistance verification

$$A := a_2 \cdot t_{pl} + n_{st} \cdot (2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}) = \left(5.038 \cdot 10^4\right) mm^2$$

$\eta_{1.c} := \dfrac{\sigma_{Ed.cmax} \cdot A}{\dfrac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.284$ < 1 OK (compression)

$\eta_{1.t} := \dfrac{\sigma_{Ed.t}}{\dfrac{f_y}{\gamma_{M1}}} = 0.311$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$\eta := 1.2$ (S355)

Effective widths for shear check (internal stiffener):

$$b_{2.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl6}}{2}\right) = 57.5\ mm$$

$$b_{3.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 195.268\ mm$$

Effective width for shear check (external stiffener):

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl6}}{2}\right) = 57.5\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 127\ mm$$

Second moment of area (internal stiffener):

$$y_{G1} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + (b_{1.1} + 2 \cdot b_{2.1} + b_{3.1}) \cdot t_{pl}} = 39.998\ mm$$

$$I_{sl1'} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G1}\right)^2 + 2 \cdot b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G1}\right)^2$$

$$I_{sl1''} := (b_{1.1} + b_{2.1}) \cdot \frac{t_{pl}^{\ 3}}{12} + (b_{1.1} + b_{2.1}) \cdot t_{pl} \cdot y_{G1}^{\ 2} + (b_{2.1} + b_{3.1}) \cdot \frac{t_{pl}^{\ 3}}{12} + (b_{2.1} + b_{3.1}) \cdot t_{pl} \cdot y_{G1}^{\ 2}$$

$$I_{sl1} := I_{sl1'} + I_{sl1''}$$

Second moment of area (external stiffener):

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + (b_{1.2} + 2 \cdot b_{2.2} + b_{3.2}) \cdot t_{pl}} = 44.156\ mm$$

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^{2} + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^{2} + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^{2} + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^{2}\right)$$

$$I_{sl2''} := (b_{1.2} + b_{2.2}) \cdot \frac{t_{pl}^{3}}{12} + (b_{1.2} + b_{2.2}) \cdot t_{pl} \cdot y_{G2}^{2} + (b_{2.2} + b_{3.2}) \cdot \frac{t_{pl}^{3}}{12} + (b_{2.1} + b_{3.1}) \cdot t_{pl} \cdot y_{G2}^{2}$$

$$I_{sl2} := I_{sl2'} + I_{sl2''}$$

$$I_{sl} := min\,(I_{sl1}, I_{sl2}) = (2.98 \cdot 10^{7})\ mm^{4}$$

Shear buckling coefficient:

$$k_{\tau sl} := \begin{cases} 9 \cdot \left(\frac{a_2}{a_1}\right)^{2} \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^{3}} & \text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^{2} \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^{3}} \ge \frac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\frac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} \\ \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\frac{I_{sl}}{a_2}} & \text{else} \end{cases} = 1.517 \cdot 10^{3}$$

$$k_{\tau} := \begin{cases} 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^{2} + k_{\tau sl} & \text{if } \frac{a_1}{a_2} \ge 1 \\ 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^{2} + k_{\tau sl} & \text{else} \end{cases} = 1.526 \cdot 10^{3}$$

Slenderness:

$$\lambda_{w1} := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau}}} = 0.105$$

Sub-panels

<u>Panel 1</u> $a_2 := b_{pl1} = 254\ mm$

Shear buckling coeff.:

$$k_{\tau.p1} := \begin{cases} 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^{2} & \text{if } \frac{a_1}{a_2} \ge 1 \\ 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^{2} & \text{else} \end{cases} = 5.405$$

Slenderness:

$$\lambda_{w.p1} := \frac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.224$$

Panel 2 $a_2 := b_{pl2} = 185\ mm$

Shear buckling coeff.:
$$k_{\tau.p2} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.374$$

Slenderness:
$$\lambda_{w.p2} := \frac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.164$$

Panel 5 $a_2 := b_{pl5} = 505\ mm$

Shear buckling coeff.:
$$k_{\tau.p5} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.595$$

Slenderness:
$$\lambda_{w.p5} := \frac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.439$$

Panel 6 $a_2 := b_{pl6} = 115\ mm$

Shear buckling coeff.:
$$k_{\tau.p6} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.353$$

Slenderness:
$$\lambda_{w.p6} := \frac{a_2}{t_{pl6} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p6}}} = 0.102$$

Final slenderness:

$$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}, \lambda_{w.p6}\right) = 0.439 \qquad < \qquad \frac{0.83}{\eta} = 0.692$$

$$\chi_w := \eta$$

Shear stress resistance verification

$$\eta_3 := \frac{\tau_{Edmax}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.065 \qquad < 1 \qquad \text{OK}$$

AXIAL-SHEAR STRESS INTERACTION

$$\eta_{1.cm} := \frac{\frac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.148$$

$$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$$

Verification $\eta_{1.cm} + \left(2 \cdot \eta_{3m} - 1\right)^2 = 1.011$ < 1 OK

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

(assuming that stiffeners act as rigid support)

$\lambda_w := \max\left(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}, \lambda_{w.p6}\right) = 0.439$ < $\frac{0.83}{\eta} = 0.692$ $\chi_w := \eta$

Verification $\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$ < 1 OK

***Box top panel** (composite plate) - alternative design*

Cross-section (1) - Anchorage of the sixth stay-cable (from abutments)

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1 + \nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 88.08\ MPa$ $\sigma_{Ed.cmin} := 3.68\ MPa$

Tension: $\sigma_{Ed.t} := 100.50\ MPa$

Shear max: $\tau_{Edmax} := 43.28\ MPa$

Shear min: $\tau_{Edmin} := 4.50\ MPa$

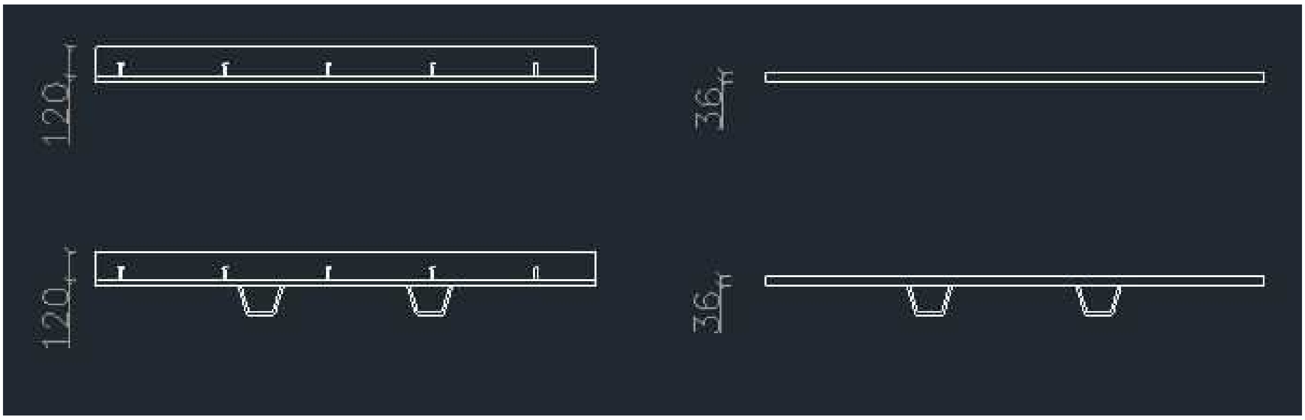
Von Mises: $\sigma_{VonMises} := 90\ MPa$

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 2000\ mm$

Plate thickness: $t_{pl} := 16\ mm + 120\ \frac{mm}{6} = 36\ mm$



STIFFENERS

Number of stiffeners: $n_{st} := 2$

Thickness: $t_{st} := 11\ mm$

Major width: $B_{st} := 185\ mm$

Minor width: $b_{st} := 108\ mm$

Depth: $d_{st} := 150\ mm$

Inclined width: $a := \sqrt{{d_{st}}^2 + \left(\frac{B_{st} - b_{st}}{2}\right)^2} = 154.862\ mm$

Angle: $\theta := \operatorname{acos}\left(\frac{d_{st}}{a}\right) = 0.251\ rad$

Center of gravity (stiffened plate):

$$y_G := \frac{n_{st} \cdot \left(b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2}\right) + 2 \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2}\cos(\theta) + \frac{t_{pl}}{2}\right)\right)}{n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}\right) + a_2 \cdot t_{pl}} = 12.561\ mm$$

Second moment of area (plate):

$$I_{pl} := a_2 \cdot \frac{{t_{pl}}^3}{12 \cdot \left(1 - \nu^2\right)} + a_2 \cdot t_{pl} \cdot \frac{{y_G}^2}{\left(1 - \nu^2\right)} = \left(2.103 \cdot 10^7\right)\ mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{{t_{st}}^3}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G\right)^2 + 2 \cdot \left(a \cdot \frac{{t_{st}}^3}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2}\cos(\theta) + \frac{t_{pl}}{2} - y_G\right)^2\right) = \left(4.922 \cdot 10^7\right)\ mm^4$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(1.195 \cdot 10^8\right)\ mm^4$$

Properties for equivalent thickness *(see Point C)* :

$$I_{BD} := I_{tot} = \left(1.195 \cdot 10^8\right)\ mm^4$$

$$A_{BD} := n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}\right) + a_2 \cdot t_{pl} = \left(8.119 \cdot 10^4\right)\ mm^2$$

$$y_{G.BD} := y_G + \frac{t_{pl}}{2} = 30.561\ mm$$

(distance b/w G of box bottom panel and bottom panel external surface)

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels

Panel 5 (INTERNAL ELEMENT)

Panel width:	$b_{pl5} := 559\ mm$
Panel thickness:	$t_{pl5} := t_{pl} = 36\ mm$
Panel stress ratio (safe-sided):	$\psi := 1$
Buckling factor:	$k_{\sigma 5} := 4$
Slenderness:	$\lambda_{p5} := \dfrac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$
Reduction factor: ('internal compression element')	$\rho_5 := \text{if } \lambda_{p5} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \dfrac{\lambda_{p5} - 0.055 \cdot (3 + \psi)}{\lambda_{p5}^{2}} \; = 1$
Effective panel width:	$b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 559\ mm$
Effective panel area:	$A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(2.012 \cdot 10^{4}\right)\ mm^{2}$

Panel 1 (INTERNAL ELEMENT)

Panel width:	$b_{pl1} := 574\ mm$
Panel thickness:	$t_{pl1} := t_{pl} = 36\ mm$
Panel stress ratio (safe-sided):	$\psi := 1$
Buckling factor:	$k_{\sigma 1} := 4$
Slenderness:	$\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}\right)}$
Reduction factor: ('internal compression element')	$\rho_1 := \text{if } \lambda_{p1} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \dfrac{\lambda_{p1} - 0.055 \cdot (3 + \psi)}{\lambda_{p1}^{2}} \; = 1$
Effective panel width:	$b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 574\ mm$
Effective panel area:	$A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(2.066 \cdot 10^{4}\right)\ mm^{2}$

Panel 2 (INTERNAL ELEMENT)

Panel width:	$b_{pl2} := B_{st} = 185\ mm$
Panel thickness:	$t_{pl2} := t_{pl} = 36\ mm$
Panel stress ratio (safe-sided):	$\psi := 1$
Buckling factor:	$k_{\sigma 2} := 4$
Slenderness:	$\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \text{if } \lambda_{p2} \le 0.673 \;\Big\|\; 1 \;\text{ else }\; \Big\|\; \frac{\lambda_{p2} - 0.055 \cdot (3 + \psi)}{\lambda_{p2}^{\;2}} = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 185\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(6.66 \cdot 10^3\right)\ mm^2$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := a = 154.862\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 3} := 4$

Slenderness: $$\lambda_{p3} := \frac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 3}}\right)}$$

Reduction factor:
('internal compression element')

$$\rho_3 := \text{if } \lambda_{p3} \le 0.673 \;\Big\|\; 1 \;\text{ else }\; \Big\|\; \frac{\lambda_{p3} - 0.055 \cdot (3 + \psi)}{\lambda_{p3}^{\;2}} = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 154.862\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(1.703 \cdot 10^3\right)\ mm^2$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := b_{st} = 108\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness: $$\lambda_{p4} := \frac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$$

Reduction factor:
('internal compression element')

$$\rho_4 := \text{if } \lambda_{p4} \le 0.673 \;\Big\|\; 1 \;\text{ else }\; \Big\|\; \frac{\lambda_{p4} - 0.055 \cdot (3 + \psi)}{\lambda_{p4}^{\;2}} = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 108\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.188 \cdot 10^3\right)\ mm^2$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(9.19 \cdot 10^3\right)\ mm^2$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(9.19 \cdot 10^{3}\right)\ mm^{2}$

Total effective section under compression:

$$A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(6.33 \cdot 10^{4}\right)\ mm^{2}$$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(6.33 \cdot 10^{4}\right)\ mm^{2}$$

Area ratio: $\delta := \dfrac{A_{sl}}{t_{pl} \cdot a_2} = 0.128$

Plate aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1$

Inertial ratio: $\gamma := \dfrac{I_{tot}}{a_2 \cdot \dfrac{t_{pl}^{\ 3}}{12\left(1-\nu^{2}\right)}} = 13.981$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1+\alpha^{2}\right)^{2} + \gamma - 1\right)}{\alpha^{2} \cdot (\psi+1) \cdot (1+\delta)} = 15.058$$

$$k_{\sigma.p} := \begin{cases} \dfrac{2 \cdot \left(\left(1+\alpha^{2}\right)^{2} + \gamma - 1\right)}{\alpha^{2} \cdot (\psi+1) \cdot (1+\delta)} & \text{if } \alpha \le \sqrt[4]{\gamma} \\ \dfrac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1) \cdot (1+\delta)} & \text{else} \end{cases} = 15.058$$

Euler plate buckling stress: $\sigma_E := \pi^{2} \cdot E \cdot \dfrac{t_{pl}^{\ 2}}{12 \cdot \left(1-\nu^{2}\right) \cdot a_2^{\ 2}} = 61.495\ MPa$

Critical elastic plate buckling stress: $\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = 926.025\ MPa$

Slenderness of stiffened plate: $\beta_{A.c} := \dfrac{A_{c.eff.loc}}{A_c} = 1$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.619$$

$$\rho := \begin{cases} 1 & \text{if } \lambda_{p\_} \le 0.673 \\ \dfrac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{\lambda_{p\_}^{\ 2}} & \text{else} \end{cases} = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener (b/w panel 1 and panel 5)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 287\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl5} = 279.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl5} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(3.165 \cdot 10^{4}\right)\ mm^{2}$$

Angle: $\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.251\ rad$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl2}} = 16.111\ mm$$

Second moment of area (column):

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^{2} + 2 \cdot b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^{2} + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^{2} + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^{2}$$

$$I_{sl2''} := b_{I.inf2} \cdot \frac{t_{pl1}^{3}}{12} + b_{I.sup2} \cdot \frac{t_{pl5}^{3}}{12} + b_{II.inf2} \cdot \frac{t_{pl2}^{3}}{12} + b_{II.sup2} \cdot \frac{t_{pl2}^{3}}{12} + b_{I.inf2} \cdot t_{pl1} \cdot y_{G.c2}^{2} + b_{I.sup2} \cdot t_{pl5} \cdot y_{G.c2}^{2} + b_{II.inf2} \cdot t_{pl2} \cdot y_{G.c2}^{2} + b_{II.sup2} \cdot t_{pl2} \cdot y_{G.c2}^{2}$$

$$I_{sl2} := I_{sl2'} + I_{sl2''}$$

Elastic critical column buckling stress: $\sigma_{cr.sl2} := \pi^{2} \cdot E \cdot \frac{I_{sl2}}{A_{sl2} \cdot a_1^{2}} = 560.202\ MPa$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $\sigma_{cr.c2} := \sigma_{cr.sl2} = 560.202\ MPa$

Slenderness of column *(column with the least critical stress is considered)*:

$$\sigma_{cr.c} := \sigma_{cr.c2} = 560.202\ MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(2.132 \cdot 10^{4}\right)\ mm^{2}$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 1.703$$

Slenderness of column: $\lambda_c := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.c}}} = 1.039$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 110.969\ mm$$

Distance: $e_1 := y_{Gst} - y_{G.c1} = 73.897\ mm$

Distance between plate and column centroids: $e_2 := y_{G.c1} = 37.071\ mm$

$$e := \max(e_1, e_2) = 73.897\ mm$$

$\alpha := 0.34$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 48.794\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.476$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot (\lambda_c - 0.2) + \lambda_c^{\ 2}\right) = 1.239$$

Reduction factor:

$$\chi_c := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^{\ 2}}} = 0.522$$

Interaction between plate and column buckling behaviours

$$\xi\_ := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = 0.653$$

$$\xi := \begin{cases} 0 & \text{if } \xi\_ \le 0 \\ 1 & \text{else if } \xi\_ \ge 1 \\ \xi\_ & \text{else} \end{cases} = 0.653$$

Final reduction factor:

$$\rho_c := (\rho - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c = 0.942$$

Effective cross-sectional area:

$$A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \frac{t_{pl1}}{2} = \left(8.032 \cdot 10^4\right) mm^2$$

<u>Axial stress resistance verification</u>

$$A := a_2 \cdot t_{pl} + n_{st} \cdot (2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}) = \left(8.119 \cdot 10^4\right) mm^2$$

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.251$$ < 1 OK (compression)

$$\eta_{1.t} := \frac{\sigma_{Ed.t}}{\frac{f_y}{\gamma_{M1}}} = 0.311$$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$\eta := 1.2$ (S355)

Effective width for shear check:

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 279.5\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 287\ mm$$

Second moment of area:

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \left(b_{1.2} + 2 \cdot b_{2.2} + b_{3.2}\right) \cdot t_{pl}} = 16.111\ mm$$

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^2\right)$$

$$I_{sl2''} := \left(b_{1.2} + b_{2.2}\right) \cdot \frac{t_{pl}^{\ 3}}{12} + \left(b_{1.2} + b_{2.2}\right) \cdot t_{pl} \cdot y_{G2}^{\ 2} + \left(b_{2.2} + b_{3.2}\right) \cdot \frac{t_{pl}^{\ 3}}{12} + \left(b_{2.1} + b_{3.1}\right) \cdot t_{pl} \cdot y_{G2}^{\ 2}$$

$$I_{sl2} := I_{sl2'} + I_{sl2''}$$

$$I_{sl} := I_{sl2} = \left(5.515 \cdot 10^7\right)\ mm^4$$

Shear buckling coefficient:

$$k_{\tau sl} := \text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \geq \frac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\frac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} \quad \begin{array}{l} \left\| 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \right. \\ \text{else} \\ \left\| \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\frac{I_{sl}}{a_2}} \right. \end{array} = 1.31 \cdot 10^3$$

$$k_{\tau} := \text{if } \frac{a_1}{a_2} \geq 1 \quad \begin{array}{l} \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \\ \text{else} \\ \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \end{array} = 1.32 \cdot 10^3$$

Slenderness:

$$\lambda_{w1} := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau}}} = 0.05$$

Sub-panels

<u>Panel 1</u> $a_2 := b_{pl1} = 574\ mm$

Shear buckling coeff.:

$$k_{\tau.p1} := \text{if } \frac{a_1}{a_2} \geq 1 \quad \begin{array}{l} \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \\ \text{else} \\ \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \end{array} = 5.669$$

Slenderness: $\lambda_{w.p1} := \dfrac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.22$

Panel 2 $a_2 := b_{pl2} = 185\ mm$

Shear buckling coeff.:

$$k_{\tau.p2} := \text{if } \frac{a_1}{a_2} \ge 1 \;\; \Big\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \;\; \text{else} \;\; \Big\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \; = 5.374$$

Slenderness: $\lambda_{w.p2} := \dfrac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.073$

Panel 5 $a_2 := b_{pl5} = 559\ mm$

Shear buckling coeff.:

$$k_{\tau.p5} := \text{if } \frac{a_1}{a_2} \ge 1 \;\; \Big\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \;\; \text{else} \;\; \Big\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \; = 5.652$$

Slenderness: $\lambda_{w.p5} := \dfrac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.215$

Final slenderness:

$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.22$ < $\dfrac{0.83}{\eta} = 0.692$

$\chi_w := \eta$

Shear stress resistance verification

$\eta_3 := \dfrac{\tau_{Edmax}}{\chi_w \cdot \dfrac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.065$ < 1 OK

AXIAL-SHEAR STRESS INTERACTION *(N/A since shear is very small)*

$$\eta_{1.cm} := \frac{\dfrac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\dfrac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.131$$

$$\eta_{3m} := \frac{\frac{(\tau_{Edmax} + \tau_{Edmin})}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$$

Verification $\eta_{1.cm} + (2 \cdot \eta_{3m} - 1)^2 = 0.993$ < 1 N/A

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

(assuming that stiffeners act as rigid support)

$\lambda_w := \max(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}) = 0.22$ < $\frac{0.83}{\eta} = 0.692$ $\chi_w := \eta$

Verification $$\eta_{3m} := \frac{\frac{(\tau_{Edmax} + \tau_{Edmin})}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$$ < 1 OK

Cross-section (2) - Midspan

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1 + \nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 280\ MPa$ $\sigma_{Ed.cmin} := 447\ kPa$

Tension: $\sigma_{Ed.t} := 200\ MPa$

Shear max: $\tau_{Edmax} := 134.4\ MPa$

Shear min: $\tau_{Edmin} := 4.43\ MPa$

Von Mises: $\sigma_{VonMises} := 263.4\ MPa$

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 2000\ mm$

Plate thickness: $t_{pl} := 16\ mm$

Center of gravity (stiffened plate):

$$y_G := \frac{b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2}\right) + 2 \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2}\right)}{b_{st} \cdot t_{st} + B_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}} = 69.977\ mm$$

Second moment of area (plate):

$$I_{pl} := a_2 \cdot \frac{t_{pl}^{\ 3}}{12 \cdot (1 - \nu^2)} + a_2 \cdot t_{pl} \cdot y_G^{\ 2} = (1.574 \cdot 10^8)\ mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{t_{st}^{3}}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G\right)^2 + 2 \cdot \left(a \cdot \frac{t_{st}^{3}}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2}\cos(\theta) + \frac{t_{pl}}{2} - y_G\right)^2\right) = \left(9.133 \cdot 10^{6}\right)\ mm^4$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(1.757 \cdot 10^{8}\right)\ mm^4$$

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels

Panel 5 (INTERNAL ELEMENT)

Panel width: $b_{pl5} := 505\ mm$

Panel thickness: $t_{pl5} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 5} := 4$

Slenderness: $\lambda_{p5} := \dfrac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_5 := \begin{cases} 1 & \text{if } \lambda_{p5} \le 0.673 \\ \dfrac{\lambda_{p5} - 0.055 \cdot (3 + \psi)}{\lambda_{p5}^{2}} & \text{else} \end{cases} = 0.993$$

Effective panel width: $b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 501.234\ mm$

Effective panel area: $A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(8.02 \cdot 10^{3}\right)\ mm^2$

Panel 6 (INTERNAL ELEMENT)

Panel width: $b_{pl6} := 115\ mm$

Panel thickness: $t_{pl6} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 6} := 4$

Slenderness: $\lambda_{p6} := \dfrac{b_{pl6}}{t_{pl6} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 6}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_6 := \begin{cases} 1 & \text{if } \lambda_{p6} \le 0.673 \\ \dfrac{\lambda_{p6} - 0.055 \cdot (3 + \psi)}{\lambda_{p6}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl6} := \rho_6 \cdot b_{pl6} = 115\ mm$

Effective panel area: $A_{eff.pl6} := b_{eff.pl6} \cdot t_{pl6} = \left(1.84 \cdot 10^{3}\right)\ mm^2$

Panel 1 (INTERNAL ELEMENT)

Panel width: $b_{pl1} := 254\ mm$

Panel thickness: $t_{pl1} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 1} := 4$

Slenderness: $\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_1 := \begin{cases} 1 & \text{if } \lambda_{p1} \le 0.673 \\ \dfrac{\lambda_{p1} - 0.055 \cdot (3+\psi)}{\lambda_{p1}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 254\ mm$

Effective panel area: $A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(4.064 \cdot 10^{3}\right)\ mm^{2}$

Panel 2 (INTERNAL ELEMENT)

Panel width: $b_{pl2} := 185\ mm$

Panel thickness: $t_{pl2} := t_{pl} = 16\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 2} := 4$

Slenderness: $\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \begin{cases} 1 & \text{if } \lambda_{p2} \le 0.673 \\ \dfrac{\lambda_{p2} - 0.055 \cdot (3+\psi)}{\lambda_{p2}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 185\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(2.96 \cdot 10^{3}\right)\ mm^{2}$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := 127\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 3} := 4$

Slenderness: $\lambda_{p3} := \dfrac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 3}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_3 := \begin{cases} 1 & \text{if } \lambda_{p3} \le 0.673 \\ \dfrac{\lambda_{p3} - 0.055 \cdot (3+\psi)}{\lambda_{p3}^{2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 127\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(1.397 \cdot 10^3\right)\ mm^2$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := 108\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness: $\lambda_{p4} := \dfrac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_4 := \begin{cases} 1 & \text{if } \lambda_{p4} \le 0.673 \\ \dfrac{\lambda_{p4} - 0.055 \cdot (3+\psi)}{\lambda_{p4}^2} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 108\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.188 \cdot 10^3\right)\ mm^2$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(7.964 \cdot 10^3\right)\ mm^2$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(7.964 \cdot 10^3\right)\ mm^2$

Total effective section under compression:

$$A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + 2 \cdot A_{eff.pl6} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(2.965 \cdot 10^4\right)\ mm^2$$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + 2 \cdot b_{pl6} \cdot t_{pl6} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(2.971 \cdot 10^4\right)\ mm^2$$

Area ratio: $\delta := \dfrac{A_{sl}}{t_{pl} \cdot a_2} = 0.249$

Plate aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1$

Inertial ratio: $\gamma := \dfrac{I_{tot}}{a_2 \cdot \dfrac{t_{pl}^3}{12\left(1-\nu^2\right)}} = 234.23$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} = 189.955$$

$$k_{\sigma.p} := \text{if } \alpha \le \sqrt[4]{\gamma} \quad \left\| \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1 \right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} \right. \quad \text{else} \quad \left\| \frac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1)\cdot(1+\delta)} \right. = 189.955$$

Euler plate buckling stress: $\sigma_E := \pi^2 \cdot E \cdot \dfrac{t_{pl}^{\ 2}}{12 \cdot \left(1-\nu^2\right) \cdot a_2^{\ 2}} = 12.147\ MPa$

Critical elastic plate buckling stress: $\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = \left(2.307 \cdot 10^3\right)\ MPa$

Slenderness of stiffened plate: $\beta_{A.c} := \dfrac{A_{c.eff.loc}}{A_c} = 0.998$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.392$$

$$\rho := \text{if } \lambda_{p\_} \le 0.673 \quad \| \ 1 \quad \text{else} \quad \left\| \frac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{\lambda_{p\_}^{\ 2}} \right. = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener 1 (b/w panel 5 and panel 6)

$b_{I.inf1} := \dfrac{3-\psi}{5-\psi} \cdot b_{pl6} = 57.5\ mm$ $\qquad b_{I.sup1} := \dfrac{2}{(5-\psi)} \cdot b_{pl5} = 252.5\ mm$

$b_{II.inf1} := \dfrac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm$ $\qquad b_{II.sup1} := \dfrac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$

$$A_{sl1} := b_{I.inf1} \cdot t_{pl6} + b_{I.sup1} \cdot t_{pl5} + b_{II.inf1} \cdot t_{pl2} + b_{II.sup1} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(1.19 \cdot 10^4\right)\ mm^2$$

Angle: $\theta := \operatorname{acos}\left(\dfrac{d_{st}}{b_{pl3}}\right) = 0.593i\ rad$

Center of gravity (column):

$$y_{G.c1} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \dfrac{t_{pl4}}{2} + \dfrac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \dfrac{b_{pl3}}{2}\cos(\theta) + \dfrac{t_{pl2}}{2}\right)}{A_{sl1}} = 34.706\ mm$$

Second moment of area (column):

$$I_{sl1'} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^2\right)$$

$$I_{sl1''} := b_{I.inf1} \cdot \frac{t_{pl1}^{\ 3}}{12} + b_{I.sup1} \cdot \frac{t_{pl6}^{\ 3}}{12} + b_{II.inf1} \cdot \frac{t_{pl2}^{\ 3}}{12} + b_{II.sup1} \cdot \frac{t_{pl2}^{\ 3}}{12} + b_{I.inf1} \cdot t_{pl1} \cdot y_{G.c1}^{\ 2} + b_{I.sup1} \cdot t_{pl6} \cdot y_{G.c1}^{\ 2} + b_{II.inf1} \cdot t_{pl2} \cdot y_{G.c1}^{\ 2} + b_{II.sup1} \cdot t_{pl2} \cdot y_{G.c1}^{\ 2}$$

$$I_{sl} := I_{sl1'} + I_{sl1''}$$

Elastic critical column buckling stress: $\sigma_{cr.sl1} := \pi^2 \cdot E \cdot \dfrac{I_{sl1}}{A_{sl1} \cdot a_1^{\,2}} = \left(1.297 \cdot 10^3\right)\,MPa$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $\sigma_{cr.c1} := \sigma_{cr.sl1} = \left(1.297 \cdot 10^3\right)\,MPa$

Column-stiffener 2 (b/w panel 1 and panel 6)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 127\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl6} = 57.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl6} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(9.894 \cdot 10^3\right)\,mm^2$$

Angle: $\theta := \operatorname{acos}\left(\dfrac{d_{st}}{b_{pl3}}\right) = 0.593\mathrm{i}\ rad$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \dfrac{t_{pl4}}{2} + \dfrac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \dfrac{b_{pl3}}{2}\cos(\theta) + \dfrac{t_{pl2}}{2}\right)}{A_{sl2}} = 41.75\ mm$$

Second moment of area (column):

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{\,3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\,3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\,3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^2\right)$$

$$I_{sl2''} := b_{I.inf2} \cdot \frac{t_{pl1}^{\,3}}{12} + b_{I.sup2} \cdot \frac{t_{pl6}^{\,3}}{12} + b_{II.inf2} \cdot \frac{t_{pl2}^{\,3}}{12} + b_{II.sup2} \cdot \frac{t_{pl2}^{\,3}}{12} + b_{I.inf2} \cdot t_{pl1} \cdot y_{G.c2}^{\,2} + b_{I.sup2} \cdot t_{pl6} \cdot y_{G.c2}^{\,2} + b_{II.inf2} \cdot t_{pl2} \cdot y_{G.c2}^{\,2} + b_{II.sup2} \cdot t_{pl2} \cdot y_{G.c2}^{\,2}$$

$$I_{sl2} := I_{sl2'} + I_{sl2''}$$

Elastic critical column buckling stress: $\sigma_{cr.sl2} := \pi^2 \cdot E \cdot \dfrac{I_{sl2}}{A_{sl2} \cdot a_1^{\,2}} = \left(1.676 \cdot 10^3\right)\,MPa$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $\sigma_{cr.c2} := \sigma_{cr.sl2} = \left(1.676 \cdot 10^3\right)\,MPa$

Slenderness of column *(column with the least critical stress is considered, i.e. column 1)*:

$$\sigma_{cr.c} := min\left(\sigma_{cr.c1}, \sigma_{cr.c2}\right) = \left(1.297 \cdot 10^3\right)\,MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := \frac{b_{eff.pl6}}{2} \cdot t_{pl6} + b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.187 \cdot 10^4\right)\,mm^2$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 0.997$$

Slenderness of column: $\lambda_c := \sqrt{\beta_{A.c} \cdot \dfrac{f_y}{\sigma_{cr.c}}} = 0.522$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \dfrac{t_{pl4}}{2} + \dfrac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \dfrac{b_{pl3}}{2}\cos(\theta) + \dfrac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 103.735\ mm$$

Distance: $e_1 := y_{Gst} - y_{G.c1} = 69.029\ mm$

Distance between plate and column centroids: $e_2 := y_{G.c1} = 34.706\ mm$

$e := \max\left(e_1, e_2\right) = 69.029\ mm$

$\alpha := 0.34$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 50.034\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.464$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + {\lambda_c}^2\right) = 0.711$$

Reduction factor:

$$\chi_c := \frac{1}{\phi + \sqrt{\phi^2 - {\lambda_c}^2}} = 0.837$$

Interaction between plate and column buckling behaviours

$$\xi\_ := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = 0.779$$

$$\xi := \begin{cases} 0 & \text{if } \xi\_ \le 0 \\ 1 & \text{else if } \xi\_ \ge 1 \\ \xi\_ & \text{else} \end{cases} = 0.779$$

Final reduction factor:

$$\rho_c := \left(\rho - \chi_c\right) \cdot \xi \cdot \left(2 - \xi\right) + \chi_c = 0.992$$

Effective cross-sectional area:

$$A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \frac{t_{pl1}}{2} = \left(3.348 \cdot 10^4\right)\ mm^2$$

<u>Axial stress resistance verification</u>

$$A := a_2 \cdot t_{pl} + n_{st} \cdot \left(2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}\right) = \left(3.996 \cdot 10^4\right)\ mm^2$$

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.942$$ < 1 OK (compression)

$$\eta_{1.t} := \frac{\sigma_{Ed.t}}{\frac{f_y}{\gamma_{M1}}} = 0.62$$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$\eta := 1.2$ (S355)

Effective widths for shear check (internal stiffener):

$$b_{2.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl6}}{2}\right) = 57.5\ mm$$

$$b_{3.1} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 195.268\ mm$$

Effective width for shear check (external stiffener):

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl6}}{2}\right) = 57.5\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 127\ mm$$

Second moment of area (internal stiffener):

$$y_{G1} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \left(b_{1.1} + 2 \cdot b_{2.1} + b_{3.1}\right) \cdot t_{pl}} = 37.599\ mm$$

$$I_{sl1} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G1}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G1}\right)^2\right) + (b$$

Second moment of area (external stiffener):

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \left(b_{1.2} + 2 \cdot b_{2.2} + b_{3.2}\right) \cdot t_{pl}} = 41.75\ mm$$

$$I_{sl2} := b_{pl4} \cdot \frac{t_{pl4}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^2\right) + (b$$

$$I_{sl} := min\left(I_{sl1}, I_{sl2}\right) = \left(3.006 \cdot 10^{7}\right)\ mm^{4}$$

Shear buckling coefficient:

$$k_{\tau sl} := \begin{aligned} &\text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \geq \frac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\frac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} \\ &\quad \left\| \; 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \right. \\ &\text{else} \\ &\quad \left\| \; \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\frac{I_{sl}}{a_2}} \right. \end{aligned} = 1.527 \cdot 10^{3}$$

$$k_{\tau} := \begin{aligned} &\text{if } \frac{a_1}{a_2} \geq 1 \\ &\quad \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \\ &\text{else} \\ &\quad \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \end{aligned} = 1.537 \cdot 10^{3}$$

Slenderness: $\lambda_{w1} := \dfrac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_\tau}} = 0.105$

Sub-panels

<u>Panel 1</u> $a_2 := b_{pl1} = 254\ mm$

Shear buckling coeff.:

$$k_{\tau.p1} := \begin{cases} \text{if } \dfrac{a_1}{a_2} \ge 1 \\ \quad 5.34 + 4 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \\ \text{else} \\ \quad 4 + 5.34 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \end{cases} = 5.405$$

Slenderness: $\lambda_{w.p1} := \dfrac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.224$

<u>Panel 2</u> $a_2 := b_{pl2} = 185\ mm$

Shear buckling coeff.:

$$k_{\tau.p2} := \begin{cases} \text{if } \dfrac{a_1}{a_2} \ge 1 \\ \quad 5.34 + 4 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \\ \text{else} \\ \quad 4 + 5.34 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \end{cases} = 5.374$$

Slenderness: $\lambda_{w.p2} := \dfrac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.164$

<u>Panel 5</u> $a_2 := b_{pl5} = 505\ mm$

Shear buckling coeff.:

$$k_{\tau.p5} := \begin{cases} \text{if } \dfrac{a_1}{a_2} \ge 1 \\ \quad 5.34 + 4 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \\ \text{else} \\ \quad 4 + 5.34 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \end{cases} = 5.595$$

Slenderness: $\lambda_{w.p5} := \dfrac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.439$

<u>Panel 6</u> $a_2 := b_{pl6} = 115\ mm$

Shear buckling coeff.: $k_{\tau.p6} := \text{if } \dfrac{a_1}{a_2} \geq 1 \; \left\| \; 5.34 + 4 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \right. \; \text{else} \; \left\| \; 4 + 5.34 \cdot \left(\dfrac{a_2}{a_1}\right)^2 \right. = 5.353$

Slenderness: $\lambda_{w.p6} := \dfrac{a_2}{t_{pl6} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p6}}} = 0.102$

Final slenderness:

$$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}, \lambda_{w.p6}\right) = 0.439 \quad < \quad \frac{0.83}{\eta} = 0.692$$

$$\chi_w := \eta$$

Shear stress resistance verification

$$\eta_3 := \frac{\tau_{Edmax}}{\chi_w \cdot \dfrac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.2$$

< 1 OK

AXIAL-SHEAR STRESS INTERACTION

$$\eta_{1.cm} := \frac{\dfrac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\dfrac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.472$$

$$\eta_{3m} := \frac{\dfrac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \dfrac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103$$

Verification $\eta_{1.cm} + \left(2 \cdot \eta_{3m} - 1\right)^2 = 1.1$ OK

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

(assuming that stiffeners act as rigid support)

$$\lambda_w := \max\left(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}, \lambda_{w.p6}\right) = 0.439 \quad < \quad \frac{0.83}{\eta} = 0.692 \qquad \chi_w := \eta$$

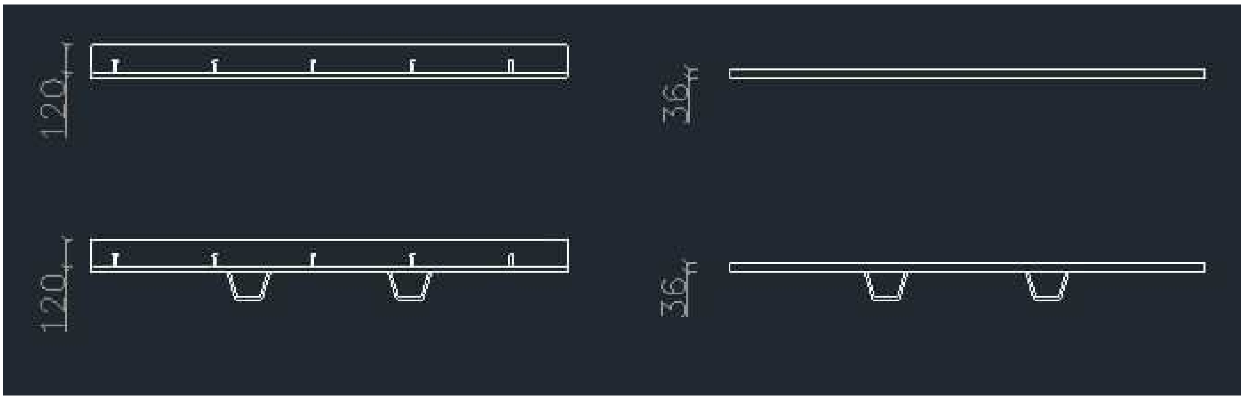
Verification

$$\eta_{3m} := \frac{\dfrac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \dfrac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103$$

< 1 OK

Box top panel (composite slab) - alternative design

Cross-section (2) - Midspan



GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1+\nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 280\ MPa$ $\sigma_{Ed.cmin} := 3.68\ MPa$

Tension: $\sigma_{Ed.t} := 200\ MPa$

Shear max: $\tau_{Edmax} := 134.4\ MPa$

Shear min: $\tau_{Edmin} := 4.43\ MPa$

Von Mises: $\sigma_{VonMises} := 263.4\ MPa$

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 2000\ mm$

Plate thickness: $t_{pl} := 16\ mm + 120\ \frac{mm}{6} = 36\ mm$

STIFFENERS

Number of stiffeners: $n_{st} := 2$

Thickness: $t_{st} := 11\ mm$

Major width: $B_{st} := 185\ mm$

Minor width: $b_{st} := 108\ mm$

Depth: $d_{st} := 150\ mm$

Inclined width: $a := \sqrt{{d_{st}}^2 + \left(\frac{B_{st} - b_{st}}{2}\right)^2} = 154.862\ mm$

Angle: $\theta := \operatorname{acos}\left(\frac{d_{st}}{a}\right) = 0.251\ rad$

Center of gravity (stiffened plate):

$$y_G := \frac{n_{st} \cdot \left(b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2} \right) + 2 \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2} \right) \right)}{n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st} \right) + a_2 \cdot t_{pl}} = 12.561 \ mm$$

Second moment of area (plate):

$$I_{pl} := a_2 \cdot \frac{t_{pl}^{\ 3}}{12 \cdot \left(1-\nu^2\right)} + a_2 \cdot t_{pl} \cdot \frac{y_G^{\ 2}}{\left(1-\nu^2\right)} = \left(2.103 \cdot 10^7\right) mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{t_{st}^{\ 3}}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G \right)^2 + 2 \cdot \left(a \cdot \frac{t_{st}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2} - y_G \right)^2 \right) = \left(4.922 \cdot 10^7\right) mm^4$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(1.195 \cdot 10^8\right) mm^4$$

Properties for equivalent thickness
(see Point C) :

$$I_{BD} := I_{tot} = \left(1.195 \cdot 10^8\right) mm^4$$

$$A_{BD} := n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st} \right) + a_2 \cdot t_{pl} = \left(8.119 \cdot 10^4\right) mm^2$$

$$y_{G.BD} := y_G + \frac{t_{pl}}{2} = 30.561 \ mm$$

(distance b/w G of box bottom panel and bottom panel external surface)

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels

Panel 5 (INTERNAL ELEMENT)

Panel width: $b_{pl5} := 559 \ mm$

Panel thickness: $t_{pl5} := t_{pl} = 36 \ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 5} := 4$

Slenderness: $\lambda_{p5} := \frac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_5 := \text{if } \lambda_{p5} \le 0.673 \ \Vert\ 1 \ \text{else} \ \Vert\ \frac{\lambda_{p5} - 0.055 \cdot (3+\psi)}{\lambda_{p5}^{\ 2}} \quad = 1$$

Effective panel width: $b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 559 \ mm$

Effective panel area: $A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(2.012 \cdot 10^4\right) mm^2$

Panel 1 (INTERNAL ELEMENT)

Panel width: $b_{pl1} := 574 \ mm$

Panel thickness: $t_{pl1} := t_{pl} = 36 \ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 1} := 4$

b_{pl1}

Slenderness: $\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_1 := \begin{array}{l} \text{if } \lambda_{p1} \le 0.673 \\ \quad \| \; 1 \\ \text{else} \\ \quad \| \; \dfrac{\lambda_{p1} - 0.055 \cdot (3 + \psi)}{\lambda_{p1}^{2}} \end{array} = 1$$

Effective panel width: $b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 574\ mm$

Effective panel area: $A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(2.066 \cdot 10^{4}\right)\ mm^{2}$

Panel 2 (INTERNAL ELEMENT)

Panel width: $b_{pl2} := B_{st} = 185\ mm$

Panel thickness: $t_{pl2} := t_{pl} = 36\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 2} := 4$

Slenderness: $\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \begin{array}{l} \text{if } \lambda_{p2} \le 0.673 \\ \quad \| \; 1 \\ \text{else} \\ \quad \| \; \dfrac{\lambda_{p2} - 0.055 \cdot (3 + \psi)}{\lambda_{p2}^{2}} \end{array} = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 185\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(6.66 \cdot 10^{3}\right)\ mm^{2}$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := a = 154.862\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 3} := 4$

Slenderness: $\lambda_{p3} := \dfrac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 3}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_3 := \begin{array}{l} \text{if } \lambda_{p3} \le 0.673 \\ \quad \| \; 1 \\ \text{else} \\ \quad \| \; \dfrac{\lambda_{p3} - 0.055 \cdot (3 + \psi)}{\lambda_{p3}^{2}} \end{array} = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 154.862\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(1.703 \cdot 10^{3}\right)\ mm^{2}$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := b_{st} = 108\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness: $\lambda_{p4} := \dfrac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_4 := \begin{cases} 1 & \text{if } \lambda_{p4} \le 0.673 \\ \dfrac{\lambda_{p4} - 0.055 \cdot (3 + \psi)}{\lambda_{p4}^{\ 2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 108\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.188 \cdot 10^{3}\right)\ mm^{2}$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(9.19 \cdot 10^{3}\right)\ mm^{2}$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(9.19 \cdot 10^{3}\right)\ mm^{2}$

Total effective section under compression:

$$A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(6.33 \cdot 10^{4}\right)\ mm^{2}$$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(6.33 \cdot 10^{4}\right)\ mm^{2}$$

Area ratio: $\delta := \dfrac{A_{sl}}{t_{pl} \cdot a_2} = 0.128$

Plate aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1$

Inertial ratio: $\gamma := \dfrac{I_{tot}}{a_2 \cdot \dfrac{t_{pl}^{\ 3}}{12\left(1 - \nu^{2}\right)}} = 13.981$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1 + \alpha^{2}\right)^{2} + \gamma - 1\right)}{\alpha^{2} \cdot (\psi + 1) \cdot (1 + \delta)} = 15.058$$

$$k_{\sigma.p} := \begin{cases} \dfrac{2 \cdot \left(\left(1 + \alpha^{2}\right)^{2} + \gamma - 1\right)}{\alpha^{2} \cdot (\psi + 1) \cdot (1 + \delta)} & \text{if } \alpha \le \sqrt[4]{\gamma} \\ \dfrac{4\left(1 + \sqrt{\gamma}\right)}{\ } & \text{else} \end{cases} = 15.058$$

Euler plate buckling stress: $$\sigma_E := \pi^2 \cdot E \cdot \frac{t_{pl}^{\ 2}}{12 \cdot (1-\nu^2) \cdot a_2^{\ 2}} = 61.495\ MPa$$

Critical elastic plate buckling stress: $$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = 926.025\ MPa$$

Slenderness of stiffened plate: $$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c} = 1$$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.619$$

$$\rho := \begin{array}{l} \text{if } \lambda_{p\_} \le 0.673 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \dfrac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{\lambda_{p\_}^{\ 2}} \end{array} = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener (b/w panel 1 and panel 5)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 287\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl5} = 279.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 92.5\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 92.5\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl5} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(3.165 \cdot 10^4\right)\ mm^2$$

Angle: $$\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.251\ rad$$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl2}} = 16.111\ mm$$

Second moment of area (column):

$$I_{sl2} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^2\right) +$$

Elastic critical column buckling stress: $$\sigma_{cr.sl2} := \pi^2 \cdot E \cdot \frac{I_{sl2}}{A_{sl2} \cdot a_1^{\ 2}} = 677.604\ MPa$$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $$\sigma_{cr.c2} := \sigma_{cr.sl2} = 677.604\ MPa$$

Slenderness of column *(column with the least critical stress is considered)*:

$$\sigma_{cr.c} := \sigma_{cr.c2} = 677.604\ MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(2.132 \cdot 10^{4}\right)\ mm^{2}$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 1.791$$

Slenderness of column: $\lambda_c := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.c}}} = 0.969$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 110.969\ mm$$

Distance: $e_1 := y_{Gst} - y_{G.c1} = 76.263\ mm$

Distance between plate and column centroids: $e_2 := y_{G.c1} = 34.706\ mm$

$$e := \max\left(e_1, e_2\right) = 76.263\ mm$$

$\alpha := 0.34$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 50.258\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.477$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + {\lambda_c}^{2}\right) = 1.152$$

Reduction factor: $\chi_c := \frac{1}{\phi + \sqrt{\phi^{2} - {\lambda_c}^{2}}} = 0.563$

Interaction between plate and column buckling behaviours

$$\xi\_ := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = 0.367$$

$$\xi := \begin{cases} 0 & \text{if } \xi\_ \le 0 \\ 1 & \text{else if } \xi\_ \ge 1 \\ \xi\_ & \text{else} \end{cases} = 0.367$$

Final reduction factor: $\rho_c := \left(\rho - \chi_c\right) \cdot \xi \cdot \left(2 - \xi\right) + \chi_c = 0.825$

Effective cross-sectional area: $A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \frac{t_{pl1}}{2} = \left(7.286 \cdot 10^{4}\right)\ mm^{2}$

Axial stress resistance verification

$$A := a_2 \cdot t_{pl} + n_{st} \cdot \left(2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}\right) = \left(8.119 \cdot 10^4\right) mm^2$$

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.879$$ < 1 OK (compression)

$$\eta_{1.t} := \frac{\sigma_{Ed.t}}{\frac{f_y}{\gamma_{M1}}} = 0.62$$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$$\eta := 1.2$$ (S355)

Effective width for shear check:

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 92.5\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 279.5\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 287\ mm$$

Second moment of area:

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \left(b_{1.2} + 2 \cdot b_{2.2} + b_{3.2}\right) \cdot t_{pl}} = 16.111\ mm$$

$$I_{sl2} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^2\right)$$

$$I_{sl} := I_{sl2} = \left(5.515 \cdot 10^7\right) mm^4$$

Shear buckling coefficient:

$$k_{\tau sl} := \begin{cases} 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} & \text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \geq \frac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\frac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} \\ \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\frac{I_{sl}}{a_2}} & \text{else} \end{cases} = 1.31 \cdot 10^3$$

$$k_\tau := \text{if } \frac{a_1}{a_2} \ge 1 \quad \begin{aligned} & \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \\ & \text{else} \\ & \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right. \end{aligned} \quad = 1.32 \cdot 10^3$$

Slenderness: $$\lambda_{w1} := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_\tau}} = 0.05$$

Sub-panels

Panel 1 $$a_2 := b_{pl1} = 574 \; mm$$

Shear buckling coeff.: $$k_{\tau.p1} := \text{if } \frac{a_1}{a_2} \ge 1 \quad \begin{aligned} & \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \\ & \text{else} \\ & \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \end{aligned} \quad = 5.669$$

Slenderness: $$\lambda_{w.p1} := \frac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.22$$

Panel 2

$$a_2 := b_{pl2} = 185 \; mm$$

Shear buckling coeff.: $$k_{\tau.p2} := \text{if } \frac{a_1}{a_2} \ge 1 \quad \begin{aligned} & \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \\ & \text{else} \\ & \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \end{aligned} \quad = 5.374$$

Slenderness: $$\lambda_{w.p2} := \frac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.073$$

Panel 5

$$a_2 := b_{pl5} = 559 \; mm$$

Shear buckling coeff.:

$$k_{\tau.p5} := \text{if } \frac{a_1}{a_2} \ge 1 \quad \begin{aligned} & \left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \\ & \text{else} \\ & \left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \end{aligned} \quad = 5.652$$

Slenderness: $$\lambda_{w.p5} := \frac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.215$$

Final slenderness:

$$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.22 \quad < \quad \frac{0.83}{\eta} = 0.692$$

$$\chi_w := \eta$$

Shear stress resistance verification

$$\eta_3 := \frac{\tau_{Edmax}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.2 \quad < 1 \quad \text{OK}$$

AXIAL-SHEAR STRESS INTERACTION *(N/A since shear is very small)*

$$\eta_{1.cm} := \frac{\frac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.445$$

$$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103$$

Verification $$\eta_{1.cm} + \left(2 \cdot \eta_{3m} - 1\right)^2 = 1.074 \quad < 1 \quad \text{N/A}$$

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

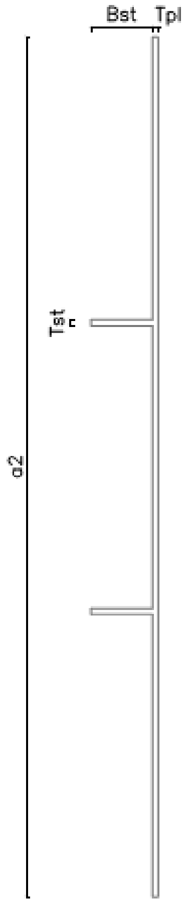
(assuming that stiffeners act as rigid support)

$$\lambda_w := \max\left(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.22 \quad < \quad \frac{0.83}{\eta} = 0.692 \qquad \chi_w := \eta$$

Verification $$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103 \quad < 1 \quad \text{OK}$$

***Box lateral panel** (stiffened with open stiffeners)*

Cross-section (1) - Anchorage of the sixth stay-cable (from abutments)



SECTION PROPERTIES

Panel width: $a_1 := 2000\ mm$

Panel height: $a_2 := 704\ mm$

Panel thickness: $t_{pl} := 16\ mm$

Stiffeners height: $b_{st} := 100\ mm$

Stiffeners thickness: $t_{st} := 10\ mm$

Number of stiffeners: $n := 2$

Stiffener area: $A_{st} := b_{st} \cdot t_{st} = 1000\ mm^2$

Inclination angle: $\theta := 60\ °$

Properties for equivalent thickness
(see Point C) :

$$a' := \frac{a_2}{3} = 234.667\ mm$$

$$y'_{G.LD} := \frac{a_2 \cdot t_{pl} \cdot \left(\frac{t_{pl}}{2}\right) + n \cdot A_{st} \cdot \left(t_{pl} + \frac{b_{st}}{2}\right)}{a_2 \cdot t_{pl} + n \cdot A_{st}} = 16.745\ mm$$

$$x'_{G.LD} := \frac{a_2}{2} = 352\ mm$$

$$I_{LDx'} := \left(a_2\,\frac{t_{pl}^{\ 3}}{12} + a_2 \cdot t_{pl} \cdot \left(\frac{t_{pl}}{2} - y'_{G.LD}\right)^2\right) \cdot \frac{1}{\left(1-\nu^2\right)} + n \cdot \left(t_{st} \cdot \frac{b_{st}^{\ 3}}{12} + t_{st} \cdot b_{st} \cdot \left(t_{pl} + \frac{b_{st}}{2} - y'_{G.LD}\right)^2\right)$$

$$I_{LDy'} := t_{pl}\,\frac{a_2^{\ 3}}{12 \cdot \left(1-\nu^2\right)} + b_{st} \cdot \frac{t_{st}^{\ 3}}{12} + b_{st} \cdot t_{st} \cdot \left(x'_{G.LD} - a'\right)^2 = \left(5.25 \cdot 10^8\right)\ mm^4$$

$$I_{p.LDx} := I_{LDx'} \cdot \left(\cos(\theta)\right)^2 + I_{LDy'} \cdot \left(\sin(\theta)\right)^2 = \left(3.957 \cdot 10^8\right)\ mm^4$$

$$I_{p.LDy} := I_{LDx'} \cdot \left(\sin(\theta)\right)^2 + I_{LDy'} \cdot \left(\cos(\theta)\right)^2 = \left(1.37 \cdot 10^8\right)\ mm^4$$

$$I_{totLD} := I_{p.LDx}$$

$$I_{LD} := I_{totLD} = \left(3.957 \cdot 10^8\right)\ mm^4$$

$$A_{LD} := n \cdot b_{st} \cdot t_{st} + a_2 \cdot t_{pl} = \left(1.326 \cdot 10^4\right)\ mm^2$$

$$y_{G.LD} := \frac{a_2}{2} \cdot \sin(\theta) + \left(\frac{t_{pl}}{2} + y'_{G.LD}\right) \cdot \cos(\theta) = 317.214\ mm$$

(distance b/w G of box lateral panel and bottom panel external surface)

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 88.08\ MPa$ $\quad \sigma_{Ed.cmin} := 3.68\ MPa$

Tension: $\sigma_{Ed.t} := 100.50\ MPa$

Shear max: $\tau_{Edmax} := 43.28\ MPa$

Shear min: $\tau_{Edmin} := 4.50\ MPa$

Von Mises: $\sigma_{VonMises} := 90\ MPa$

Longitudinal stiffeners

Stress ratio:
(safe-sided) $\psi := 1$

0.578

Stiffener buckling factor: $k_{\sigma.st} := \dfrac{0.578}{(0.34+\psi)} = 0.431$

Slenderness: $\lambda_{st} := \dfrac{b_{st}}{t_{st}} \cdot \dfrac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.st}}} = 0.659$

Reduction factor:
('outstand compression element')

$$\rho_{st} := \begin{array}{l} \text{if } \lambda_{st} \le 0.748 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \dfrac{\lambda_{st} - 0.188}{\lambda_{st}^{2}} \end{array} = 1$$

Effective area of stiffener: $A_{L.eff} := \rho_{st} \cdot A_{st} = 1000\ mm^2$

Sub-panels

Stress ratio:
(safe-sided)
$\psi_1 := 1$

Sub-panel height: $b_1 := \dfrac{a_2}{(n+1)} - \dfrac{t_{st}}{2} = 0.23\ m$

Sub-panel buckling factor: $k_{\sigma 1} := \dfrac{8.2}{(1.05+\psi_1)} = 4$

Slenderness: $\lambda_{pl} := \dfrac{b_1}{28.4 \cdot t_{pl} \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}} = 0.311$

Reduction factor:
('internal compression element')

$$\rho_1 := \begin{array}{l} \text{if } \lambda_{pl} \le 0.673 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \dfrac{\lambda_{pl} - 0.055 \cdot (3+\psi_1)}{\lambda_{pl}^{2}} \end{array} = 1$$

Gross width: $b_{1sup} := \dfrac{2}{(5-\psi_1)} \cdot b_1 = 0.115\ m$ $\qquad b_{1inf} := \dfrac{(3-\psi_1)}{(5-\psi_1)} \cdot b_1 = 0.115\ m$

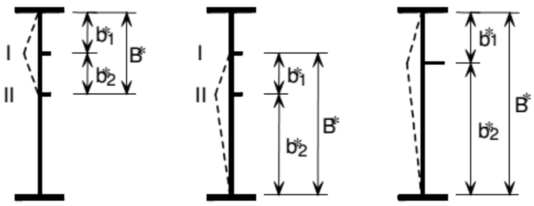
Effective width: $b_{1.eff} := b_1 \cdot \rho_1 = 0.23\ m$

$$b_{1sup.eff} := \frac{2}{(5-\psi_1)} \cdot b_{1.eff} = 0.115\ m$$

$$b_{1inf.eff} := \frac{(3-\psi_1)}{(5-\psi_1)} \cdot b_{1.eff} = 0.115\ m$$

NOTE: sub-panel checks are the same for all the three elements due to the symmetric layout of the siffeners along the lateral box panel

Critical plate buckling stress



Stiffener I Stiffener II Lumped stiffener

	Stiffener I	Stiffener II	Lumped stiffener
Cross-sectional area	$A_{s\ell,I}$	$A_{s\ell,II}$	$A_{s\ell,I} + A_{s\ell,II}$
Second moment of area	$I_{s\ell,I}$	$I_{s\ell,II}$	$I_{s\ell,I} + I_{s\ell,II}$

Stiffener 1:

Gross area: $A_{sl.1} := (b_{1inf} + b_{1sup} + t_{st}) \cdot t_{pl} + b_{st} \cdot t_{st} = 4834.667\ mm^2$

Center of gravity $x_{sl.1} := \dfrac{b_{st} \cdot t_{st} \cdot \left(\dfrac{b_{st}}{2} + \dfrac{t_{pl}}{2}\right)}{A_{sl.1}} = 0.012\ m$

Second moment of area

$$I_{sl.1} := \frac{(b_{1sup} + b_{1inf} + t_{st}) \cdot t_{pl}^{3}}{12} + \frac{t_{st} \cdot b_{st}^{3}}{12} + (b_{1sup} + b_{1inf} + t_{st}) \cdot t_{pl} \cdot x_{sl.1}^{2} + t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl.1}\right)^{2} = \left(3.583 \cdot 10^{-6}\right)\ m^4$$

Stiffener 2 : same as Stiffener 1

Lumped stiffener:

Gross area: $A_{sl.lumped} := 2 \cdot A_{sl.1} = 0.01\ m^2$

Second moment of area: $I_{sl.lumped} := 2 \cdot I_{sl.1} = \left(7.167 \cdot 10^{-6}\right)\ m^4$

Position: $h_{lumped} := \dfrac{a_2}{2} = 0.352\ m$

CASES 1/2: buckling of a single stiffener

Dimensions:
(refer to image above)

$b_I := \dfrac{a_2}{n+1}$

$b_{II} := b_I$

$B_1 := b_I + b_{II} = 0.469\ m$

$$a_{c1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.1} \cdot b_I^{2} \cdot b_{II}^{2}}{t_{pl}^{3} \cdot B_1}} = 1.187\ m$$

Elastic buckling stress:

$$\sigma_{cr.sl1} := \text{if } a_1 \ge a_{c1} \quad \left\| \frac{1.05 \cdot E}{A_{sl.1}} \cdot \frac{\sqrt{I_{sl.1} \cdot t_{pl}^{3} \cdot B_1}}{b_I \cdot b_{II}} \right. \quad \text{else} \quad \left\| \frac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_1^{2}} + \frac{E \cdot t_{pl}^{3} \cdot B_1 \cdot a_1^{2}}{4 \cdot \pi^2 \cdot (1-\nu^2) \cdot A_{sl.1} \cdot b_I^{2} \cdot b_{II}^{2}} \right. = \left(2.174 \cdot 10^{3}\right)\ MPa$$

Elastic plate buckling stress: $\sigma_{cr.p1} := \dfrac{a_2}{(a_2 - b_I)} \cdot \sigma_{cr.sl1} = \left(3.261 \cdot 10^{3}\right)\ MPa$

CASE 3: buckling of the lumped stiffener

Dimensions:
(refer to image above)

$b_{I.lumped} := \dfrac{a_2}{2}$

$b_{II.lumped} := b_{I.lumped}$

$B_{1.lumped} := b_{I.lumped} + b_{II.lumped} = 0.704\ m$

$$a_{c.lumped} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.lumped} \cdot b_{I.lumped}^{2} \cdot b_{II.lumped}^{2}}{t_{pl}^{3} \cdot B_{1.lumped}}} = 1.914\ m$$

Elastic buckling stress:

$$\sigma_{cr.sl.lumped} := \text{if } a_1 \ge a_{c.lumped} \;\; \left\| \frac{1.05 \cdot E}{A_{sl.1}} \cdot \frac{\sqrt{I_{sl.lumped} \cdot t_{pl}^{\;3} \cdot B_1}}{b_{I.lumped} \cdot b_{II.lumped}} \right. \;\; \text{else} \;\; \left\| \frac{\pi^2 \cdot E \cdot I_{sl.lumped}}{A_{sl.lumped} \cdot a_1^{\;2}} + \frac{E \cdot t_{pl}^{\;3} \cdot B_{1.lumped} \cdot a_1^{\;2}}{4 \cdot \pi^2 \cdot \left(1-\nu^2\right) \cdot A_{sl.lumped} \cdot b_{I.lumped}^{\;2} \cdot b_{II.lumped}^{\;2}} \right. = \left(1.366 \cdot 10^3\right) \, MPa$$

Elastic plate buckling stress:

$$\sigma_{cr.p.lumped} := \frac{a_2}{\left(a_2 - h_{lumped}\right)} \cdot \sigma_{cr.sl.lumped} = \left(2.733 \cdot 10^3\right) \, MPa$$

Critical buckling stress:

$$\sigma_{cr.p} := min\left(\sigma_{cr.p.lumped}, \sigma_{cr.p1}\right) = \left(2.733 \cdot 10^3\right) \, MPa$$

Plate type behaviour:

Compression gross area:

$$A_c := t_{pl} \cdot \left(a_2 - 2\, b_{1sup}\right) + n \cdot b_{st} \cdot t_{st} = 0.01 \, m^2$$

Effective area of stiffeners: $\Sigma A_{sl.eff} := 2 \cdot \rho_{st} \cdot \left(b_{st} \cdot t_{st}\right) = 0.002 \, m^2$

Effective compression area:

$$A_{c.eff.loc1} := \Sigma A_{sl.eff} + \left(2 \cdot b_{1sup.eff} + 2 \cdot b_{1inf.eff} + 2 \cdot t_{st}\right) \cdot t_{pl} = 0.01 \, m^2$$

Area ratio: $\beta_{A.c} := \dfrac{A_{c.eff.loc1}}{A_c} = 1.008$

Slenderness: $\lambda_p := \sqrt{\dfrac{\beta_{A.c} \cdot f_y}{\sigma_{cr.p}}} = 0.362$

Sub-panel reduction factor:
('internal compression element')

$$\rho := \text{if } \lambda_p \le 0.673 \;\; \left\| 1 \right. \;\; \text{else} \;\; \left\| \frac{\lambda_p - 0.055 \cdot \left(3 + \psi_1\right)}{\lambda_p^{\;2}} \right. = 1$$

Column type buckling behaviour:

Gross area of the stiffener + adjacent parts of plate:

$$A_{sl.1} := \left(b_{1sup} + b_{1inf} + t_{st}\right) \cdot t_{pl} + \frac{\Sigma A_{sl.eff}}{2 \cdot \rho_{st}} = 0.005 \, m^2$$

Effective area of the stiffener + adjacent parts of plate:

$$A_{sl.1.eff} := \left(b_{1sup.eff} + b_{1inf.eff} + t_{st}\right) \cdot t_{pl} + \frac{\Sigma A_{sl.eff}}{2} = 0.005 \, m^2$$

Area ratio: $\beta_{A.c.1} := \dfrac{A_{sl.1.eff}}{A_{sl.1}} = 1$

$$\sigma_{cr.sl} := \frac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_2^{\;2}} = \left(3.1 \cdot 10^3\right) \, MPa$$

Elastic buckling stress: $\sigma_{cr.c} := \dfrac{a_2}{\left(a_2 - b_I\right)} \cdot \sigma_{cr.sl} = \left(4.649 \cdot 10^3\right) \, MPa$

Slenderness: $\lambda_c := \sqrt{\dfrac{\beta_{A.c.1} \cdot f_y}{\sigma_{cr.c}}} = 0.276$

Distance b/w G of plate and G of column, and b/w G of stiffener and G of column

$$e_1 := \frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl.1} = 0.046\ m$$

$$e_2 := x_{sl.1} = 0.012\ m$$

$$e := \max\left(e_1, e_2\right) = 0.046\ m$$

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 0.027\ m$$

$$\alpha_E := 0.49 + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.642$$

$$\phi := 0.5 \cdot \left(1 + \alpha_E \cdot \left(\lambda_c - 0.2\right) + {\lambda_c}^2\right) = 0.563$$

Reduction factor: $$\chi_c := \left(\phi + \left(\phi^2 - {\lambda_c}^2\right)^{0.5}\right)^{-1} = 0.95$$

Interaction between plate and column buckling behaviour

$$\xi_1 := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = -0.412$$

$$\xi := \begin{array}{l} \text{if } \xi_1 \le 0 \\ \quad \| \ 0 \\ \text{else if } 0 < \xi_1 < 1 \\ \quad \| \ \xi_1 \\ \text{else} \\ \quad \| \ 1 \end{array} = 0$$

Final reduction factor: $$\rho_c := \left(\rho - \chi_c\right) \cdot \xi \cdot \left(2 - \xi\right) + \chi_c = 0.95$$

Effective properties

Effective area of stiffeners+ sub-panels:

$$A_{c.eff.loc} := \begin{array}{l} \text{if } \dfrac{\rho_c \cdot f_y}{\gamma_{M0} \cdot \sigma_{Ed.cmax}} > 1 \\ \quad \| \ A_{c.eff.loc1} \\ \text{else} \\ \quad \| \ \dfrac{\rho_c \cdot f_y \cdot A_{sl.1}}{\gamma_{M0} \cdot \sigma_{Ed.cmax}} \end{array} = 0.01\ m^2$$

Effective global area: $$A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{1inf.eff} \cdot t_{pl} = 0.013\ m^2$$

Gross global area: $$A_c := a_2 \cdot t_{pl} + n \cdot b_{st} \cdot t_{st} = 0.013\ m^2$$

Area reduction ratio: $$\frac{A_{c.eff}}{A_c} = 0.969$$ >0.5 OK

Axial stress resistance verification

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax} \cdot \gamma_{M0}}{f_y} \cdot \frac{A_c}{A_{c.eff}} = 0.256$$ < 1 OK

Shear stress resistance verification

Stiffened panel

Gross area: $$A_{sl} := 30 \cdot \varepsilon \cdot {t_{pl}}^2 + t_{pl} \cdot t_{st} + b_{st} \cdot t_{st} = 0.007\ m^2$$

Center of gravity: $$x_{sl} := \frac{b_{st} \cdot t_{st} \cdot \left(\frac{b_{st} + t_{pl}}{2}\right)}{A_{sl}} = 0.008\ m$$

Second moment of area:

$$I_{sl} := \frac{(30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl}^{3}}{12} + \frac{t_{st} \cdot b_{st}^{3}}{12} + (30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl} \cdot x_{sl}^{2} + t_{st} \cdot b_{st} \cdot \left(\frac{t_{pl} + b_{st}}{2} - x_{sl}\right)^{2} = (3.88 \cdot 10^{-6})\ m^{4}$$

Aspect ratio: $\alpha := \frac{a_1}{a_2} = 2.841$

Shear buckling coefficient: $k_{\tau.sl1} := 9 \cdot \left(\frac{1}{\alpha}\right)^{2} \cdot \sqrt[4]{\left(\frac{n \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}\right)^{3}} = 2.343$

$$k_{\tau.sl2} := \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\left(\frac{n \cdot I_{sl}}{a_2}\right)} = 2.921$$

$$k_{\tau.sl} := \max\left(k_{\tau.sl1}, k_{\tau.sl2}\right) = 2.921$$

$$k_t := \begin{cases} 4.1 + \dfrac{6.3 + 0.18 \cdot \dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}}{\alpha^{2}} + 2.2 \cdot \sqrt[3]{\dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}} & \text{if } \alpha < 3 \\ 5.34 + 4 \cdot \left(\dfrac{1}{\alpha}\right)^{2} + k_{\tau.sl} & \text{else} \end{cases} = 8.001$$

Slenderness: $\lambda_w := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.511$

Single panel (stiffeners divide plate into three equal sub-panels)

Panel aspect ratio: $\alpha := \frac{a_1}{\left(\frac{a_2}{n+1}\right)} = 8.523$

Shear buckling coefficient: $k_{tl.st} := 0$

$$k_t := \begin{cases} 5.34 + 4 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^{2} & \text{if } \alpha \ge 1 \\ 4 + 5.34 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^{2} & \text{else} \end{cases} = 5.395$$

Slenderness: $\lambda_{w1} := \frac{a_2}{(n+1) \cdot t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.208$

$\lambda_w = 0.511$ > $\lambda_{w1} = 0.208$ column section buckling is critical

$\eta := 1.2$

$$\chi_w := \begin{cases} \eta & \text{if } \lambda_w < \dfrac{0.83}{\eta} \\ \dfrac{0.83}{\lambda_w} & \text{else if } \dfrac{0.83}{\eta} \le \lambda_w < 1.08 \\ \dfrac{1.37}{(0.7 + \lambda_w)} & \text{else} \end{cases} = 1.2$$

$$\eta_3 := \frac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \chi_w} = 0.194$$ < 1 OK

Interaction of axial and shear stresses verification

$$\eta_{1.c} + \left(2 \cdot \left(\frac{\sqrt{3} \cdot (\tau_{Edmax} - \tau_{Edmin}) \cdot \gamma_{M1}}{f_y \cdot \chi_w}\right) - 1\right)^2 = 0.682$$ < 1 OK

$\sigma_{VonMises} = 90\ MPa$ < $\frac{f_y}{\gamma_{M0}} = 355\ MPa$ OK

Torsional buckling of stiffeners with open section

Second moments of area: $I_y := \frac{b_{st}^{\ 3} \cdot t_{st}}{3} = \left(3.333 \cdot 10^{-6}\right)\ m^4$ $I_z := \frac{b_{st} \cdot t_{st}^{\ 3}}{12} = \left(8.333 \cdot 10^{-9}\right)\ m^4$

Polar second moment of area: $I_p := I_y + I_z = \left(3.342 \cdot 10^{-6}\right)\ m^4$

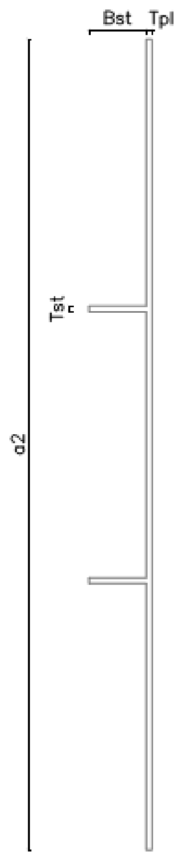
Torsional constant $I_t := \frac{b_{st} \cdot t_{st}^{\ 3}}{3} = \left(3.333 \cdot 10^{-8}\right)\ m^4$

$$\frac{I_t}{I_p} = 0.00998$$

$MinVal := 5.3 \cdot \frac{f_y}{E} = 0.00896$ $\frac{MinVal}{\frac{I_t}{I_p}} = 0.898$ < 1 OK

Cross-section (2) - Midspan

SECTION PROPERTIES



Panel width: $a_1 := 2000\ mm$

Panel height: $a_2 := 693\ mm$

Panel thickness: $t_{pl} = 16\ mm$

Stiffeners height: $b_{st} := 100\ mm$

Stiffeners thickness: $t_{st} := 10\ mm$

Number of stiffeners: $n := 2$

Stiffener area: $A_{st} := b_{st} \cdot t_{st} = 1000\ mm^2$

Equivalent shell thickness:
(from analysis model) $t_{eq} := t_{pl} + \frac{n \cdot t_{st} \cdot b_{st}}{a_2} = 0.01889\ m$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 280\ MPa$ $\sigma_{Ed.cmin} := 447\ kPa$

Tension: $\sigma_{Ed.t} := 200\ MPa$

Shear max: $\tau_{Edmax} := 134.4\ MPa$

Shear min: $\tau_{Edmin} := 4.43\ MPa$

Von Mises: $\sigma_{VonMises} := 263.4\ MPa$

Longitudinal stiffeners

Stress ratio:
(safe-sided) $\psi := 1$

Stiffener buckling factor: $k_{\sigma.st} := \frac{0.578}{(0.34 + \psi)} = 0.431$

Slenderness: $$\lambda_{st} := \frac{b_{st}}{t_{st}} \cdot \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.st}}} = 0.659$$

Reduction factor:
('outstand compression element')

$$\rho_{st} := \text{if } \lambda_{st} \le 0.748 \;\; \Big\| \; 1 \;\; \text{else} \;\; \Big\| \; \frac{\lambda_{st} - 0.188}{\lambda_{st}^{2}} \qquad = 1$$

Effective area of stiffener: $$A_{L.eff} := \rho_{st} \cdot A_{st} = 1000 \; mm^{2}$$

Sub-panels

Stress ratio:
(safe-sided)

$$\psi_1 := 1$$

Sub-panel height: $$b_1 := \frac{a_2}{(n+1)} - \frac{t_{st}}{2} = 0.226 \; m$$

Sub-panel buckling factor: $$k_{\sigma 1} := \frac{8.2}{(1.05 + \psi_1)} = 4$$

Slenderness: $$\lambda_{pl} := \frac{b_1}{28.4 \cdot t_{pl} \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}} = 0.306$$

Reduction factor:
('internal compression element')

$$\rho_1 := \text{if } \lambda_{pl} \le 0.673 \;\; \Big\| \; 1 \;\; \text{else} \;\; \Big\| \; \frac{\lambda_{pl} - 0.055 \cdot (3 + \psi_1)}{\lambda_{pl}^{2}} \qquad = 1$$

Gross width: $$b_{1sup} := \frac{2}{(5 - \psi_1)} \cdot b_1 = 0.113 \; m \qquad b_{1inf} := \frac{(3 - \psi_1)}{(5 - \psi_1)} \cdot b_1 = 0.113 \; m$$

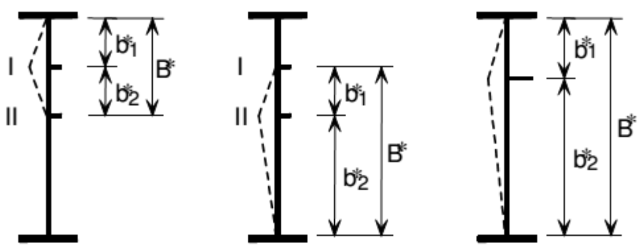
Effective width: $$b_{1.eff} := b_1 \cdot \rho_1 = 0.226 \; m$$

$$b_{1sup.eff} := \frac{2}{(5 - \psi_1)} \cdot b_{1.eff} = 0.113 \; m$$

$$b_{1inf.eff} := \frac{(3 - \psi_1)}{(5 - \psi_1)} \cdot b_{1.eff} = 0.113 \; m$$

NOTE: sub-panel checks are the same for all the three elements due to the symmetric layout of the sifeners along the lateral box panel

Critical plate buckling stress



Stiffener I — Stiffener II — Lumped stiffener

	Stiffener I	Stiffener II	Lumped stiffener
Cross-sectional area	$A_{s\ell,I}$	$A_{s\ell,II}$	$A_{s\ell,I} + A_{s\ell,II}$
Second moment of area	$I_{s\ell,I}$	$I_{s\ell,II}$	$I_{s\ell,I} + I_{s\ell,II}$

Stiffener 1:

Gross area: $A_{sl.1} := (b_{1inf} + b_{1sup} + t_{st}) \cdot t_{pl} + b_{st} \cdot t_{st} = 4776\ mm^2$

Center of gravity $x_{sl.1} := \dfrac{b_{st} \cdot t_{st} \cdot \left(\dfrac{b_{st}}{2} + \dfrac{t_{pl}}{2}\right)}{A_{sl.1}} = 0.012\ m$

Second moment of area

$$I_{sl.1} := \frac{(b_{1sup} + b_{1inf} + t_{st}) \cdot t_{pl}^3}{12} + \frac{t_{st} \cdot b_{st}^3}{12} + (b_{1sup} + b_{1inf} + t_{st}) \cdot t_{pl} \cdot x_{sl.1}^2 + t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl.1}\right)^2 = \left(3.574 \cdot 10^{-6}\right)\ m^4$$

Stiffener 2 : same as Stiffener 1

Lumped stiffener:

Gross area: $A_{sl.lumped} := 2 \cdot A_{sl.1} = 0.01\ m^2$

Second moment of area: $I_{sl.lumped} := 2 \cdot I_{sl.1} = \left(7.147 \cdot 10^{-6}\right)\ m^4$

Position: $h_{lumped} := \dfrac{a_2}{2} = 0.347\ m$

CASES 1/2: buckling of a single stiffener

Dimensions:
(refer to image above) $b_I := \dfrac{a_2}{n+1}$

$b_{II} := b_I$

$B_1 := b_I + b_{II} = 0.462\ m$

$$a_{c1} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.1} \cdot b_I^2 \cdot b_{II}^2}{t_{pl}^3 \cdot B_1}} = 1.173\ m$$

Elastic buckling stress:

$$\sigma_{cr.sl1} := \begin{cases} \dfrac{1.05 \cdot E}{A_{sl.1}} \cdot \dfrac{\sqrt{I_{sl.1} \cdot t_{pl}^3 \cdot B_1}}{b_I \cdot b_{II}} & \text{if } a_1 \ge a_{c1} \\ \dfrac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_1^2} + \dfrac{E \cdot t_{pl}^3 \cdot B_1 \cdot a_1^2}{4 \cdot \pi^2 \cdot (1-\nu^2) \cdot A_{sl.1} \cdot b_I^2 \cdot b_{II}^2} & \text{else} \end{cases} = \left(2.25 \cdot 10^3\right)\ MPa$$

Elastic plate buckling stress: $\sigma_{cr.p1} := \dfrac{a_2}{(a_2 - b_I)} \cdot \sigma_{cr.sl1} = \left(3.375 \cdot 10^3\right)\ MPa$

CASE 3: buckling of the lumped stiffener

Dimensions:
(refer to image above) $b_{I.lumped} := \dfrac{a_2}{2}$

$b_{II.lumped} := b_{I.lumped}$

$B_{1.lumped} := b_{I.lumped} + b_{II.lumped} = 0.693\ m$

$$a_{c.lumped} := 4.33 \cdot \sqrt[4]{\frac{I_{sl.lumped} \cdot b_{I.lumped}^2 \cdot b_{II.lumped}^2}{t_{pl}^3 \cdot B_{1.lumped}}} = 1.89\ m$$

Elastic buckling stress:

$$\sigma_{cr.sl.lumped} := \begin{cases} \dfrac{1.05 \cdot E}{A_{sl.1}} \cdot \dfrac{\sqrt{I_{sl.lumped} \cdot t_{pl}^{3} \cdot B_1}}{b_{I.lumped} \cdot b_{II.lumped}} & \text{if } a_1 \ge a_{c.lumped} \\ \dfrac{\pi^2 \cdot E \cdot I_{sl.lumped}}{A_{sl.lumped} \cdot a_1^{2}} + \dfrac{E \cdot t_{pl}^{3} \cdot B_{1.lumped} \cdot a_1^{2}}{4 \cdot \pi^2 \cdot (1-\nu^2) \cdot A_{sl.lumped} \cdot b_{I.lumped}^{2} \cdot b_{II.lumped}^{2}} & \text{else} \end{cases} = (1.414 \cdot 10^3)\ MPa$$

Elastic plate buckling stress: $$\sigma_{cr.p.lumped} := \frac{a_2}{(a_2 - h_{lumped})} \cdot \sigma_{cr.sl.lumped} = (2.828 \cdot 10^3)\ MPa$$

Critical buckling stress:

$$\sigma_{cr.p} := min\,(\sigma_{cr.p.lumped}, \sigma_{cr.p1}) = (2.828 \cdot 10^3)\ MPa$$

Plate type behaviour:

Compression gross area:

$$A_c := t_{pl} \cdot (a_2 - 2\ b_{1sup}) + n \cdot b_{st} \cdot t_{st} = 0.009\ m^2$$

Effective area of stiffeners: $$\Sigma A_{sl.eff} := 2 \cdot \rho_{st} \cdot (b_{st} \cdot t_{st}) = 0.002\ m^2$$

Effective compression area:

$$A_{c.eff.loc1} := \Sigma A_{sl.eff} + (2 \cdot b_{1sup.eff} + 2 \cdot b_{1inf.eff} + 2 \cdot t_{st}) \cdot t_{pl} = 0.01\ m^2$$

Area ratio: $$\beta_{A.c} := \frac{A_{c.eff.loc1}}{A_c} = 1.008$$

Slenderness: $$\lambda_p := \sqrt{\frac{\beta_{A.c} \cdot f_y}{\sigma_{cr.p}}} = 0.356$$

Sub-panel reduction factor:
('internal compression element')

$$\rho := \begin{cases} 1 & \text{if } \lambda_p \le 0.673 \\ \dfrac{\lambda_p - 0.055 \cdot (3 + \psi_1)}{\lambda_p^{2}} & \text{else} \end{cases} = 1$$

Column type buckling behaviour:

Gross area of the stiffener + adjacent parts of plate:

$$A_{sl.1} := (b_{1sup} + b_{1inf} + t_{st}) \cdot t_{pl} + \frac{\Sigma A_{sl.eff}}{2 \cdot \rho_{st}} = 0.005\ m^2$$

Effective area of the stiffener + adjacent parts of plate:

$$A_{sl.1.eff} := (b_{1sup.eff} + b_{1inf.eff} + t_{st}) \cdot t_{pl} + \frac{\Sigma A_{sl.eff}}{2} = 0.005\ m^2$$

Area ratio: $$\beta_{A.c.1} := \frac{A_{sl.1.eff}}{A_{sl.1}} = 1$$

$$\sigma_{cr.sl} := \frac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_2^{2}} = (3.229 \cdot 10^3)\ MPa$$

Elastic buckling stress: $$\sigma_{cr.c} := \frac{a_2}{(a_2 - b_I)} \cdot \sigma_{cr.sl} = (4.844 \cdot 10^3)\ MPa$$

Slenderness: $\lambda_c := \sqrt{\dfrac{\beta_{A.c.1} \cdot f_y}{\sigma_{cr.c}}} = 0.271$

Distance b/w G of plate and G of column, and b/w G of stiffener and G of column

$$e_1 := \frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl.1} = 0.046\ m$$

$$e_2 := x_{sl.1} = 0.012\ m$$

$$e := \max\left(e_1, e_2\right) = 0.046\ m$$

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 0.027\ m$$

$$\alpha_E := 0.49 + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.641$$

$$\phi := 0.5 \cdot \left(1 + \alpha_E \cdot \left(\lambda_c - 0.2\right) + \lambda_c^{\ 2}\right) = 0.559$$

Reduction factor: $\chi_c := \left(\phi + \left(\phi^2 - \lambda_c^{\ 2}\right)^{0.5}\right)^{-1} = 0.954$

Interaction between plate and column buckling behaviour

$$\xi_1 := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = -0.416$$

$$\xi := \begin{cases} 0 & \text{if } \xi_1 \le 0 \\ \xi_1 & \text{else if } 0 < \xi_1 < 1 \\ 1 & \text{else} \end{cases} = 0$$

Final reduction factor: $\rho_c := \left(\rho - \chi_c\right) \cdot \xi \cdot \left(2 - \xi\right) + \chi_c = 0.954$

Effective properties

Effective area of stiffeners+ sub-panels:

$$A_{c.eff.loc} := \begin{cases} A_{c.eff.loc1} & \text{if } \dfrac{\rho_c \cdot f_y}{\gamma_{M0} \cdot \sigma_{Ed.cmax}} > 1 \\ \dfrac{\rho_c \cdot f_y \cdot A_{sl.1}}{\gamma_{M0} \cdot \sigma_{Ed.cmax}} & \text{else} \end{cases} = 0.01\ m^2$$

Effective global area: $A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{1inf.eff} \cdot t_{pl} = 0.013\ m^2$

Gross global area: $A_c := a_2 \cdot t_{pl} + n \cdot b_{st} \cdot t_{st} = 0.013\ m^2$

Area reduction ratio: $\dfrac{A_{c.eff}}{A_c} = 0.972$ >0.5 OK

Axial stress resistance verification

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax} \cdot \gamma_{M0}}{f_y} \cdot \frac{A_c}{A_{c.eff}} = 0.811$$ < 1 OK

Shear stress resistance verification

Stiffened panel

Gross area: $A_{sl} := 30 \cdot \varepsilon \cdot t_{pl}^{2} + t_{pl} \cdot t_{st} + b_{st} \cdot t_{st} = 0.007\ m^2$

Center of gravity: $x_{sl} := \dfrac{b_{st} \cdot t_{st} \cdot \left(\dfrac{b_{st} + t_{pl}}{2}\right)}{A_{sl}} = 0.008\ m$

Second moment of area:

$$I_{sl} := \frac{(30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl}^{3}}{12} + \frac{t_{st} \cdot b_{st}^{3}}{12} + (30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl} \cdot x_{sl}^{2} + t_{st} \cdot b_{st} \cdot \left(\frac{t_{pl} + b_{st}}{2} - x_{sl}\right)^{2} = \left(3.88 \cdot 10^{-6}\right)\ m^4$$

Aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 2.886$

Shear buckling coefficient: $k_{\tau.sl1} := 9 \cdot \left(\dfrac{1}{\alpha}\right)^{2} \cdot \sqrt[4]{\left(\dfrac{n \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}\right)^{3}} = 2.297$

$$k_{\tau.sl2} := \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\left(\frac{n \cdot I_{sl}}{a_2}\right)} = 2.936$$

$$k_{\tau.sl} := \max\left(k_{\tau.sl1}, k_{\tau.sl2}\right) = 2.936$$

$$k_t := \begin{cases} 4.1 + \dfrac{6.3 + 0.18 \cdot \dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}}{\alpha^{2}} + 2.2 \cdot \sqrt[3]{\dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}} & \text{if } \alpha < 3 \\ 5.34 + 4 \cdot \left(\dfrac{1}{\alpha}\right)^{2} + k_{\tau.sl} & \text{else} \end{cases} = 7.992$$

Slenderness: $\lambda_w := \dfrac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.504$

Single panel (stiffeners divide plate into three equal sub-panels)

Panel aspect ratio: $\alpha := \dfrac{a_1}{\left(\dfrac{a_2}{n+1}\right)} = 8.658$

Shear buckling coefficient: $k_{tl.st} := 0$

$$k_t := \begin{cases} 5.34 + 4 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^{2} & \text{if } \alpha \geq 1 \\ 4 + 5.34 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^{2} & \text{else} \end{cases} = 5.393$$

Slenderness: $\lambda_{w1} := \dfrac{a_2}{(n+1) \cdot t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.204$

$\lambda_w = 0.504$ > $\lambda_{w1} = 0.204$ column section buckling is critical

0.83

$\eta := 1.2$

$$\chi_w := \begin{array}{l} \text{if } \lambda_w < \dfrac{0.83}{\eta} \\ \quad \| \ \eta \\ \text{else if } \dfrac{0.83}{\eta} \le \lambda_w < 1.08 \\ \quad \| \ \dfrac{0.83}{\lambda_w} \\ \text{else} \\ \quad \| \ \dfrac{1.37}{(0.7+\lambda_w)} \end{array} = 1.2$$

$$\eta_3 := \frac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \chi_w} = 0.601$$ < 1 OK

Interaction of axial and shear stresses verification

$$\eta_{1.c} + \left(2 \cdot \left(\frac{\sqrt{3} \cdot (\tau_{Edmax} - \tau_{Edmin}) \cdot \gamma_{M1}}{f_y \cdot \chi_w}\right) - 1\right)^2 = 0.838$$ < 1 OK

$\sigma_{VonMises} = 263.4\ MPa$ < $\dfrac{f_y}{\gamma_{M0}} = 355\ MPa$ OK

Torsional buckling of stiffeners with open section

Second moments of area: $I_y := \dfrac{{b_{st}}^3 \cdot t_{st}}{3} = (3.333 \cdot 10^{-6})\ m^4$ $I_z := \dfrac{b_{st} \cdot {t_{st}}^3}{12} = (8.333 \cdot 10^{-9})\ m^4$

Polar second moment of area: $I_p := I_y + I_z = (3.342 \cdot 10^{-6})\ m^4$

Torsional constant $I_t := \dfrac{b_{st} \cdot {t_{st}}^3}{3} = (3.333 \cdot 10^{-8})\ m^4$

$\dfrac{I_t}{I_p} = 0.00998$

$MinVal := 5.3 \cdot \dfrac{f_y}{E} = 0.00896$ $\dfrac{MinVal}{\dfrac{I_t}{I_p}} = 0.898$ < 1 OK

Box bottom panel ***(stiffened by trapezoidal stiffeners)***

Cross-section (1) - Anchorage of the sixth stay-cable (from abutments)

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\varepsilon := \sqrt{\dfrac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \dfrac{E}{2 \cdot (1+\nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.cmax} := 88.08\ MPa$ $\sigma_{Ed.cmin} := 3.68\ MPa$

Tension: $\sigma_{Ed.t} := 100.50\ MPa$

Shear max: $\tau_{Edmax} := 43.28\ MPa$

Shear min: $\tau_{Edmin} := 4.50\ MPa$

Von Mises: $\sigma_{VonMises} := 90\ MPa$

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 1370\ mm$

Plate thickness: $t_{pl} := 18\ mm$

STIFFENERS

Number of stiffeners: $n_{st} := 2$

Thickness: $t_{st} := 11\ mm$

Major width: $B_{st} := 282\ mm$

Minor width: $b_{st} := 165\ mm$

Depth: $d_{st} := 196\ mm$

Inclined width: $a := \sqrt{d_{st}^2 + \left(\frac{B_{st} - b_{st}}{2}\right)^2} = 204.544\ mm$

Angle: $\theta := \operatorname{acos}\left(\frac{d_{st}}{a}\right) = 0.29\ rad$

Center of gravity (stiffened plate):

$$y_G := \frac{n_{st} \cdot \left(b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2}\right) + 2 \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2}\right)\right)}{n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}\right) + a_2 \cdot t_{pl}} = 45.245\ mm$$

Second moment of area (plate):

$$I_{pl} := a_2 \cdot \frac{t_{pl}^3}{12 \cdot \left(1 - \nu^2\right)} + a_2 \cdot t_{pl} \cdot \frac{y_G^2}{\left(1 - \nu^2\right)} = \left(5.621 \cdot 10^7\right)\ mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{t_{st}^3}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G\right)^2 + 2 \cdot \left(a \cdot \frac{t_{st}^3}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2} \cos(\theta) + \frac{t_{pl}}{2} - y_G\right)^2\right)$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(1.796 \cdot 10^8\right)\ mm^4$$

Properties for equivalent thickness
(see Point C) :

$$I_{BD} := I_{tot} = \left(1.796 \cdot 10^8\right)\ mm^4$$

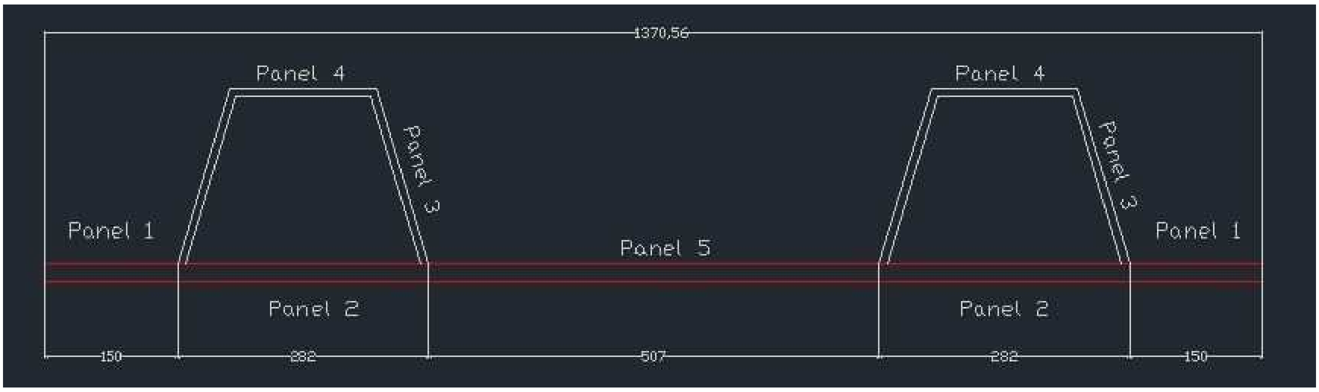
$$A_{BD} := n_{st} \cdot \left(b_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}\right) + a_2 \cdot t_{pl} = \left(3.729 \cdot 10^4\right)\ mm^2$$

$$y_{G.BD} := y_G + \frac{t_{pl}}{2} = 54.245\ mm$$

(distance b/w G of box bottom panel and bottom panel external surface)

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels



Panel 5 (INTERNAL ELEMENT)

Panel width: $b_{pl5} := 507\ mm$

Panel thickness: $t_{pl5} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 5} := 4$

Slenderness: $\lambda_{p5} := \dfrac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_5 := \begin{cases} 1 & \text{if } \lambda_{p5} \le 0.673 \\ \dfrac{\lambda_{p5} - 0.055 \cdot (3 + \psi)}{\lambda_{p5}^{\,2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 507\ mm$

Effective panel area: $A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(9.126 \cdot 10^3\right)\ mm^2$

Panel 1 (INTERNAL ELEMENT)

Panel width: $b_{pl1} := 150\ mm$

Panel thickness: $t_{pl1} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 1} := 4$

Slenderness: $\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 1}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_1 := \begin{cases} 1 & \text{if } \lambda_{p1} \le 0.673 \\ \dfrac{\lambda_{p1} - 0.055 \cdot (3 + \psi)}{\lambda_{p1}^{\,2}} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 150\ mm$

Effective panel area: $A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(2.7 \cdot 10^3\right)\ mm^2$

Panel 2 (INTERNAL ELEMENT)

Panel width: $b_{pl2} := B_{st} = 282\ mm$

Panel thickness: $t_{pl2} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 2} := 4$

Slenderness: $\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \text{if } \lambda_{p2} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p2} - 0.055 \cdot (3 + \psi)}{\lambda_{p2}^{\ 2}} \; = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 282\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(5.076 \cdot 10^3\right)\ mm^2$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := a = 204.544\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 3} := 4$

Slenderness: $\lambda_{p3} := \dfrac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 3}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_3 := \text{if } \lambda_{p3} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p3} - 0.055 \cdot (3 + \psi)}{\lambda_{p3}^{\ 2}} \; = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 204.544\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(2.25 \cdot 10^3\right)\ mm^2$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := b_{st} = 165\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness: $\lambda_{p4} := \dfrac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_4 := \text{if } \lambda_{p4} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p4} - 0.055 \cdot (3 + \psi)}{\lambda_{p4}^{\ 2}} \; = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 165\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.815 \cdot 10^3\right)\ mm^2$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(1.263 \cdot 10^4\right)\ mm^2$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.263 \cdot 10^4\right)\ mm^2$

Total effective section under compression:

$$A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(3.461 \cdot 10^4\right)\ mm^2$$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(3.461 \cdot 10^4\right)\ mm^2$$

Area ratio: $\delta := \dfrac{A_{sl}}{t_{pl} \cdot a_2} = 0.512$

Plate aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1.46$

Inertial ratio: $\gamma := \dfrac{I_{tot}}{a_2 \cdot \dfrac{t_{pl}^{\ 3}}{12\left(1-\nu^2\right)}} = 245.452$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} = 78.896$$

$$k_{\sigma.p} := \begin{array}{l} \text{if } \alpha \le \sqrt[4]{\gamma} \\ \quad \left\| \dfrac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} \right. \\ \text{else} \\ \quad \left\| \dfrac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1) \cdot (1+\delta)} \right. \end{array} = 78.896$$

Euler plate buckling stress: $\sigma_E := \pi^2 \cdot E \cdot \dfrac{t_{pl}^{\ 2}}{12 \cdot \left(1-\nu^2\right) \cdot a_2^{\ 2}} = 32.764\ MPa$

Critical elastic plate buckling stress: $\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = \left(2.585 \cdot 10^3\right)\ MPa$

Slenderness of stiffened plate: $\beta_{A.c} := \dfrac{A_{c.eff.loc}}{A_c} = 1$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.371$$

$$\rho := \text{if } \lambda_{p\_} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{{\lambda_{p\_}}^{2}} = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener (b/w panel 1 and panel 5)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 75\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl5} = 253.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 141\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 141\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl5} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(1.73 \cdot 10^{4}\right)\ mm^{2}$$

Angle:

$$\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.29\ rad$$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl2}} = 48.751\ mm$$

Second moment of area (column):

$$I_{sl2} := b_{pl4} \cdot \frac{{t_{pl4}}^{3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^{2} + 2 \cdot \left(b_{pl3} \cdot \frac{{t_{pl3}}^{3}}{12} \cdot (\cos(\theta))^{2} + t_{pl3} \cdot \frac{{b_{pl3}}^{3}}{12} \cdot (\sin(\theta))^{2} + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^{2}\right) +$$

Elastic critical column buckling stress:

$$\sigma_{cr.sl2} := \pi^{2} \cdot E \cdot \frac{I_{sl2}}{A_{sl2} \cdot {a_1}^{2}} = \left(3.011 \cdot 10^{3}\right)\ MPa$$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1-------->

$$\sigma_{cr.c2} := \sigma_{cr.sl2} = \left(3.011 \cdot 10^{3}\right)\ MPa$$

Slenderness of column *(column with the least critical stress is considered)*:

$$\sigma_{cr.c} := \sigma_{cr.c2} = \left(3.011 \cdot 10^{3}\right)\ MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.595 \cdot 10^{4}\right)\ mm^{2}$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 1.34$$

Slenderness of column:

$$\lambda_c := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.c}}} = 0.398$$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 133.586\ mm$$

Distance: $e_1 := y_{Gst} - y_{G.c1} = 98.88\ mm$

Distance between plate and column centroids: $e_2 := y_{G.c1} = 34.706\ mm$

$$e := \max\left(e_1, e_2\right) = 98.88\ mm$$

$\alpha := 0.34$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 50.258\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.517$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + {\lambda_c}^2\right) = 0.63$$

Reduction factor: $\chi_c := \dfrac{1}{\phi + \sqrt{\phi^2 - {\lambda_c}^2}} = 0.894$

Interaction between plate and column buckling behaviours

$$\xi\_ := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = -0.142$$

$$\xi := \begin{array}{l} \text{if } \xi\_ \le 0 \\ \quad \| \ 0 \\ \text{else if } \xi\_ \ge 1 \\ \quad \| \ 1 \\ \text{else} \\ \quad \| \ \xi\_ \end{array} = 0$$

Final reduction factor: $\rho_c := \left(\rho - \chi_c\right) \cdot \xi \cdot \left(2 - \xi\right) + \chi_c = 0.894$

Effective cross-sectional area: $A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \dfrac{t_{pl1}}{2} = \left(3.363 \cdot 10^4\right)\ mm^2$

Axial stress resistance verification

$$A := a_2 \cdot t_{pl} + n_{st} \cdot \left(2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}\right) = \left(3.729 \cdot 10^4\right)\ mm^2$$

$\eta_{1.c} := \dfrac{\sigma_{Ed.cmax} \cdot A}{\dfrac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.275$ < 1 OK (compression)

$\eta_{1.t} := \dfrac{\sigma_{Ed.t}}{\dfrac{f_y}{\gamma_{M1}}} = 0.311$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$$\eta := 1.2 \qquad \text{(S355)}$$

Effective width for shear check:

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 141\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 219.676\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 75\ mm$$

Second moment of area:

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \left(b_{1.2} + 2 \cdot b_{2.2} + b_{3.2}\right) \cdot t_{pl}} = 50.529\ mm$$

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2}\cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^2\right)$$

$$I_{sl2''} := \left(b_{1.2} + b_{2.2}\right) \cdot \frac{t_{pl}^{\ 3}}{12} + \left(b_{1.2} + b_{2.2}\right) \cdot t_{pl} \cdot y_{G2}^{\ 2} + \left(b_{2.2} + b_{3.2}\right) \cdot \frac{t_{pl}^{\ 3}}{12} + \left(b_{2.1} + b_{3.1}\right) \cdot t_{pl} \cdot y_{G2}^{\ 2}$$

$$I_{sl} := I_{sl2'} + I_{sl2''} = \left(8.605 \cdot 10^7\right)\ mm^4$$

Shear buckling coefficient:

$$k_{\tau sl} := \begin{cases} 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\dfrac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} & \text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\dfrac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \ge \dfrac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\dfrac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} \\ \dfrac{2.1}{t_{pl}} \cdot \sqrt[3]{\dfrac{I_{sl}}{a_2}} & \text{else} \end{cases} = 1.917 \cdot 10^3$$

$$k_{\tau} := \begin{cases} 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} & \text{if } \frac{a_1}{a_2} \ge 1 \\ 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} & \text{else} \end{cases} = 1.925 \cdot 10^3$$

Slenderness:

$$\lambda_{w1} := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau}}} = 0.057$$

Sub-panels

Panel 1 $a_2 := b_{pl1} = 150\ mm$

Shear buckling coeff.:
$$k_{\tau.p1} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.363$$

Slenderness:
$$\lambda_{w.p1} := \frac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.118$$

Panel 2 $a_2 := b_{pl2} = 282\ mm$

Shear buckling coeff.:
$$k_{\tau.p2} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.42$$

Slenderness:
$$\lambda_{w.p2} := \frac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.221$$

Panel 5 $a_2 := b_{pl5} = 507\ mm$

Shear buckling coeff.:
$$k_{\tau.p5} := \text{if } \frac{a_1}{a_2} \geq 1 \;\; \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \;\; \text{else} \;\; \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.597$$

Slenderness:
$$\lambda_{w.p5} := \frac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.391$$

Final slenderness:

$$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.391 \quad < \quad \frac{0.83}{\eta} = 0.692$$

$$\chi_w := \eta$$

Shear stress resistance verification

$$\eta_3 := \frac{\tau_{Edmax}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.065$$ < 1 OK

AXIAL-SHEAR STRESS INTERACTION *(N/A since shear is very small)*

$$\eta_{1.cm} := \frac{\frac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.143$$

$$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$$

Verification $\eta_{1.cm} + \left(2 \cdot \eta_{3m} - 1\right)^2 = 1.006$ < 1 N/A

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

(assuming that stiffeners act as rigid support)

$\lambda_w := \max\left(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.391$ < $\frac{0.83}{\eta} = 0.692$ $\chi_w := \eta$

Verification $$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.036$$ < 1 OK

Cross-section (2) - Midspan

GENERAL DATA

Safety factors:		$\gamma_{M0} := 1.0$	$\gamma_{M1} := 1.1$
Steel type:	S355		
Yielding strength:		$f_y := 355\ MPa$	$\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$
Rupture strength:		$f_u := 510\ MPa$	
Elastic moduli:		$E := 210000\ MPa$	$\nu := 0.3$ $\quad G := \frac{E}{2 \cdot (1 + \nu)} = 80769\ MPa$

MAXIMUM STRESSES

Compression:	$\sigma_{Ed.cmax} := 280\ MPa$	$\sigma_{Ed.cmin} := 447\ kPa$
Tension:	$\sigma_{Ed.t} := 200\ MPa$	
Shear max:	$\tau_{Edmax} := 134.4\ MPa$	
Shear min:	$\tau_{Edmin} := 4.43\ MPa$	
Von Mises:	$\sigma_{VonMises} := 263.4\ MPa$	

PLATE GEOMETRY

Plate length: $a_1 := 2000\ mm$

Plate width: $a_2 := 1370\ mm$

Plate thickness: $t_{pl} := 18\ mm$

STIFFENERS

Number of stiffeners: $n_{st} := 2$

Thickness: $t_{st} := 11\ mm$

Major width: $B_{st} := 282\ mm$

Minor width: $b_{st} := 165\ mm$

Depth: $d_{st} := 196\ mm$

Inclined width: $a := \sqrt{{d_{st}}^2 + \left(\frac{B_{st} - b_{st}}{2}\right)^2} = 204.544\ mm$

Angle: $\theta := \mathrm{acos}\left(\frac{d_{st}}{a}\right) = 0.29\ rad$

Center of gravity (stiffened plate):

$$y_G := \frac{b_{st} \cdot t_{st} \cdot \left(d_{st} - \frac{t_{st}}{2} + \frac{t_{pl}}{2}\right) + 2 \cdot a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2}\cos(\theta) + \frac{t_{pl}}{2}\right)}{b_{st} \cdot t_{st} + B_{st} \cdot t_{st} + 2 \cdot a \cdot t_{st}} = 89.582\ mm$$

Second moment of area (plate):

$$I_{pl} := a_2 \cdot \frac{{t_{pl}}^3}{12 \cdot (1 - \nu^2)} + a_2 \cdot t_{pl} \cdot \frac{{y_G}^2}{(1 - \nu^2)} = \left(2.182 \cdot 10^8\right)\ mm^4$$

Second moment of area (stiffener):

$$I_{st} := b_{st} \cdot \frac{{t_{st}}^3}{12} + b_{st} \cdot t_{st} \cdot \left(d_{st} + \frac{t_{pl}}{2} - \frac{t_{st}}{2} - y_G\right)^2 + 2 \cdot \left(a \cdot \frac{{t_{st}}^3}{12} \cdot (\cos(\theta))^2 + t_{st} \cdot \frac{a^3}{12} \cdot (\sin(\theta))^2 + a \cdot t_{st} \cdot \left(d_{st} - \frac{a}{2}\cos(\theta) + \frac{t_{pl}}{2} - y_G\right)^2\right)$$

Second moment of area (stiffened plate):

$$I_{tot} := I_{pl} + n_{st} \cdot I_{st} = \left(2.675 \cdot 10^8\right)\ mm^4$$

AXIAL RESISTANCE VERIFICATION

Calculation of effective areas of sub-panels

Panel 5 (INTERNAL ELEMENT)

Panel width: $b_{pl5} := 507\ mm$

Panel thickness: $t_{pl5} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 5} := 4$

Slenderness: $\lambda_{p5} := \frac{b_{pl5}}{t_{pl5} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 5}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_5 := \begin{cases} 1 & \text{if } \lambda_{p5} \le 0.673 \\ \frac{\lambda_{p5} - 0.055 \cdot (3 + \psi)}{{\lambda_{p5}}^2} & \text{else} \end{cases} = 1$$

Effective panel width: $b_{eff.pl5} := \rho_5 \cdot b_{pl5} = 507\ mm$

Effective panel area: $A_{eff.pl5} := b_{eff.pl5} \cdot t_{pl5} = \left(9.126 \cdot 10^3\right)\ mm^2$

Panel 1 (INTERNAL ELEMENT)

Panel width: $b_{pl1} := 150\ mm$

Panel thickness: $t_{pl1} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma1} := 4$

Slenderness: $\lambda_{p1} := \dfrac{b_{pl1}}{t_{pl1} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma1}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_1 := \text{if } \lambda_{p1} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p1} - 0.055 \cdot (3 + \psi)}{\lambda_{p1}^{2}} \quad = 1$$

Effective panel width: $b_{eff.pl1} := \rho_1 \cdot b_{pl1} = 150\ mm$

Effective panel area: $A_{eff.pl1} := b_{eff.pl1} \cdot t_{pl1} = \left(2.7 \cdot 10^3\right)\ mm^2$

Panel 2 (INTERNAL ELEMENT)

Panel width: $b_{pl2} := B_{st} = 282\ mm$

Panel thickness: $t_{pl2} := t_{pl} = 18\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma2} := 4$

Slenderness: $\lambda_{p2} := \dfrac{b_{pl2}}{t_{pl2} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma2}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_2 := \text{if } \lambda_{p2} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p2} - 0.055 \cdot (3 + \psi)}{\lambda_{p2}^{2}} \quad = 1$$

Effective panel width: $b_{eff.pl2} := \rho_2 \cdot b_{pl2} = 282\ mm$

Effective panel area: $A_{eff.pl2} := b_{eff.pl2} \cdot t_{pl2} = \left(5.076 \cdot 10^3\right)\ mm^2$

Panel 3 (INTERNAL ELEMENT)

Panel width: $b_{pl3} := a = 204.544\ mm$

Panel thickness: $t_{pl3} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma3} := 4$

Slenderness: $\lambda_{p3} := \dfrac{b_{pl3}}{t_{pl3} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma3}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_3 := \text{if } \lambda_{p3} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p3} - 0.055 \cdot (3 + \psi)}{\lambda_{p3}^2} = 1$$

Effective panel width: $b_{eff.pl3} := \rho_3 \cdot b_{pl3} = 204.544\ mm$

Effective panel area: $A_{eff.pl3} := b_{eff.pl3} \cdot t_{pl3} = \left(2.25 \cdot 10^3\right)\ mm^2$

Panel 4 (INTERNAL ELEMENT)

Panel width: $b_{pl4} := b_{st} = 165\ mm$

Panel thickness: $t_{pl4} := t_{st} = 11\ mm$

Panel stress ratio (safe-sided): $\psi := 1$

Buckling factor: $k_{\sigma 4} := 4$

Slenderness: $\lambda_{p4} := \dfrac{b_{pl4}}{t_{pl4} \cdot \left(28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma 4}}\right)}$

Reduction factor:
('internal compression element')

$$\rho_4 := \text{if } \lambda_{p4} \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_{p4} - 0.055 \cdot (3 + \psi)}{\lambda_{p4}^2} = 1$$

Effective panel width: $b_{eff.pl4} := \rho_4 \cdot b_{pl4} = 165\ mm$

Effective panel area: $A_{eff.pl4} := b_{eff.pl4} \cdot t_{pl4} = \left(1.815 \cdot 10^3\right)\ mm^2$

PLATE TYPE BUCKLING BEHAVIOUR

Sum of gross areas of stiffeners: $A_{sl} := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} = \left(1.263 \cdot 10^4\right)\ mm^2$

Sum of effective areas of stiffeners: $A_{sl.eff} := n_{st} \cdot b_{eff.pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.263 \cdot 10^4\right)\ mm^2$

Total effective section under compression:

$$A_{c.eff.loc} := A_{sl.eff} + A_{eff.pl5} + n_{st} \cdot A_{eff.pl2} + A_{eff.pl1} = \left(3.461 \cdot 10^4\right)\ mm^2$$

Total gross section under compression:

$$A_c := n_{st} \cdot b_{pl4} \cdot t_{pl4} + 2 \cdot n_{st} \cdot b_{pl3} \cdot t_{pl3} + b_{pl5} \cdot t_{pl5} + n_{st} \cdot b_{pl2} \cdot t_{pl2} + b_{pl1} \cdot t_{pl1} = \left(3.461 \cdot 10^4\right)\ mm^2$$

Area ratio: $\delta := \dfrac{A_{sl}}{t_{pl} \cdot a_2} = 0.512$

Plate aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1.46$

Inertial ratio:

$$\gamma := \frac{I_{tot}}{a_2 \cdot \frac{t_{pl}^{\ 3}}{12\left(1-\nu^2\right)}} = 365.564$$

$$k_{\sigma.p} := \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} = 116.167$$

$$k_{\sigma.p} := \text{if } \alpha \le \sqrt[4]{\gamma} \; \left\| \; \frac{2 \cdot \left(\left(1+\alpha^2\right)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi+1) \cdot (1+\delta)} \right. \; \text{else} \; \left\| \; \frac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1) \cdot (1+\delta)} \right. = 116.167$$

Euler plate buckling stress:

$$\sigma_E := \pi^2 \cdot E \cdot \frac{t_{pl}^{\ 2}}{12 \cdot \left(1-\nu^2\right) \cdot a_2^{\ 2}} = 32.764\ MPa$$

Critical elastic plate buckling stress:

$$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = \left(3.806 \cdot 10^3\right)\ MPa$$

Slenderness of stiffened plate:

$$\beta_{A.c} := \frac{A_{c.eff.loc}}{A_c} = 1$$

$$\lambda_{p\_} := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.p}}} = 0.305$$

$$\rho := \text{if } \lambda_{p\_} \le 0.673 \; \| \; 1 \; \text{else} \; \left\| \; \frac{\lambda_{p\_} - 0.055 \cdot (3+\psi)}{\lambda_{p\_}^{\ 2}} \right. = 1$$

COLUMN TYPE BUCKLING BEHAVIOUR

Column-stiffener (b/w panel 1 and panel 5)

$$b_{I.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl1} = 75\ mm \qquad b_{I.sup2} := \frac{2}{(5-\psi)} \cdot b_{pl5} = 253.5\ mm$$

$$b_{II.inf2} := \frac{3-\psi}{5-\psi} \cdot b_{pl2} = 141\ mm \qquad b_{II.sup2} := \frac{2}{5-\psi} \cdot b_{pl2} = 141\ mm$$

$$A_{sl2} := b_{I.inf2} \cdot t_{pl1} + b_{I.sup2} \cdot t_{pl5} + b_{II.inf2} \cdot t_{pl2} + b_{II.sup2} \cdot t_{pl2} + 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} = \left(1.73 \cdot 10^4\right)\ mm^2$$

Angle:

$$\theta := \operatorname{acos}\left(\frac{d_{st}}{b_{pl3}}\right) = 0.29\ rad$$

Center of gravity (column):

$$y_{G.c2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{A_{sl2}} = 48.751\ mm$$

Second moment of area (column):

$$I_{sl2} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G.c1}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G.c1}\right)^2\right) +$$

Elastic critical column buckling stress: $\sigma_{cr.sl2} := \pi^2 \cdot E \cdot \frac{I_{sl2}}{A_{sl2} \cdot a_1^{\ 2}} = \left(3.011 \cdot 10^3\right)\ MPa$

Extrapolation of elastic critical stress to the edge of panel

ψ =1 --------> bc=b.sl1--------> $\sigma_{cr.c2} := \sigma_{cr.sl2} = \left(3.011 \cdot 10^3\right)\ MPa$

Slenderness of column *(column with the least critical stress is considered)*:

$$\sigma_{cr.c} := \sigma_{cr.c2} = \left(3.011 \cdot 10^3\right)\ MPa$$

Effective cross-sectional area of stiffener:

$$A_{sl1.eff} := b_{eff.pl2} \cdot t_{pl2} + b_{eff.pl5} \cdot \frac{t_{pl5}}{2} + b_{eff.pl4} \cdot t_{pl4} + 2 \cdot b_{eff.pl3} \cdot t_{pl3} = \left(1.595 \cdot 10^4\right)\ mm^2$$

$$\beta_{A.c} := \frac{A_{sl1.eff}}{A_{sl1}} = 1.34$$

Slenderness of column: $\lambda_c := \sqrt{\beta_{A.c} \cdot \frac{f_y}{\sigma_{cr.c}}} = 0.398$

Calculation of distance between stiffener and column centroids

$$y_{Gst} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4}} = 133.586\ mm$$

Distance: $e_1 := y_{Gst} - y_{G.c1} = 98.88\ mm$

Distance between plate and column centroids: $e_2 := y_{G.c1} = 34.706\ mm$

$$e := \max\left(e_1, e_2\right) = 98.88\ mm$$

$\alpha := 0.34$ (curve b for closed section stiffeners)

$$i := \sqrt{\frac{I_{sl1}}{A_{sl1}}} = 50.258\ mm$$

$$\alpha_e := \alpha + \frac{0.09}{\left(\frac{i}{e}\right)} = 0.517$$

$$\phi := 0.5 \cdot \left(1 + \alpha_e \cdot \left(\lambda_c - 0.2\right) + \lambda_c^{\ 2}\right) = 0.63$$

Reduction factor: $\chi_c := \frac{1}{\phi + \sqrt{\phi^2 - \lambda_c^{\ 2}}} = 0.894$

Interaction between plate and column buckling behaviours

$$\xi\_ := \frac{\sigma_{cr.p}}{\sigma_{cr.c}} - 1 = 0.264$$

$$\xi := \text{if } \xi\_ \le 0 \; \| \; 0 \;\; \text{else if } \xi\_ \ge 1 \; \| \; 1 \;\; \text{else} \; \| \; \xi\_ \quad = 0.264$$

Final reduction factor: $\rho_c := \langle \rho - \chi_c \rangle \cdot \xi \cdot (2 - \xi) + \chi_c = 0.942$

Effective cross-sectional area: $A_{c.eff} := \rho_c \cdot A_{c.eff.loc} + 2 \cdot b_{eff.pl1} \cdot \frac{t_{pl1}}{2} = \left(3.531 \cdot 10^4\right)\ mm^2$

Axial stress resistance verification

$$A := a_2 \cdot t_{pl} + n_{st} \cdot \langle 2 \cdot b_{pl3} \cdot t_{pl3} + b_{pl4} \cdot t_{pl4} \rangle = \left(3.729 \cdot 10^4\right)\ mm^2$$

$\eta_{1.c} := \dfrac{\sigma_{Ed.cmax} \cdot A}{\dfrac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.833$ < 1 OK (compression)

$\eta_{1.t} := \dfrac{\sigma_{Ed.t}}{\dfrac{f_y}{\gamma_{M1}}} = 0.62$ < 1 OK (tension)

SHEAR RESISTANCE VERIFICATION

$\eta := 1.2$ (S355)

Effective width for shear check:

$$b_{2.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl2}}{2}\right) = 141\ mm$$

$$b_{1.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl5}}{2}\right) = 219.676\ mm$$

$$b_{3.2} := min\left(15 \cdot \varepsilon \cdot t_{pl}, \frac{b_{eff.pl1}}{2}\right) = 75\ mm$$

Second moment of area:

$$y_{G2} := \frac{b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} - \frac{t_{pl4}}{2} + \frac{t_{pl2}}{2}\right) + 2 \cdot b_{pl3} \cdot t_{pl3} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2}\right)}{b_{pl4} \cdot t_{pl4} + 2 \cdot b_{pl3} \cdot t_{pl3} + \langle b_{1.2} + 2 \cdot b_{2.2} + b_{3.2} \rangle \cdot t_{pl}} = 50.529\ mm$$

$$I_{sl2'} := b_{pl4} \cdot \frac{t_{pl4}^{\ 3}}{12} + b_{pl4} \cdot t_{pl4} \cdot \left(d_{st} + \frac{t_{pl2}}{2} - \frac{t_{pl4}}{2} - y_{G2}\right)^2 + 2 \cdot \left(b_{pl3} \cdot \frac{t_{pl3}^{\ 3}}{12} \cdot (\cos(\theta))^2 + t_{pl3} \cdot \frac{b_{pl3}^{\ 3}}{12} \cdot (\sin(\theta))^2 + b_{pl3} \cdot t_{st} \cdot \left(d_{st} - \frac{b_{pl3}}{2} \cos(\theta) + \frac{t_{pl2}}{2} - y_{G2}\right)^2\right)$$

$$I_{sl2''} := \langle b_{1.2} + b_{2.2} \rangle \cdot \frac{t_{pl}^{\ 3}}{12} + \langle b_{1.2} + b_{2.2} \rangle \cdot t_{pl} \cdot y_{G2}^{\ 2} + \langle b_{2.2} + b_{3.2} \rangle \cdot \frac{t_{pl}^{\ 3}}{12} + \langle b_{2.1} + b_{3.1} \rangle \cdot t_{pl} \cdot y_{G2}^{\ 2}$$

$$I_{sl} := I_{sl2'} + I_{sl2''} = \left(8.605 \cdot 10^7\right)\ mm^4$$

Shear buckling coefficient:

$$k_{\tau sl} := \text{if } 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \geq \frac{2.1}{\frac{t_{pl}}{mm}} \cdot \sqrt[3]{\frac{\frac{I_{sl}}{mm^4}}{\frac{a_2}{mm}}} = 1.917 \cdot 10^3$$

$$\left\| \; 9 \cdot \left(\frac{a_2}{a_1}\right)^2 \cdot \sqrt[4]{\left(\frac{\frac{I_{sl}}{mm^4}}{t_{pl} \cdot \frac{a_2}{mm^2}}\right)^3} \right.$$

else

$$\left\| \; \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\frac{I_{sl}}{a_2}} \right.$$

$$k_{\tau} := \text{if } \frac{a_1}{a_2} \geq 1 = 1.925 \cdot 10^3$$

$$\left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right.$$

else

$$\left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 + k_{\tau sl} \right.$$

Slenderness: $$\lambda_{w1} := \frac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau}}} = 0.057$$

Sub-panels

Panel 1 $$a_2 := b_{pl1} = 150\ mm$$

Shear buckling coeff.: $$k_{\tau.p1} := \text{if } \frac{a_1}{a_2} \geq 1 = 5.363$$

$$\left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right.$$

else

$$\left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right.$$

Slenderness: $$\lambda_{w.p1} := \frac{a_2}{t_{pl1} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p1}}} = 0.118$$

Panel 2 $$a_2 := b_{pl2} = 282\ mm$$

Shear buckling coeff.: $$k_{\tau.p2} := \text{if } \frac{a_1}{a_2} \geq 1 = 5.42$$

$$\left\| \; 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right.$$

else

$$\left\| \; 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right.$$

Slenderness: $$\lambda_{w.p2} := \frac{a_2}{t_{pl2} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p2}}} = 0.221$$

Panel 5 $a_2 := b_{pl5} = 507\ mm$

Shear buckling coeff.: $$k_{\tau.p5} := \text{if } \frac{a_1}{a_2} \geq 1 \left\| 5.34 + 4 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. \text{ else } \left\| 4 + 5.34 \cdot \left(\frac{a_2}{a_1}\right)^2 \right. = 5.597$$

Slenderness: $$\lambda_{w.p5} := \frac{a_2}{t_{pl5} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_{\tau.p5}}} = 0.391$$

Final slenderness:

$$\lambda_w := \max\left(\lambda_{w1}, \lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.391 \qquad < \qquad \frac{0.83}{\eta} = 0.692$$

$$\chi_w := \eta$$

Shear stress resistance verification

$$\eta_3 := \frac{\tau_{Edmax}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.2$$ < 1 OK

AXIAL-SHEAR STRESS INTERACTION

$$\eta_{1.cm} := \frac{\frac{\left(\sigma_{Ed.cmax} + \sigma_{Ed.cmin}\right)}{2} \cdot A}{\frac{f_y}{\gamma_{M0}} \cdot A_{c.eff}} = 0.417$$

$$\eta_{3m} := \frac{\frac{\left(\tau_{Edmax} + \tau_{Edmin}\right)}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103$$

Verification $$\eta_{1.cm} + \left(2 \cdot \eta_{3m} - 1\right)^2 = 1.046$$ < 1 OK

ADDITIONAL VERIFICATION OF SHEAR BUCKLING OF THE SUB-PANELS

(assuming that stiffeners act as rigid support)

$$\lambda_w := \max\left(\lambda_{w.p1}, \lambda_{w.p2}, \lambda_{w.p5}\right) = 0.391 \qquad < \qquad \frac{0.83}{\eta} = 0.692 \qquad \chi_w := \eta$$

Verification

$$\eta_{3m} := \frac{\frac{(\tau_{Edmax} + \tau_{Edmin})}{2}}{\chi_w \cdot \frac{f_y}{\gamma_{M1}} \cdot \sqrt{3}} = 0.103$$

< 1 OK

B. Cantilevered panels verification

***Cantilevered panel** (stiffened by open-section stiffeners)*

Cross-section (1) - Anchorage of the sixth stay-cable (from abutments)

GENERAL DATA

Safety factors $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $f_{yd} := \frac{f_y}{\gamma_{M0}} = 355\ MPa$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1+\nu)} = (8.077 \cdot 10^{10})\ Pa$

Stiffeners height: $b_{st} := 140\ mm$

Stiffeners thickness: $t_{st} := 14\ mm$

Number of longitudinal stiffeners:

$n_{st} := 6$

MAXIMUM STRESSES

Compression: $\sigma_{Ed.c} := 88.08\ MPa$

Tension: $\sigma_{Ed.t} := 100.5\ MPa$

Shear max: $\tau_{Edmax} := 43.28\ MPa$

Shear min: $\tau_{Edmin} := 4.5\ MPa$

Von Mises: $\sigma_{VonMises} := 90\ MPa$

Stress ratio: $\sigma_1 := 1$ $\sigma_2 := 1$ $\psi := \frac{\sigma_2}{\sigma_1} = 1$

Stiffener buckling coefficient: $k_{\sigma.st} := \frac{0.578}{(0.34+\psi)} = 0.431$

Stiffener slenderness $\lambda_{st} := \frac{b_{st}}{t_{st}} \cdot \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.st}}} = 0.659$

Reduction factor

('outstand compression element')

$$\rho_{st} := \begin{cases} 1 & \text{if } \lambda_{st} \le 0.748 \\ \frac{\lambda_{st} - 0.188}{\lambda_{st}^{2}} & \text{else} \end{cases} = 1$$

$b_{eff} := \rho_{st} \cdot b_{st} = 140\ mm$ $b_{e1} := \frac{2 \cdot b_{eff}}{(5-\psi)} = 70\ mm$ $b_{e2} := b_{eff} - b_{e1} = 70\ mm$

$$A_{L.eff.stiff} := (b_{e1} + b_{e2}) \cdot t_{st} = (1.96 \cdot 10^3)\ mm^2$$

PANEL GEOMETRIC PROPERTIES

Panel width: $a_1 := 2000\ mm$

Panel height: $a_2 := 2000\ mm$

Panel thickness: $t_{pl} := 20\ mm$

Equivalent thickness to be used in the model:

$$t_{eq.cant} := \frac{a_2 \cdot t_{pl} + n_{st} \cdot b_{st} \cdot t_{st}}{a_2} = 25.88\ mm$$

$$I_{eq.cant} := a_2 \cdot \frac{t_{eq.cant}^{\ 3}}{12 \cdot (1 - \nu^2)} = (3.175 \cdot 10^6)\ mm^4$$

Panel aspect ratio: $\alpha := \frac{a_1}{a_2} = 1$

Stress ratio: $\psi_p := 1$

Elastic plate buckling stress: $\sigma_E := \pi^2 \cdot E \cdot \frac{t_{pl}^{\ 2}}{12 \cdot (1 - \nu^2) \cdot a_2^{\ 2}} = 18.98\ MPa$

Plate second moment of area: $I_p := a_2 \cdot \frac{t_{pl}^{\ 3}}{12 \cdot (1 - \nu^2)} = (1.465 \cdot 10^6)\ mm^4$

Center of gravity of the whole stiffened plate:

$$x_{sl} := \frac{n_{st} \cdot t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2}\right)}{t_{pl} \cdot a_2 + n_{st} \cdot t_{st} \cdot b_{st}} = 18.176\ mm$$

Second moment of area of the whole stiffened plate:

$$I_{sl} := n_{st} \cdot \left(t_{st} \cdot \frac{b_{st}^{\ 3}}{12} + t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl}\right)^2\right) + I_p + t_{pl} \cdot a_2 \cdot x_{sl}^{\ 2}$$

$$I_{cant} := I_{sl} = (7.884 \cdot 10^7)\ mm^4$$

$$\gamma := \frac{I_{sl}}{I_p} = 53.806 \qquad \gamma^{\frac{1}{4}} = 2.708$$

Plate area $A_p := a_2 \cdot t_{pl} = (4 \cdot 10^4)\ mm^2$

Area of the whole stiffened plate:

$$A_{sl} := t_{pl} \cdot a_2 + n_{st} \cdot t_{st} \cdot b_{st} = (5.176 \cdot 10^4)\ mm^2$$

$$\delta := \frac{A_{sl}}{A_p} = 1.294$$

Since $\alpha < \gamma^{\frac{1}{4}}$ ---->

$$k_{\sigma.p} := \frac{2 \cdot \left((1 + \alpha^2)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi + 1) \cdot (1 + \delta)} = 24.763$$

$$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = 470.002\ MPa$$

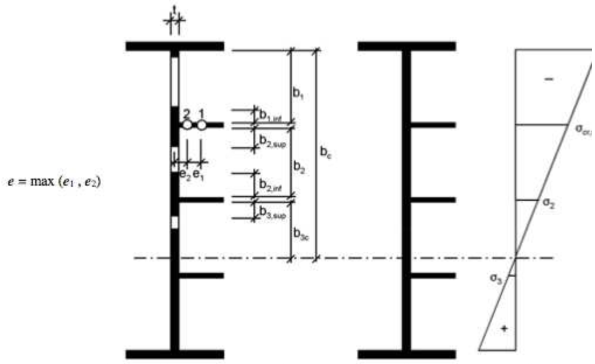
Column type behaviour:

Panel slenderness: $\lambda_p := \frac{a_2}{t_{pl}} \cdot \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.p}}} = 0.87$

Panel reduction factor:

$$\rho_p := \text{if } \lambda_p \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_p - 0.055 \cdot (3+\psi_p)}{\lambda_p^{\,2}} \quad = 0.859$$

internal compression element



	width for gross area	width for effective area according to Table 4.1	condition for ψ_i
$b_{1,\text{inf}}$	$\frac{3-\psi_1}{5-\psi_1} b_1$	$\frac{3-\psi_1}{5-\psi_1} b_{1,\text{eff}}$	$\psi_1 = \frac{\sigma_{cr,sl,1}}{\sigma_{cr,p}} > 0$
$b_{2,\text{sup}}$	$\frac{2}{5-\psi_2} b_2$	$\frac{2}{5-\psi_2} b_{2,\text{eff}}$	$\psi_2 = \frac{\sigma_2}{\sigma_{cr,sl,1}} > 0$
$b_{2,\text{inf}}$	$\frac{3-\psi_2}{5-\psi_2} b_2$	$\frac{3-\psi_2}{5-\psi_2} b_{2,\text{eff}}$	$\psi_2 > 0$
$b_{3,\text{sup}}$	0,4 b_{3c}	0,4 $b_{3c,\text{eff}}$	$\psi_3 = \frac{\sigma_3}{\sigma_2} < 0$

Figure A.1: Notations for longitudinally stiffened plates

Area ratio:

$$\beta_{A.c} := \frac{A_{c.eff.loc1}}{A_c} = 0.276$$

Slenderness:

$$\lambda_p := \sqrt{\frac{\beta_{A.c} \cdot f_y}{\sigma_{cr.p}}} = 0.457$$

Sub-panel reduction factor:
('internal compression element')

$$\rho_{subp} := \text{if } \lambda_p \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_p - 0.055 \cdot (3+\psi_1)}{\lambda_p^{\,2}} \quad = 1$$

Sub-panel stress ratio:

$$\psi_{sub.p} := 1$$

Sub-panel widths:

$$b_{pl} := \frac{a_2}{n_{st}+1} - \frac{t_{st}}{2} = 278.714 \; mm \qquad b_{pl.eff} := \rho_{subp} \cdot b_{pl} = 278.714 \; mm$$

Gross widths:

$$b_{1.inf} := \frac{3-\psi_{sub.p}}{5-\psi_{sub.p}} \cdot b_{pl} = 139.357 \; mm$$

$$b_{2.sup} := \frac{2}{(5-\psi_{sub.p})} \cdot b_{pl} = 139.357 \; mm$$

Effective widths:

$$b_{1.inf.eff} := \frac{3-\psi_{sub.p}}{5-\psi_{sub.p}} \cdot b_{pl.eff} = 139.357 \; mm$$

$$b_{2.sup.eff} := \frac{2}{(5-\psi_{sub.p})} \cdot b_{pl.eff} = 139.357\ mm$$

Gross area of stiff. + adjacent parts:

$$A_{sl.1} := b_{st} \cdot t_{st} + (b_{1.inf} + b_{2.sup} + t_{st}) \cdot t_{pl} = \left(7.814 \cdot 10^{3}\right)\ mm^{2}$$

Effective area of stiff. + adjacent parts:

$$A_{sl.1.eff} := A_{L.eff.stiff} + (b_{1.inf.eff} + b_{2.sup.eff} + t_{st}) \cdot t_{pl} = \left(7.814 \cdot 10^{3}\right)\ mm^{2}$$

Center of gravity of gross cross section of stiff.+ adjacent plates:

$$y_G := \frac{t_{pl} \cdot b_{pl} \cdot \left(b_{st} + \frac{t_{pl}}{2}\right) + b_{st} \cdot t_{st} \cdot \frac{b_{st}}{2}}{t_{pl} \cdot b_{pl} + t_{st} \cdot b_{st}} = 129.188\ mm$$

Second moment of area of gross cross section of stiff.+ adjacent plates:

$$I_{sl.1} := b_{pl} \cdot \frac{t_{pl}^{3}}{12} + b_{pl} \cdot t_{pl} \cdot \left(b_{st} + \frac{t_{pl}}{2} - y_G\right)^{2} + t_{st} \cdot \frac{b_{st}^{3}}{12} + t_{st} \cdot b_{st} \cdot \left(y_G - \frac{b_{st}}{2}\right)^{2}$$

$$\sigma_{cr.sl} := \frac{\pi^{2} \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_1^{2}} = 839.99\ MPa$$ ($\psi = 1$ -----> bc = bsl,1)

$$\sigma_{cr.c} := \sigma_{cr.sl}$$

Area ratio:

$$\beta_{A.c} := \frac{A_{sl.1.eff}}{A_{sl.1}} = 1$$

$$e_1 := y_G - \frac{b_{st}}{2} = 59.188\ mm$$

$$e_2 := b_{st} + \frac{t_{pl}}{2} - y_G = 20.812\ mm$$

$$e := \max(e_1, e_2) = 59.188\ mm$$

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 40.263\ mm$$

$$\alpha_e := 0.49 + \frac{0.09}{\frac{i}{e}} = 0.622$$

Column slenderness:

$$\lambda_c := \sqrt{\frac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.sl}}} = 0.65$$

$$\Phi := 0.5 \cdot \left(1 + \alpha_e \cdot (\lambda_c - 0.2) + \lambda_c^{2}\right) = 0.851$$

$$\chi_c := \frac{1}{\Phi + \sqrt{\Phi^{2} - \lambda_c^{2}}} = 0.714$$ < 1

$$\xi := \frac{\sigma_{cr.p}}{\sigma_{cr.sl}} - 1 = -0.44$$ < 0 -> = 0

$$\xi := 0$$

$$\rho_c := (\rho_p - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c = 0.714$$

Effective reduced area of stiffener + adjacent parts of panel:

$$A_i := \left(\rho_c \cdot \left(n_{st} \cdot A_{L.eff.stiff} + \left(n_{st}+1\right) \cdot \rho_{subp} \cdot b_{pl} \cdot t_{pl}\right)\right) \cdot 1 = \left(3.624 \cdot 10^{4}\right)\ mm^2$$

$$A := a_2 \cdot t_{pl} + n_{st} \cdot t_{st} \cdot b_{st} = \left(5.176 \cdot 10^{4}\right)\ mm^2$$

Area reduction ratio: $\dfrac{A_i}{A} = 0.7$ >0.5 OK

Axial stress resistance verification

$\eta_{1.c} := \dfrac{\sigma_{Ed.c} \cdot A}{f_{yd} \cdot A_i} = 0.354$ < 1 OK (compression)

$\eta_{1.t} := \dfrac{\sigma_{Ed.t}}{f_{yd}} = 0.283$ < 1 OK (tension)

Shear stress resistance verification

Stiffened panel

Gross area: $A_{sl} := 30 \cdot \varepsilon \cdot t_{pl}^{2} + t_{pl} \cdot t_{st} + b_{st} \cdot t_{st} = 0.012\ m^2$

Center of gravity: $x_{sl} := \dfrac{b_{st} \cdot t_{st} \cdot \left(\dfrac{b_{st}+t_{pl}}{2}\right)}{A_{sl}} = 0.013\ m$

Second moment of area:

$$I_{sl} := \frac{\left(30 \cdot \varepsilon \cdot t_{pl} + t_{st}\right) \cdot t_{pl}^{3}}{12} + \frac{t_{st} \cdot b_{st}^{3}}{12} + \left(30 \cdot \varepsilon \cdot t_{pl} + t_{st}\right) \cdot t_{pl} \cdot x_{sl}^{2} + t_{st} \cdot b_{st} \cdot \left(\frac{t_{pl}+b_{st}}{2} - x_{sl}\right)^{2} = \left(1.403 \cdot 10^{-5}\right)\ m^4$$

Aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1$

Shear buckling coefficient: $k_{\tau.sl1} := 9 \cdot \left(\dfrac{1}{\alpha}\right)^{2} \cdot \sqrt[4]{\left(\dfrac{n \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}\right)^{3}} = 13.717$

$$k_{\tau.sl2} := \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\left(\frac{n \cdot I_{sl}}{a_2}\right)} = 2.533$$

$$k_{\tau.sl} := \max\left(k_{\tau.sl1}, k_{\tau.sl2}\right) = 13.717$$

$$k_t := \begin{cases} 4.1 + \dfrac{6.3 + 0.18 \cdot \dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}}{\alpha^{2}} + 2.2 \cdot \sqrt[3]{\dfrac{2 \cdot I_{sl}}{t_{pl}^{3} \cdot a_2}} & \text{if } \alpha < 3 \\ 5.34 + 4 \cdot \left(\dfrac{1}{\alpha}\right)^{2} + k_{\tau.sl} & \text{else} \end{cases} = 13.369$$

Slenderness: $\lambda_w := \dfrac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.899$

Single panel (stiffeners divide plate into three equal sub-panels)

a_2

Panel aspect ratio: $\alpha := \dfrac{a_1}{\left(\dfrac{a_2}{n+1}\right)} = 3$

Shear buckling coefficient: $k_{tl.st} := 0$

$$k_t := \begin{cases} \text{if } \alpha \ge 1 \\ \quad 5.34 + 4 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^2 \\ \text{else} \\ \quad 4 + 5.34 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^2 \end{cases} = 5.784$$

Slenderness: $\lambda_{w1} := \dfrac{a_2}{(n+1) \cdot t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.455$

$\lambda_w = 0.899$ > $\lambda_{w1} = 0.455$ column section buckling is critical

$$\chi_w := \begin{cases} \text{if } \lambda_w < \dfrac{0.83}{\eta} \\ \quad \eta \\ \text{else if } \dfrac{0.83}{\eta} \le \lambda_w < 1.08 \\ \quad \dfrac{0.83}{\lambda_w} \\ \text{else} \\ \quad \dfrac{1.37}{(0.7 + \lambda_w)} \end{cases} = 0.923$$

$\eta_3 := \dfrac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \chi_w} = 0.252$ < 1 OK

Interaction of axial and shear stresses verification

$\eta_{1.c} + \left(2 \cdot \left(\dfrac{\sqrt{3} \cdot (\tau_{Edmax} - \tau_{Edmin}) \cdot \gamma_{M1}}{f_y \cdot \chi_w}\right) - 1\right)^2 = 0.656$ < 1 OK

$\sigma_{VonMises} = 90\ MPa$ < $\dfrac{f_y}{\gamma_{M1}} = 322.727\ MPa$ OK

Torsional buckling of stiffeners with open section

Second moments of area: $I_y := \dfrac{b_{st}^{\ 3} \cdot t_{st}}{3} = \left(1.281 \cdot 10^{-5}\right)\ m^4$ $I_z := \dfrac{b_{st} \cdot t_{st}^{\ 3}}{12} = \left(3.201 \cdot 10^{-8}\right)\ m^4$

Polar second moment of area: $I_p := I_y + I_z = \left(1.284 \cdot 10^{-5}\right)\ m^4$

Torsional constant $I_t := \dfrac{b_{st} \cdot t_{st}^{\ 3}}{3} = \left(1.281 \cdot 10^{-7}\right)\ m^4$

$\dfrac{I_t}{I_p} = 0.00998$

$MinVal := 5.3 \cdot \frac{f_y}{E} = 0.00896$ $\qquad \frac{MinVal}{\frac{I_t}{I_p}} = 0.898$ < 1 OK

Cross-section (2) - Midspan

GENERAL DATA

Safety factors $\quad \gamma_{M0} := 1.0 \quad \gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $\quad f_y := 355\ MPa \quad f_{yd} := \frac{f_y}{\gamma_{M0}} = 355\ MPa \quad \varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $\quad f_u := 510\ MPa$

Elastic moduli: $\quad E := 210000\ MPa \quad \nu := 0.3 \quad G := \frac{E}{2 \cdot (1+\nu)} = \left(8.077 \cdot 10^{10}\right)\ Pa$

Stiffeners height: $\quad b_{st} := 160\ mm$

Stiffeners thickness: $\quad t_{st} := 20\ mm$

Number of longitudinal stiffeners:

$n_{st} := 6$

MAXIMUM STRESSES

Compression: $\quad \sigma_{Ed.c} := 280\ MPa$

Tension: $\quad \sigma_{Ed.t} := 200\ MPa$

Shear max: $\quad \tau_{Edmax} := 134.4\ MPa$

Shear min: $\quad \tau_{Edmin} := 4.43\ MPa$

Von Mises: $\quad \sigma_{VonMises} := 263.4\ MPa$

Stress ratio: $\quad \sigma_1 := 1 \quad \sigma_2 := 1 \quad \psi := \frac{\sigma_2}{\sigma_1} = 1$

Stiffener buckling coefficient: $\quad k_{\sigma.st} := \frac{0.578}{(0.34+\psi)} = 0.431$

Stiffener slenderness $\quad \lambda_{st} := \frac{b_{st}}{t_{st}} \cdot \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.st}}} = 0.527$

Reduction factor

('outstand compression element')

$$\rho_{st} := \begin{cases} 1 & \text{if } \lambda_{st} \le 0.748 \\ \frac{\lambda_{st} - 0.188}{\lambda_{st}^2} & \text{else} \end{cases} = 1$$

$b_{eff} := \rho_{st} \cdot b_{st} = 160\ mm \qquad b_{e1} := \frac{2 \cdot b_{eff}}{(5-\psi)} = 80\ mm \qquad b_{e2} := b_{eff} - b_{e1} = 80\ mm$

$$A_{L.eff.stiff} := \left(b_{e1} + b_{e2}\right) \cdot t_{st} = \left(3.2 \cdot 10^3\right)\ mm^2$$

PANEL GEOMETRIC PROPERTIES

Panel width: $\quad a_1 := 2000\ mm$

Panel height: $\quad a_2 := 2000\ mm$

Panel thickness: $\quad t_{pl} := 20\ mm$

Panel aspect ratio: $\alpha := \frac{a_1}{a_2} = 1$

Stress ratio: $\psi_p := 1$

Elastic plate buckling stress: $\sigma_E := \pi^2 \cdot E \cdot \frac{t_{pl}^2}{12 \cdot (1-\nu^2) \cdot a_2^2} = 18.98\ MPa$

Plate second moment of area: $I_p := a_2 \cdot \frac{t_{pl}^3}{12 \cdot (1-\nu^2)} = (1.465 \cdot 10^6)\ mm^4$

Center of gravity of the whole stiffened plate: $x_{sl} := \frac{n_{st} \cdot t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2}\right)}{t_{pl} \cdot a_2 + n_{st} \cdot t_{st} \cdot b_{st}} = 29.189\ mm$

Second moment of area of the whole stiffened plate:

$$I_{sl} := n_{st} \cdot \left(t_{st} \cdot \frac{b_{st}^3}{12} + t_{st} \cdot b_{st} \cdot \left(\frac{b_{st}}{2} + \frac{t_{pl}}{2} - x_{sl}\right)^2\right) + I_p + t_{pl} \cdot a_2 \cdot x_{sl}^2$$

$\gamma := \frac{I_{sl}}{I_p} = 100.673$ $\qquad \gamma^{\frac{1}{4}} = 3.168$

Plate area $\qquad A_p := a_2 \cdot t_{pl} = (4 \cdot 10^4)\ mm^2$

Area of the whole stiffened plate:

$A_{sl} := t_{pl} \cdot a_2 + n_{st} \cdot t_{st} \cdot b_{st} = (5.92 \cdot 10^4)\ mm^2$

$\delta := \frac{A_{sl}}{A_p} = 1.48$

Since $\alpha < \gamma^{\frac{1}{4}}$ ----> $k_{\sigma.p} := \frac{2 \cdot \left((1+\alpha^2)^2 + \gamma - 1\right)}{\alpha^2 \cdot (\psi + 1) \cdot (1 + \delta)} = 41.804$

$\sigma_{cr.p} := k_{\sigma.p} \cdot \sigma_E = 793.434\ MPa$

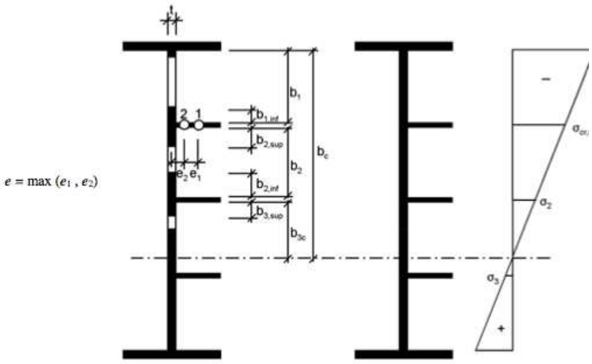
Column type behaviour:

Panel slenderness: $\lambda_p := \frac{a_2}{t_{pl}} \cdot \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma.p}}} = 0.669$

Panel reduction factor:

$$\rho_p := \begin{cases} 1 & \text{if } \lambda_p \le 0.673 \\ \frac{\lambda_p - 0.055 \cdot (3 + \psi_p)}{\lambda_p^2} & \text{else} \end{cases} = 1$$

internal compression element



	width for gross area	width for effective area according to Table 4.1	condition for ψ_i
$b_{1,\text{inf}}$	$\frac{3-\psi_1}{5-\psi_1} b_1$	$\frac{3-\psi_1}{5-\psi_1} b_{1,\text{eff}}$	$\psi_1 = \frac{\sigma_{cr,sl,1}}{\sigma_{cr,p}} > 0$
$b_{2,\text{sup}}$	$\frac{2}{5-\psi_2} b_2$	$\frac{2}{5-\psi_2} b_{2,\text{eff}}$	$\psi_2 = \frac{\sigma_2}{\sigma_{cr,sl,1}} > 0$
$b_{2,\text{inf}}$	$\frac{3-\psi_2}{5-\psi_2} b_2$	$\frac{3-\psi_2}{5-\psi_2} b_{2,\text{eff}}$	$\psi_2 > 0$
$b_{3,\text{sup}}$	0,4 b_{3c}	0,4 $b_{3c,\text{eff}}$	$\psi_3 = \frac{\sigma_3}{\sigma_2} < 0$

Figure A.1: Notations for longitudinally stiffened plates

Area ratio: $\beta_{A.c} := \frac{A_{c.eff.loc1}}{A_c} = 0.276$

Slenderness: $\lambda_p := \sqrt{\frac{\beta_{A.c} \cdot f_y}{\sigma_{cr.p}}} = 0.351$

Sub-panel reduction factor:
('internal compression element')

$$\rho_{subp} := \text{if } \lambda_p \le 0.673 \;\; \| \; 1 \;\; \text{else} \;\; \| \; \frac{\lambda_p - 0.055 \cdot (3 + \psi_1)}{\lambda_p^{\,2}} \quad = 1$$

Sub-panel stress ratio: $\psi_{sub.p} := 1$

Sub-panel widths: $b_{pl} := \frac{a_2}{n_{st}+1} - \frac{t_{st}}{2} = 275.714\ mm$ $\quad b_{pl.eff} := \rho_{subp} \cdot b_{pl} = 275.714\ mm$

Gross widths: $b_{1.inf} := \frac{3-\psi_{sub.p}}{5-\psi_{sub.p}} \cdot b_{pl} = 137.857\ mm$

$$b_{2.sup} := \frac{2}{(5-\psi_{sub.p})} \cdot b_{pl} = 137.857\ mm$$

Effective widths:

$$b_{1.inf.eff} := \frac{3-\psi_{sub.p}}{5-\psi_{sub.p}} \cdot b_{pl.eff} = 137.857\ mm$$

$$b_{2.sup.eff} := \frac{2}{(5-\psi_{sub.p})} \cdot b_{pl.eff} = 137.857\ mm$$

Gross area of stiff. + adjacent parts:

$$A_{sl.1} := b_{st} \cdot t_{st} + (b_{1.inf} + b_{2.sup} + t_{st}) \cdot t_{pl} = (9.114 \cdot 10^3)\ mm^2$$

Effective area of stiff. + adjacent parts:

$$A_{sl.1.eff} := A_{L.eff.stiff} + (b_{1.inf.eff} + b_{2.sup.eff} + t_{st}) \cdot t_{pl} = (9.114 \cdot 10^3)\ mm^2$$

Center of gravity of gross cross section of stiff.+ adjacent plates:

$$y_G := \frac{t_{pl} \cdot b_{pl} \cdot \left(b_{st} + \frac{t_{pl}}{2}\right) + b_{st} \cdot t_{st} \cdot \frac{b_{st}}{2}}{t_{pl} \cdot b_{pl} + t_{st} \cdot b_{st}} = 136.951\ mm$$

Second moment of area of gross cross section of stiff.+ adjacent plates:

$$I_{sl.1} := b_{pl} \cdot \frac{t_{pl}^3}{12} + b_{pl} \cdot t_{pl} \cdot \left(b_{st} + \frac{t_{pl}}{2} - y_G\right)^2 + t_{st} \cdot \frac{b_{st}^3}{12} + t_{st} \cdot b_{st} \cdot \left(y_G - \frac{b_{st}}{2}\right)^2$$

$$\sigma_{cr.sl} := \frac{\pi^2 \cdot E \cdot I_{sl.1}}{A_{sl.1} \cdot a_1^2} = (1.331 \cdot 10^3)\ MPa$$ ($\psi = 1$ -----> bc = bsl,1)

$$\sigma_{cr.c} := \sigma_{cr.sl}$$

Area ratio:

$$\beta_{A.c} := \frac{A_{sl.1.eff}}{A_{sl.1}} = 1$$

$$e_1 := y_G - \frac{b_{st}}{2} = 56.951\ mm$$

$$e_2 := b_{st} + \frac{t_{pl}}{2} - y_G = 33.049\ mm$$

$$e := \max(e_1, e_2) = 56.951\ mm$$

$$i := \sqrt{\frac{I_{sl.1}}{A_{sl.1}}} = 50.683\ mm$$

$$\alpha_e := 0.49 + \frac{0.09}{\frac{i}{e}} = 0.591$$

Column slenderness:

$$\lambda_c := \sqrt{\frac{\beta_{A.c} \cdot f_{yd}}{\sigma_{cr.sl}}} = 0.516$$

$$\Phi := 0.5 \cdot \left(1 + \alpha_e \cdot (\lambda_c - 0.2) + \lambda_c^2\right) = 0.727$$

$$\chi_c := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda_c^2}} = 0.807$$ < 1

$$\xi := \frac{\sigma_{cr.p}}{\sigma_{cr.sl}} - 1 = -0.404$$ < 0 -> = 0

$$\xi := 0$$

$$\rho_c := (\rho_p - \chi_c) \cdot \xi \cdot (2 - \xi) + \chi_c = 0.807$$

Effective reduced area of stiffener + adjacent parts of panel:

$$A_i := (\rho_c \cdot (n_{st} \cdot A_{L.eff.stiff} + (n_{st} + 1) \cdot \rho_{subp} \cdot b_{pl} \cdot t_{pl})) \cdot 1 = (4.667 \cdot 10^4)\ mm^2$$

$$A := a_2 \cdot t_{pl} + n_{st} \cdot t_{st} \cdot b_{st} = (5.92 \cdot 10^4)\ mm^2$$

Area reduction ratio:

$$\frac{A_i}{A} = 0.788$$ >0.5 OK

Axial stress resistance verification

$\eta_{1.c} := \dfrac{\sigma_{Ed.c} \cdot A}{f_{yd} \cdot A_i} = 1$ < 1 OK (compression)

$\eta_{1.t} := \dfrac{\sigma_{Ed.t}}{f_{yd}} = 0.563$ < 1 OK (tension)

Shear stress resistance verification

Stiffened panel

Gross area: $A_{sl} := 30 \cdot \varepsilon \cdot t_{pl}^{\ 2} + t_{pl} \cdot t_{st} + b_{st} \cdot t_{st} = 0.013\ m^2$

Center of gravity: $x_{sl} := \dfrac{b_{st} \cdot t_{st} \cdot \left(\dfrac{b_{st} + t_{pl}}{2}\right)}{A_{sl}} = 0.022\ m$

Second moment of area:

$$I_{sl} := \frac{(30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl}^{\ 3}}{12} + \frac{t_{st} \cdot b_{st}^{\ 3}}{12} + (30 \cdot \varepsilon \cdot t_{pl} + t_{st}) \cdot t_{pl} \cdot x_{sl}^{\ 2} + t_{st} \cdot b_{st} \cdot \left(\frac{t_{pl} + b_{st}}{2} - x_{sl}\right)^2 = \left(2.688 \cdot 10^{-5}\right) m^4$$

Aspect ratio: $\alpha := \dfrac{a_1}{a_2} = 1$

Shear buckling coefficient: $k_{\tau.sl1} := 9 \cdot \left(\dfrac{1}{\alpha}\right)^2 \cdot \sqrt[4]{\left(\dfrac{n \cdot I_{sl}}{t_{pl}^{\ 3} \cdot a_2}\right)^3} = 22.335$

$$k_{\tau.sl2} := \frac{2.1}{t_{pl}} \cdot \sqrt[3]{\left(\frac{n \cdot I_{sl}}{a_2}\right)} = 3.145$$

$$k_{\tau.sl} := \max\left(k_{\tau.sl1}, k_{\tau.sl2}\right) = 22.335$$

$$k_t := \begin{cases} 4.1 + \dfrac{6.3 + 0.18 \cdot \dfrac{2 \cdot I_{sl}}{t_{pl}^{\ 3} \cdot a_2}}{\alpha^2} + 2.2 \cdot \sqrt[3]{\dfrac{2 \cdot I_{sl}}{t_{pl}^{\ 3} \cdot a_2}} & \text{if } \alpha < 3 \\ 5.34 + 4 \cdot \left(\dfrac{1}{\alpha}\right)^2 + k_{\tau.sl} & \text{else} \end{cases} = 14.3$$

Slenderness: $\lambda_w := \dfrac{a_2}{t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.869$

Single panel (stiffeners divide plate into three equal sub-panels)

Panel aspect ratio: $\alpha := \dfrac{a_1}{\left(\dfrac{a_2}{n+1}\right)} = 3$

Shear buckling coefficient: $k_{tl.st} := 0$

$$k_t := \begin{cases} 5.34 + 4 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^2 & \text{if } \alpha \ge 1 \\ 4 + 5.34 \cdot \left(\dfrac{a_2}{3 \cdot a_1}\right)^2 & \text{else} \end{cases} = 5.784$$

Slenderness: $\lambda_{w1} := \dfrac{a_2}{(n+1) \cdot t_{pl} \cdot 37.4 \cdot \varepsilon \cdot \sqrt{k_t}} = 0.455$

$\lambda_w = 0.869$ > $\lambda_{w1} = 0.455$ column section buckling is critical

$$\chi_w := \text{if } \lambda_w < \frac{0.83}{\eta} \;\Big\|\; \eta \;\; \text{else if } \frac{0.83}{\eta} \le \lambda_w < 1.08 \;\Big\|\; \frac{0.83}{\lambda_w} \;\; \text{else } \Big\|\; \frac{1.37}{(0.7+\lambda_w)} \; = 0.955$$

$\eta_3 := \dfrac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \chi_w} = 0.755$ < 1 OK

Interaction of axial and shear stresses verification

$\eta_{1.c} + \left(2 \cdot \left(\dfrac{\sqrt{3} \cdot (\tau_{Edmax} - \tau_{Edmin}) \cdot \gamma_{M1}}{f_y \cdot \chi_w}\right) - 1\right)^2 = 1.213$ < 1 OK

$\sigma_{VonMises} = 263.4 \; MPa$ < $\dfrac{f_y}{\gamma_{M1}} = 322.727 \; MPa$ OK

Torsional buckling of stiffeners with open section

Second moments of area: $I_y := \dfrac{b_{st}^{\;3} \cdot t_{st}}{3} = \left(2.731 \cdot 10^{-5}\right) m^4$ $I_z := \dfrac{b_{st} \cdot t_{st}^{\;3}}{12} = \left(1.067 \cdot 10^{-7}\right) m^4$

Polar second moment of area: $I_p := I_y + I_z = \left(2.741 \cdot 10^{-5}\right) m^4$

Torsional constant $I_t := \dfrac{b_{st} \cdot t_{st}^{\;3}}{3} = \left(4.267 \cdot 10^{-7}\right) m^4$

$\dfrac{I_t}{I_p} = 0.01556$

$MinVal := 5.3 \cdot \dfrac{f_y}{E} = 0.00896$ $\dfrac{MinVal}{\dfrac{I_t}{I_p}} = 0.576$ < 1 OK

C. Equivalent cross-section

Equivalent thicknesses of cross-sectional panels to be used in the FE model

Equivalent box top panel shell (same area)

$$b_{eqTD} := 2000\ mm$$

$$t_{eqTD} := \frac{A_{TD}}{b_{eqTD}} = 25.19\ mm$$

Equivalent box lateral panel shell (same area)

$$b_{eqLD} := 580\ mm$$

$$t_{eqLD} := \frac{A_{LD}}{b_{eqLD}} = 22.869\ mm$$

Equivalent box bottom panel shell (same area)

$$b_{eqBD} := 1370\ mm$$

$$t_{eqBD} := \frac{A_{BD}}{b_{eqBD}} = 27.219\ mm$$

Global effective section center of gravity coordinates

Distance b/w center of gravity and bottom panel external surface

$$y_G := \frac{\left(A_{TD} \cdot y_{G.TD} + 2 \cdot A_{LD} \cdot y_{G.LD} + A_{BD} \cdot y_{G.BD}\right)}{A_{TD} + 2 \cdot A_{LD} + A_{BD}} = 292.203\ mm$$

Correspondance b/w equivalent cross-section panels' stiffnesses and effective panels' stiffenesses

Box top panel second moment of area with respect to global effective section's G

$$I_1 := I_{TD} + A_{TD} \cdot \left(y_{G.TD} - y_G\right)^2 = \left(1.481 \cdot 10^9\right)\ mm^4$$

Box lateral panels second moment of area with respect to global effective section's G

$$I_2 := 2 \cdot \left(I_{LD} + A_{LD} \cdot \left(y_{G.LD} - y_G\right)^2\right) = \left(8.08 \cdot 10^8\right)\ mm^4$$

Box bottom panel second moment of area with respect to global effective section's G

$$I_3 := I_{BD} + A_{BD} \cdot \left(y_{G.BD} - y_G\right)^2 = \left(2.291 \cdot 10^9\right)\ mm^4$$

Global equivalent section center of gravity coordinates

$$y_{Geq} := \frac{b_{eqTD} \cdot t_{eqTD} \cdot \left(h - \frac{t_{eqTD}}{2}\right) + 2 \cdot b_{eqLD} \cdot t_{eqLD} \cdot \left(\frac{b_{eqLD}}{2} \cdot \sin(\theta) + \frac{t_{eqLD}}{2} \cos(\theta)\right) + b_{eqBD} \cdot t_{eqBD} \cdot \left(\frac{t_{eqBD}}{2}\right)}{b_{eqTD} \cdot t_{eqTD} + 2 \cdot b_{eqLD} \cdot t_{eqLD} + b_{eqBD} \cdot t_{eqBD}} = 241.281\ mm$$

Box top panel second moment of area with respect to global equivalent section's G

$$I_{1eq} := \left(b_{eqTD} \cdot \frac{t_{eqTD}^{\ 3}}{12} + b_{eqTD} \cdot t_{eqTD} \cdot \left(h - \frac{t_{eqTD}}{2} - y_{Geq} \right)^2 \right) \cdot \frac{1}{\left(1 - \nu^2\right)} = \left(3.357 \cdot 10^9\right) \ mm^4$$

Box lateral panel second moment of area with respect to global equivalent section's G

Momento inerzia baricentrico nel sdr locale (inclinato)

$$I_{x'eq} := b_{eqLD} \cdot \frac{t_{eqLD}^{\ 3}}{12 \left(1 - \nu^2\right)} = \left(6.353 \cdot 10^5\right) \ mm^4 \qquad I_{y'eq} := t_{eqLD} \cdot \frac{b_{eqLD}^{\ 3}}{12 \left(1 - \nu^2\right)} = \left(4.086 \cdot 10^8\right) \ mm^4$$

$$I_{xeq} := I_{x'eq} \cdot (\cos(\theta))^2 + I_{y'eq} \cdot (\sin(\theta))^2 = \left(3.401 \cdot 10^7\right) \ mm^4$$

$$I_{2eq} := 2 \cdot \left(I_{xeq} + b_{eqLD} \cdot t_{eqLD} \cdot \frac{\left(\frac{b_{eqLD}}{2} \cdot \sin(\theta) + \frac{t_{eqLD}}{2} \cdot \cos(\theta) - y_{Geq} \right)^2}{1 - \nu^2} \right) = \left(7.012 \cdot 10^8\right) \ mm^4$$

Momento inerzia pannello inferiore rispetto all'asse baricentrico della sezione globale (shell equivalenti):

$$I_{3eq} := \left(b_{eqBD} \cdot \frac{t_{eqBD}^{\ 3}}{12} + b_{eqBD} \cdot t_{eqBD} \cdot \left(y_{Geq} - \frac{t_{eqBD}}{2} \right)^2 \right) \cdot \frac{1}{\left(1 - \nu^2\right)} = \left(2.127 \cdot 10^9\right) \ mm^4$$

Riepilogo spessori equivalenti:

$t_{eqTD} = 25.19 \ mm$ $\qquad$ $t_{eqLD} = 22.869 \ mm$ $\qquad$ $t_{eqBD} = 27.219 \ mm$ $\qquad$ $t_{eq.cant} = 25.88 \ mm$

Rapporti di rigidezze *(rigidezza effettiva/rigidezza equivalente)*

$$R_{TD} := \frac{I_1}{I_{1eq}} = 0.441 \qquad R_{LD} := \frac{I_2}{I_{2eq}} = 1.152 \qquad R_{BD} := \frac{I_3}{I_{3eq}} = 1.077$$

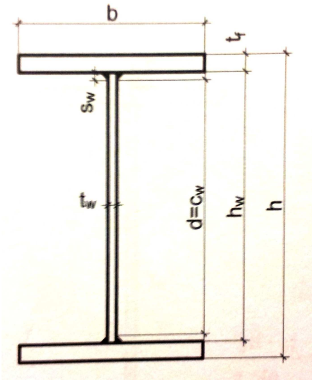
$$R_{cant} := \frac{I_{cant}}{I_{eq.cant}} = 24.833$$

*Use an equivalent thickness that has a I=25*Ieq.cant, because GSA cannot accept thicnkess factor >200%*

-------->

$$t' := \sqrt[3]{12 \cdot \left(1 - \nu^2\right) \cdot \frac{I_{cant}}{2000 \ mm}} = 75.505 \ mm$$

D. Diaphragm verification



CROSS-SECTION PROPERTIES

Total depth: $h := 130\ mm$

Flange width: $b_f := 100\ mm$

Flange thickness: $t_f := 20\ mm$

Web thickness: $t_w := 20\ mm$

Web depth: $h_w := h - 2 \cdot t_f = 90\ mm$

Cross-sectional area: $A := 2 \cdot b_f \cdot t_f + h_w \cdot t_w = 0.006\ m^2$

Second moments of area :

$$I_y := 2 \cdot \left(\frac{b_f \cdot t_f^{\ 3}}{12} + b_f \cdot t_f \cdot \left(\frac{h}{2} - \frac{t_f}{2} \right)^2 \right) + \frac{t_w \cdot h_w^{\ 3}}{12} = \left(1.345 \cdot 10^7\right)\ mm^4$$

$$I_z := 2 \cdot \frac{t_f \cdot b_f^{\ 3}}{12} + \frac{h_w \cdot t_w^{\ 3}}{12} = \left(3.393 \cdot 10^6\right)\ mm^4$$

Section moduli: $W_{el.y} := I_y \cdot \frac{2}{h} = 206897.4359\ mm^3$ $\quad W_{el.z} := I_z \cdot \frac{2}{b_f} = \left(67.867 \cdot 10^3\right)\ mm^3$

Radii of gyration: $i_y := \sqrt{\frac{I_y}{A}} = 48.153\ mm$ $\quad i_z := \sqrt{\frac{I_z}{A}} = 24.188\ mm$

Polar radius of gyration: $i_c := \sqrt{i_y^{\ 2} + i_z^{\ 2}} = 53.886\ mm$

Torsional constant: $I_t := \frac{1}{3} \cdot \left(h_w \cdot t_w^{\ 3} + 2 \cdot b_f \cdot t_f^{\ 3}\right) = \left(7.733 \cdot 10^{-7}\right)\ m^4$

Warping constant: $I_w := \frac{t_f \cdot (h - t_f)^2 \cdot b_f^{\ 3}}{24} = \left(1.008 \cdot 10^{-8}\right)\ m^6$

Maximum length: $l := 2000\ mm$

Imperfection factor: $\alpha_y := 0.34$ $\quad \alpha_z := 0.49$

Unrestrained length coefficient: $\beta_y := 1$ $\quad \beta_z := 1$

Unrestrained lenghts: $l_{cr.y} := \beta_y \cdot l = 2000\ mm$ $\quad l_{cr.z} := \beta_z \cdot l = 2000\ mm$

Critical torsional buckling length: $l_{cr.T} := l = 2000\ mm$

ULS FORCES:

$N_c := -128.1\ kN$

$N_t := 310.2\ kN$

$N := \max\left(\text{abs}\left(N_c\right), \text{abs}\left(N_t\right)\right) = 310.2\ kN$

$V_2 := 36.54\ kN$

$V_3 := 12.09\ kN$

$M_2 := 3.73\ kN \cdot m$

$M_3 := 42.81\ kN \cdot m$

$T := 0.52\ kN \cdot m$

ULS Resistance verification - Elastic behaviour

$$\sigma_{x1} := \frac{N}{A} + \frac{M_2}{W_{el.z}} + \frac{M_3}{W_{el.y}} = 315.358\ MPa$$

$$\sigma_{x2} := \frac{N}{A} + \frac{M_2}{W_{el.z}} - \frac{M_3}{W_{el.y}} = -98.471\ MPa$$

$$\sigma_{x3} := \frac{N}{A} - \frac{M_2}{W_{el.z}} + \frac{M_3}{W_{el.y}} = 205.436\ MPa$$

$$\sigma_{x4} := \frac{N}{A} - \frac{M_2}{W_{el.z}} - \frac{M_3}{W_{el.y}} = -208.392\ MPa$$

$$\sigma_x := \max\left(\text{abs}\left(\sigma_{x1}\right), \text{abs}\left(\sigma_{x2}\right), \text{abs}\left(\sigma_{x3}\right), \text{abs}\left(\sigma_{x4}\right)\right) = 315.358\ MPa$$

$$\tau_z := \frac{V_2}{t_w \cdot h_w} = 20.3\ MPa$$

$$\tau_y := \frac{V_3}{2 \cdot t_f \cdot b_f} = 3.023\ MPa$$

$$\left(\frac{\sigma_x \cdot \gamma_{M0}}{f_y}\right)^2 + 3 \cdot \left(\frac{\left(\tau_z + \tau_y\right) \cdot \gamma_{M0}}{f_y}\right)^2 = 0.802$$ < 1 OK

Buckling verification

Plane buckling - Y axis

Elastic critical buckling load: $$N_{cr.y} := \frac{\pi^2 \cdot E \cdot I_y}{l_{cr.y}^{\ 2}} = 6968\ kN$$

Slenderness: $$\lambda_y := \sqrt{\frac{A \cdot f_y}{N_{cr.y}}} = 0.544$$

$$\phi_y := 0.5 \cdot \left(1 + \alpha_y \cdot \left(\lambda_y - 0.2\right) + \lambda_y^{\ 2}\right) = 0.706$$

Reduction factor: $$\chi_y := \text{if } \lambda_y < 0.2 \;\; \Big\| \; 1 \;\; \text{else} \;\; \Big\| \; \frac{1}{\phi_y + \sqrt{\phi_y^{\ 2} - \lambda_y^{\ 2}}} = 0.864$$

Buckling resistance: $$N_{b.y.Rd} := \frac{\chi_y \cdot A \cdot f_y}{\gamma_{M1}} = 1618\ kN$$

$$\frac{\text{abs}\left(N_c\right)}{N_{b.y.Rd}} = 0.079$$ < 1 OK

Plane buckling- Z axis

Elastic critical buckling load: $$N_{cr.z} := \frac{\pi^2 \cdot E \cdot I_z}{l_{cr.z}^{\ 2}} = 1758\ kN$$

Slenderness: $$\lambda_z := \sqrt{\frac{A \cdot f_y}{N_{cr.z}}} = 1.082$$

$$\phi_z := 0.5 \cdot \left(1 + \alpha_z \cdot \left(\lambda_z - 0.2\right) + \lambda_z^{\ 2}\right) = 1.302$$

Reduction factor: $$\chi_z := \text{if } \lambda_z < 0.2 \;\; \Big\| \; 1 \;\; \text{else} \;\; \Big\| \; \frac{1}{\phi_z + \sqrt{\phi_z^{\ 2} - \lambda_z^{\ 2}}} = 0.494$$

Buckling resistance: $$N_{b.z.Rd} := \frac{\chi_z \cdot A \cdot f_y}{\gamma_{M1}} = 924\ kN$$

$$\frac{\text{abs}\left(N_c\right)}{N_{b.z.Rd}} = 0.139$$ < 1 OK

Torsional and flexural-torsional buckling - Y axis

Torsional buckling load:

$$N_{cr.T} := \frac{1}{i_c^{\ 2}} \cdot \left(G \cdot I_t + \frac{\pi^2 \cdot E \cdot I_w}{l_{cr.T}^{\ 2}} \right) = 23310\ kN$$

Flexural-torsional buckling load:

$$N_{cr.TF} := \frac{i_c^{\ 2}}{2 \cdot \left(i_y^{\ 2} + i_z^{\ 2} \right)} \cdot \left(N_{cr.y} + N_{cr.T} - \sqrt{\left(N_{cr.y} + N_{cr.T} \right)^2 - 4 \cdot N_{cr.y} \cdot N_{cr.T} \cdot \frac{i_y^{\ 2} + i_z^{\ 2}}{i_c^{\ 2}}} \right) = 6968\ kN$$

Critical load: $N_{cr.Tf} := min\left(N_{cr.T}, N_{cr.TF} \right) = 6968\ kN$

Slenderness: $\lambda_T := \sqrt{\frac{A \cdot f_y}{N_{cr.Tf}}} = 0.544$

$$\phi_T := 0.5 \cdot \left(1 + \alpha_y \cdot \left(\lambda_T - 0.2 \right) + \lambda_T^{\ 2} \right) = 0.706$$

Reduction factor:

$$\chi := \text{if } \lambda_T < 0.2 \ \ \| \ 1 \ \ \text{else} \ \ \| \ \frac{1}{\phi_T + \sqrt{\phi_T^{\ 2} - \lambda_T^{\ 2}}} \quad = 0.864$$

Buckling resistance: $N_{b.Rd} := \frac{\chi \cdot A \cdot f_y}{\gamma_{M1}} = 1618\ kN$

$\frac{\text{abs}\left(N_c \right)}{N_{b.Rd}} = 0.079$ < 1 OK

Torsional and flexural-torsional buckling - Z axis

Torsional buckling load:

$$N_{cr.T} := \frac{1}{i_c^{\ 2}} \cdot \left(G \cdot I_t + \frac{\pi^2 \cdot E \cdot I_w}{l_{cr.T}^{\ 2}} \right) = 23310\ kN$$

Flexural-torsional buckling load:

$$N_{cr.TF} := \frac{i_c^{\ 2}}{2 \cdot \left(i_y^{\ 2} + i_z^{\ 2} \right)} \cdot \left(N_{cr.z} + N_{cr.T} - \sqrt{\left(N_{cr.z} + N_{cr.T} \right)^2 - 4 \cdot N_{cr.z} \cdot N_{cr.T} \cdot \frac{i_y^{\ 2} + i_z^{\ 2}}{i_c^{\ 2}}} \right) = 1758\ kN$$

Critical load: $N_{cr.Tf} := min\left(N_{cr.T}, N_{cr.TF} \right) = 1758\ kN$

Slenderness: $\lambda_T := \sqrt{\frac{A \cdot f_y}{N_{cr.Tf}}} = 1.082$

$$\phi_T := 0.5 \cdot \left(1 + \alpha_z \cdot \left(\lambda_T - 0.2 \right) + \lambda_T^{\ 2} \right) = 1.302$$

Reduction factor:

$$\chi := \text{if } \lambda_T < 0.2 \ \ \| \ 1 \ \ \text{else} \ \ \| \ \frac{1}{\phi_T + \sqrt{\phi_T^{\ 2} - \lambda_T^{\ 2}}} \quad = 0.494$$

Buckling resistance: $N_{b.Rd} := \frac{\chi \cdot A \cdot f_y}{\gamma_{M1}} = 924\ kN$

$\frac{\text{abs}\left(N_c \right)}{N_{b.Rd}} = 0.139$ < 1 OK

E. Cantilever plate verification

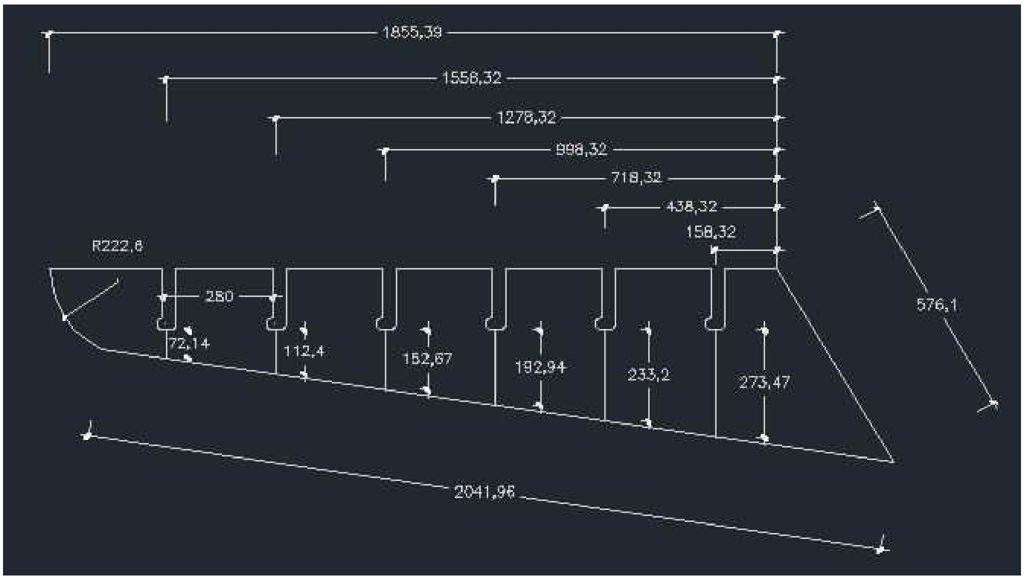
Plate 1

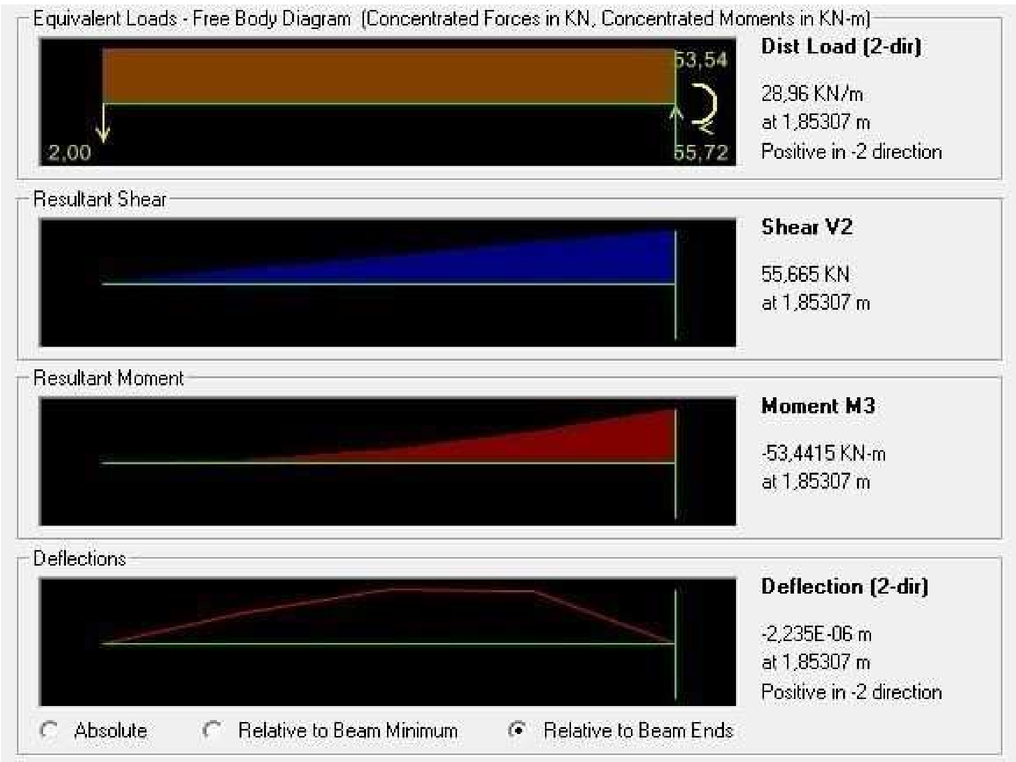
GENERAL DATA

Web thickness: $t_w := 25\ mm$

Design UDL on the cantilever: $q := 28.96\ \frac{kN}{m}$

Design concentrated load (balustrade) at the tip of the cantilever: $Q := 2\ kN$





Analytical expressions of bending moment and shear acting on the cantilever

$$M(x) := \left(-53.54 + 55.72 \cdot x - 28.96 \cdot \frac{x^2}{2}\right) kN \cdot m$$

$$V(x) := (-55.72 + 28.96 \cdot x)\ kN$$

FLEXURE RESISTANCE VERIFICATION

Verification has been carried out for the reduced cross-sections located beneath each cut-out

Moment values	Beam depths	Section moduli
$M_1 := M(0.15832) = -45.081\ kN \cdot m$	$h_1 := 273.47\ mm$	$W_1 := t_w \cdot \frac{h_1^2}{6} = (3.116 \cdot 10^5)\ mm^3$
$M_2 := M(0.43832) = -31.899\ kN \cdot m$	$h_2 := 233.20\ mm$	$W_2 := t_w \cdot \frac{h_2^2}{6} = (2.266 \cdot 10^5)\ mm^3$
$M_3 := M(0.71832) = -20.987\ kN \cdot m$	$h_3 := 192.94\ mm$	$W_3 := t_w \cdot \frac{h_3^2}{6} = (1.551 \cdot 10^5)\ mm^3$
$M_4 := M(0.99832) = -12.345\ kN \cdot m$	$h_4 := 152.67\ mm$	$W_4 := t_w \cdot \frac{h_4^2}{6} = (9.712 \cdot 10^4)\ mm^3$
$M_5 := M(1.278) = -5.98\ kN \cdot m$	$h_5 := 112.40\ mm$	$W_5 := t_w \cdot \frac{h_5^2}{6} = (5.264 \cdot 10^4)\ mm^3$
$M_6 := M(1.558) = -1.876\ kN \cdot m$	$h_6 := 72.14\ mm$	$W_6 := t_w \cdot \frac{h_6^2}{6} = (2.168 \cdot 10^4)\ mm^3$

Verification:

$$\sigma_{max} := \max\left(\mathrm{abs}\left(\frac{M_1}{W_1}\right), \mathrm{abs}\left(\frac{M_2}{W_2}\right), \mathrm{abs}\left(\frac{M_3}{W_3}\right), \mathrm{abs}\left(\frac{M_4}{W_4}\right), \mathrm{abs}\left(\frac{M_5}{W_5}\right), \mathrm{abs}\left(\frac{M_6}{W_6}\right)\right) = 144.673\ MPa$$

$$\frac{\sigma_{max} \cdot \gamma_{M0}}{f_y} = 0.408 \qquad < 1 \qquad \text{OK}$$

SHEAR RESISTANCE VERIFICATION

Verification has been carried out for the reduced cross-sections located beneath each cut-out

Shear values

$$V_1 := V(0.15832) = -51.135\ kN$$

$$V_2 := V(0.43832) = -43.026\ kN$$

$$V_3 := V(0.71832) = -34.917\ kN$$

$$V_4 := V(0.99832) = -26.809\ kN$$

$$V_5 := V(1.278) = -18.709\ kN$$

$$V_6 := V(1.558) = -10.6\ kN$$

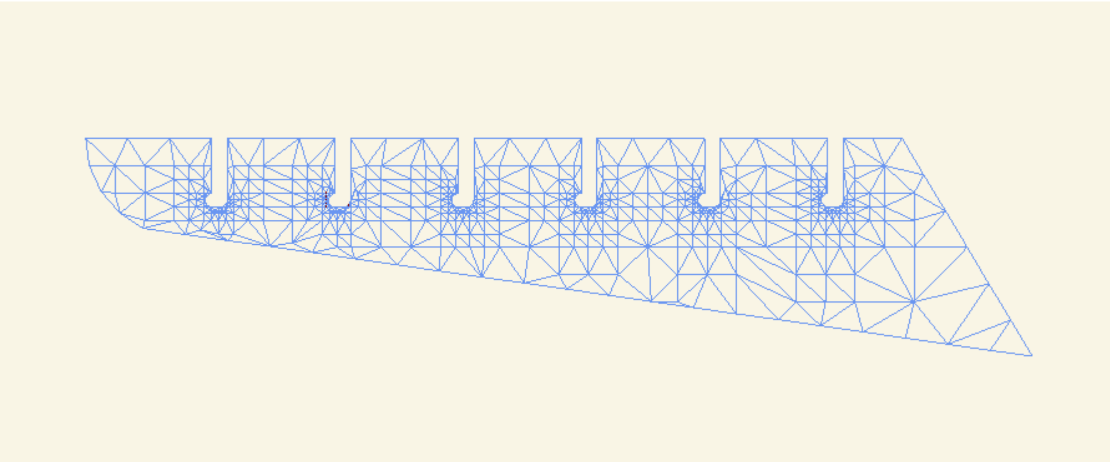
$$\tau_{max} := \max\left(\mathrm{abs}\left(\frac{V_1}{t_w \cdot h_1}\right), \mathrm{abs}\left(\frac{V_2}{t_w \cdot h_2}\right), \mathrm{abs}\left(\frac{V_3}{t_w \cdot h_3}\right), \mathrm{abs}\left(\frac{V_4}{t_w \cdot h_4}\right), \mathrm{abs}\left(\frac{V_5}{t_w \cdot h_5}\right), \mathrm{abs}\left(\frac{V_6}{t_w \cdot h_6}\right)\right) = 7.479\ MPa$$

$$\frac{\tau_{max} \cdot \gamma_{M0}}{f_y} = 0.021 \quad < \ 1 \quad \text{OK}$$

FLEXURAL-TORSIONAL BUCKLING VERIFICATION

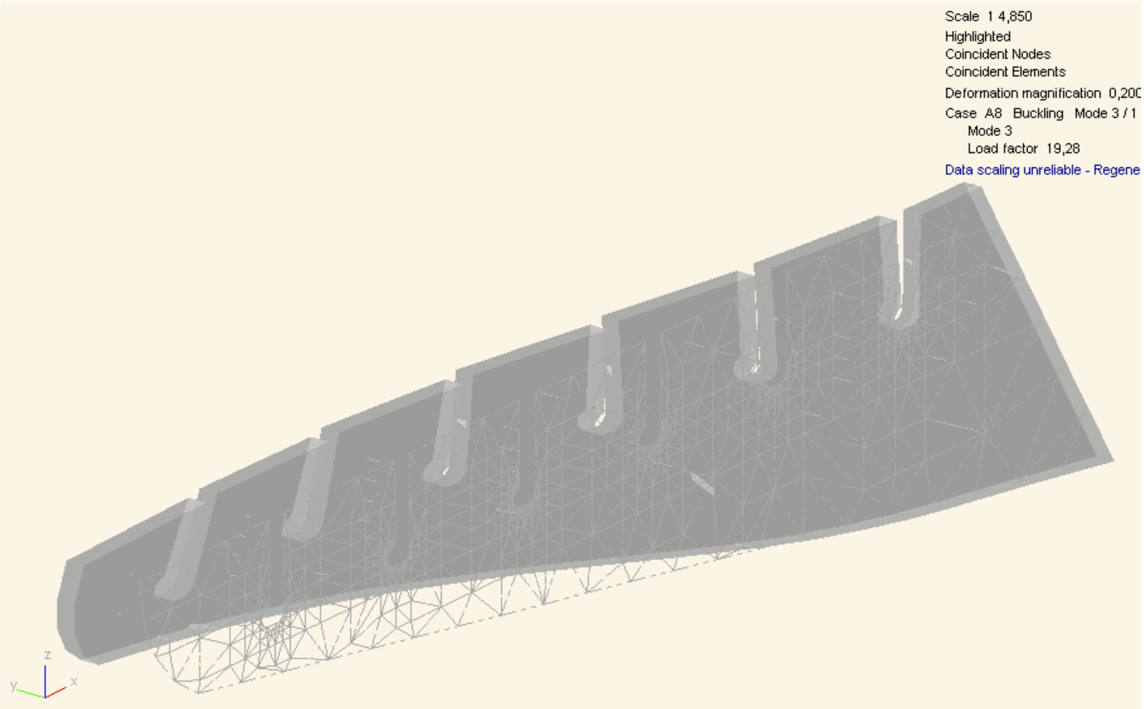
A FEM analysis has been conducted to assess the flexural-torsional buckling resistance of the cantilever beams.

Beam model mesh

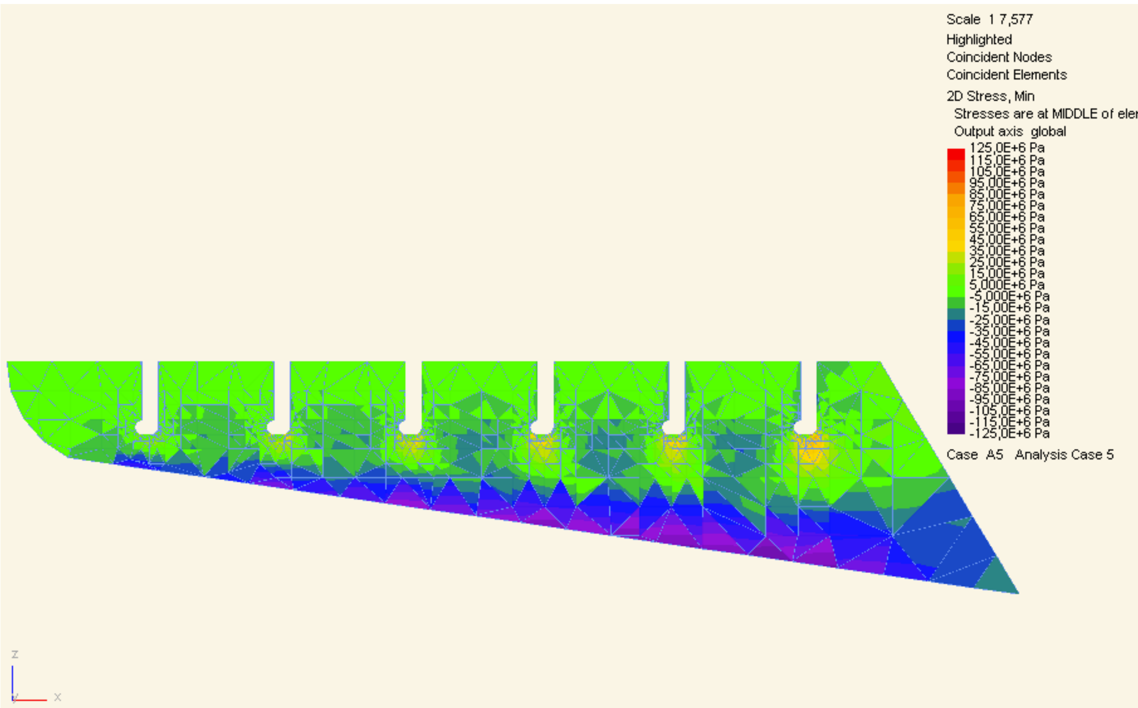


LT-Buckling mode 1 (Load factor = 7.382)

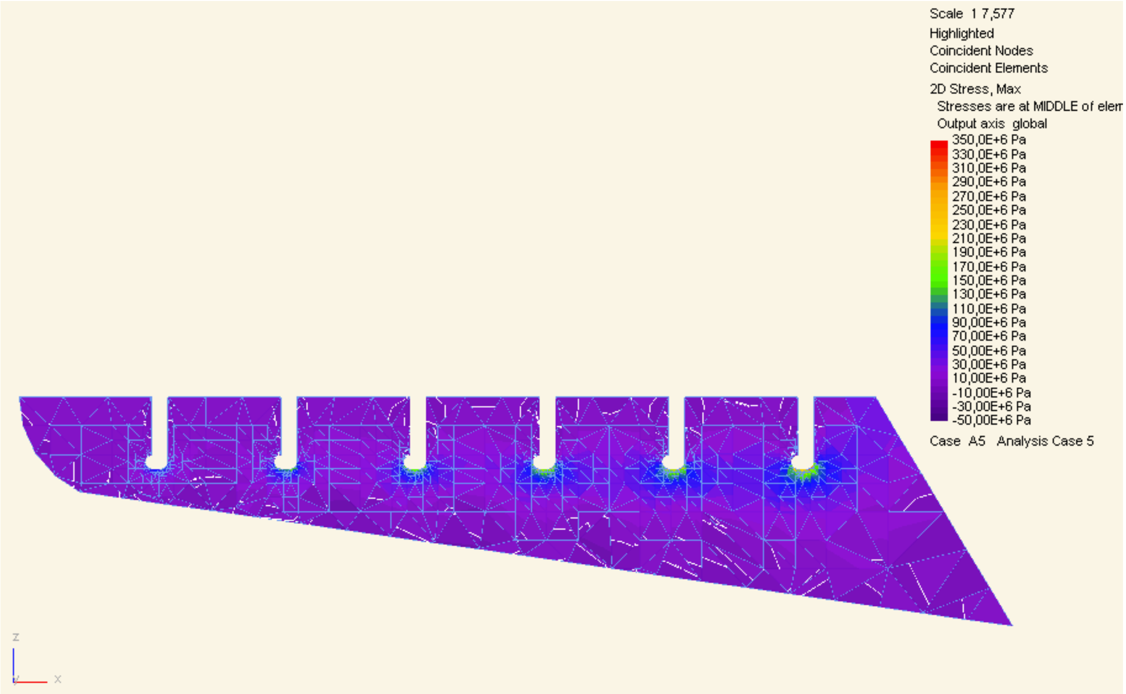
LT-Buckling mode 2 (Load factor = 19.28)



Maximum absolute compression stress $\sigma_{Ed.cmax} := 102.6\ MPa$



Maximum tension stress $\sigma_{Ed.t} := 333\ MPa$ *(peak value at cut-out bottom)*

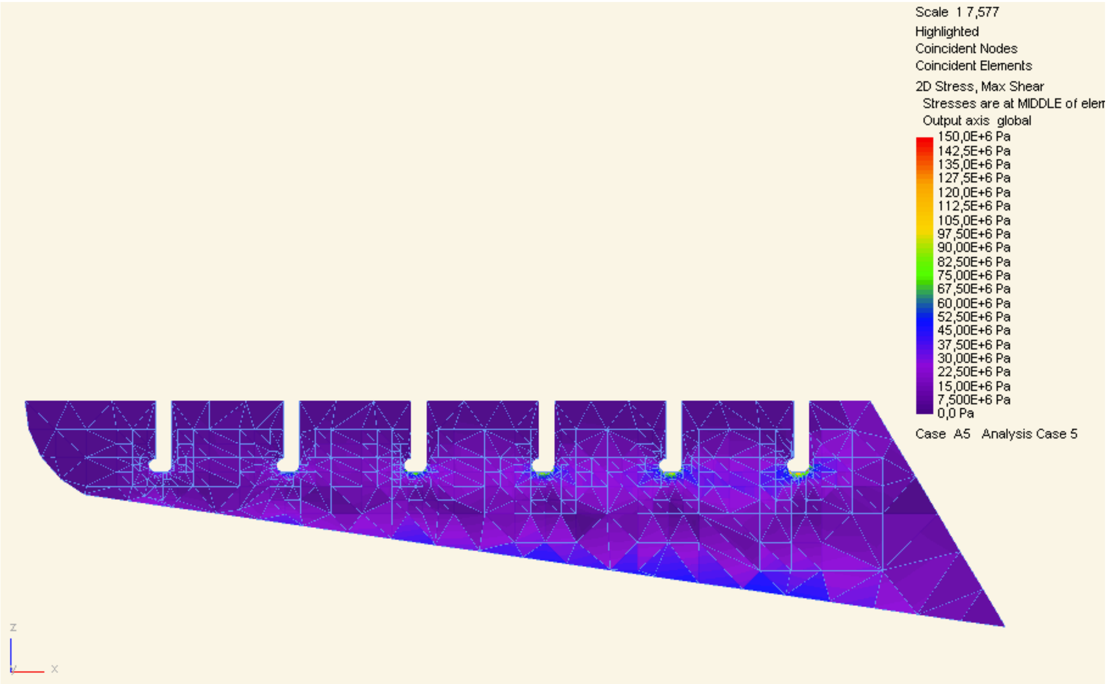


Axial stress verification:

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax}}{\frac{f_y}{\gamma_{M0}}} = 0.289$$ < 1 OK

$$\eta_{1.t} := \frac{\sigma_{Ed.t}}{\frac{f_y}{\gamma_{M1}}} = 1.032$$ < 1 OK *(it's okay since a conservative load has been applied)*

Maximum absolute shear stress $\tau_{Edmax} := 141.2\ MPa$



Shear stress verification:

$$\eta_3 := \frac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \eta} = 0.632 \qquad < 1 \qquad \text{OK}$$

shear buckling coefficient =1 since: $\frac{h_1}{t_w} = 10.939 \qquad < \qquad 72 \cdot \frac{\varepsilon}{\eta} = 48.817$

Axial-shear stress interaction $\sigma_{VonMises} := 307.2\ MPa$

Verification: $\eta := \sigma_{VonMises} \cdot \frac{\gamma_{M0}}{f_y} = 0.865 \qquad < 1 \qquad \text{OK}$

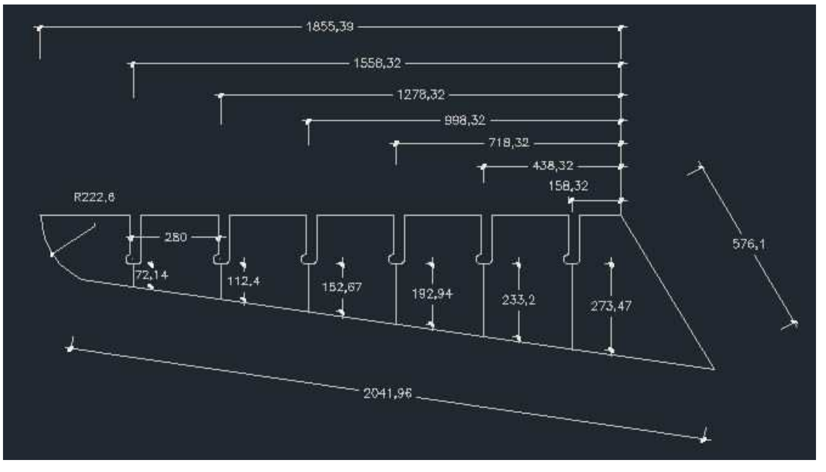
Plate 2

GENERAL DATA

Web thickness: $t_w := 25\ mm$

Design UDL on the cantilever: $q := 28.96\ \frac{kN}{m}$

Design concentrated load (balustrade) at the tip of the cantilever: $Q := 2\ kN$



Analytical expressions of bending moment and shear acting on the cantilever

$$M(x) := \left(-53.54 + 55.72 \cdot x - 28.96 \cdot \frac{x^2}{2}\right) kN \cdot m$$

$$V(x) := \left(-55.72 + 28.96 \cdot x\right) kN$$

FLEXURE RESISTANCE VERIFICATION

Verification has been carried out for the reduced cross-sections located beneath each cut-out

Moment values	Beam depths	Section moduli
$M_1 := M(0.15832) = -45.081\ kN \cdot m$	$h_1 := 273.47\ mm$	$W_1 := t_w \cdot \frac{h_1^{\ 2}}{6} = \left(3.116 \cdot 10^5\right) mm^3$
$M_2 := M(0.43832) = -31.899\ kN \cdot m$	$h_2 := 233.20\ mm$	$W_2 := t_w \cdot \frac{h_2^{\ 2}}{6} = \left(2.266 \cdot 10^5\right) mm^3$
$M_3 := M(0.71832) = -20.987\ kN \cdot m$	$h_3 := 192.94\ mm$	$W_3 := t_w \cdot \frac{h_3^{\ 2}}{6} = \left(1.551 \cdot 10^5\right) mm^3$
$M_4 := M(0.99832) = -12.345\ kN \cdot m$	$h_4 := 152.67\ mm$	$W_4 := t_w \cdot \frac{h_4^{\ 2}}{6} = \left(9.712 \cdot 10^4\right) mm^3$
$M_5 := M(1.278) = -5.98\ kN \cdot m$	$h_5 := 112.40\ mm$	$W_5 := t_w \cdot \frac{h_5^{\ 2}}{6} = \left(5.264 \cdot 10^4\right) mm^3$
$M_6 := M(1.558) = -1.876\ kN \cdot m$	$h_6 := 72.14\ mm$	$W_6 := t_w \cdot \frac{h_6^{\ 2}}{6} = \left(2.168 \cdot 10^4\right) mm^3$

Verification:

$$\sigma_{max} := \max\left(\mathrm{abs}\left(\frac{M_1}{W_1}\right), \mathrm{abs}\left(\frac{M_2}{W_2}\right), \mathrm{abs}\left(\frac{M_3}{W_3}\right), \mathrm{abs}\left(\frac{M_4}{W_4}\right), \mathrm{abs}\left(\frac{M_5}{W_5}\right), \mathrm{abs}\left(\frac{M_6}{W_6}\right)\right) = 144.673\ MPa$$

$$\frac{\sigma_{max} \cdot \gamma_{M0}}{f_y} = 0.408 \qquad < 1 \qquad \text{OK}$$

SHEAR RESISTANCE VERIFICATION

Verification has been carried out for the reduced cross-sections located beneath each cut-out

Shear values

$$V_1 := V(0.15832) = -51.135\ kN$$

$$V_2 := V(0.43832) = -43.026\ kN$$

$$V_3 := V(0.71832) = -34.917\ kN$$

$$V_4 := V(0.99832) = -26.809\ kN$$

$$V_5 := V(1.278) = -18.709\ kN$$

$$V_6 := V(1.558) = -10.6\ kN$$

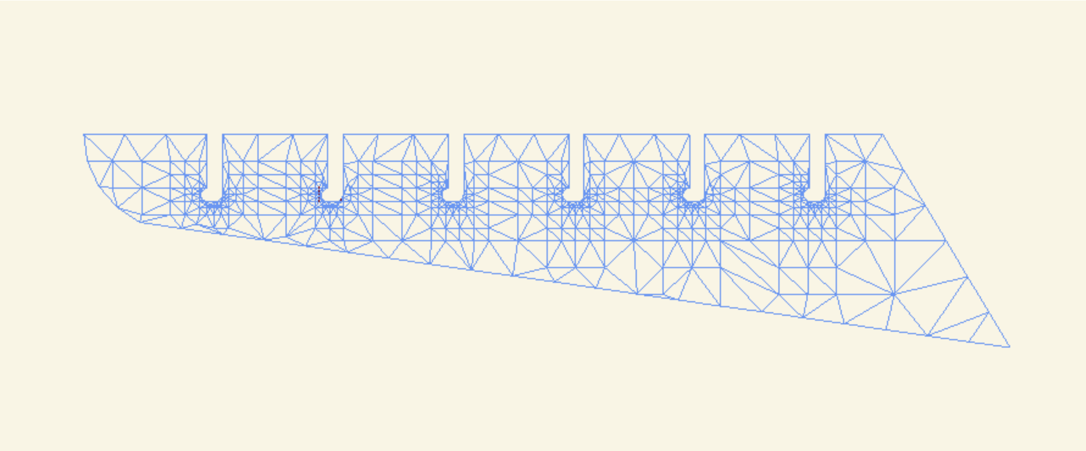
$$\tau_{max} := \max\left(\mathrm{abs}\left(\frac{V_1}{t_w \cdot h_1}\right), \mathrm{abs}\left(\frac{V_2}{t_w \cdot h_2}\right), \mathrm{abs}\left(\frac{V_3}{t_w \cdot h_3}\right), \mathrm{abs}\left(\frac{V_4}{t_w \cdot h_4}\right), \mathrm{abs}\left(\frac{V_5}{t_w \cdot h_5}\right), \mathrm{abs}\left(\frac{V_6}{t_w \cdot h_6}\right)\right) = 7.479\ MPa$$

$$\frac{\tau_{max} \cdot \gamma_{M0}}{f_y} = 0.021 \qquad < \ 1 \qquad \text{OK}$$

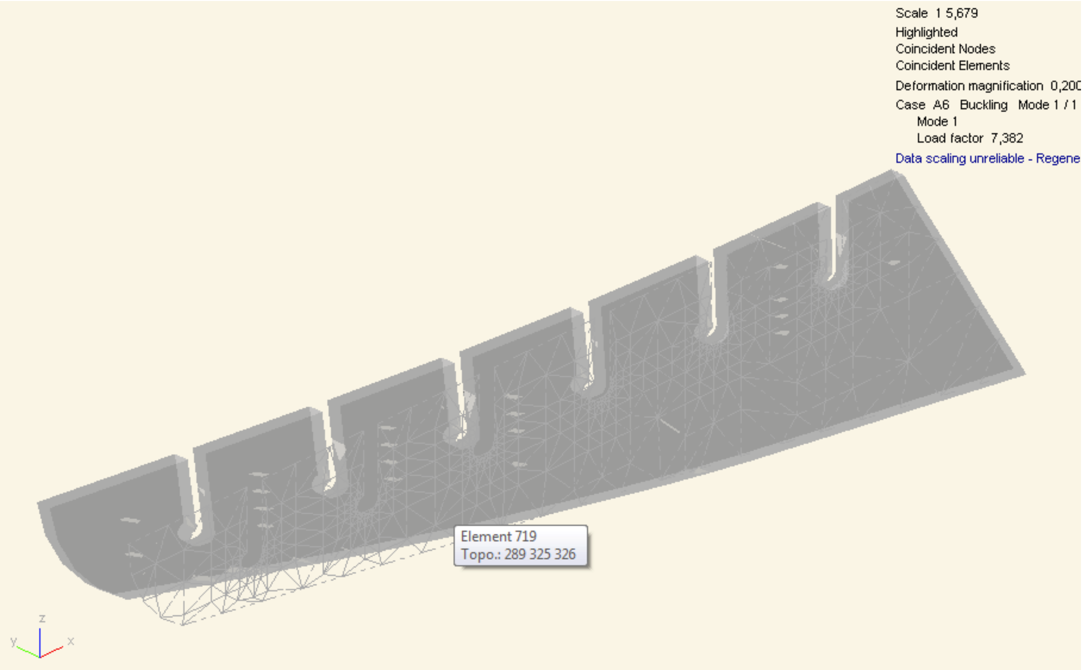
FLEXURAL-TORSIONAL BUCKLING VERIFICATION

A FEM analysis has been conducted to assess the flexural-torsional buckling resistance of the cantilever beams.

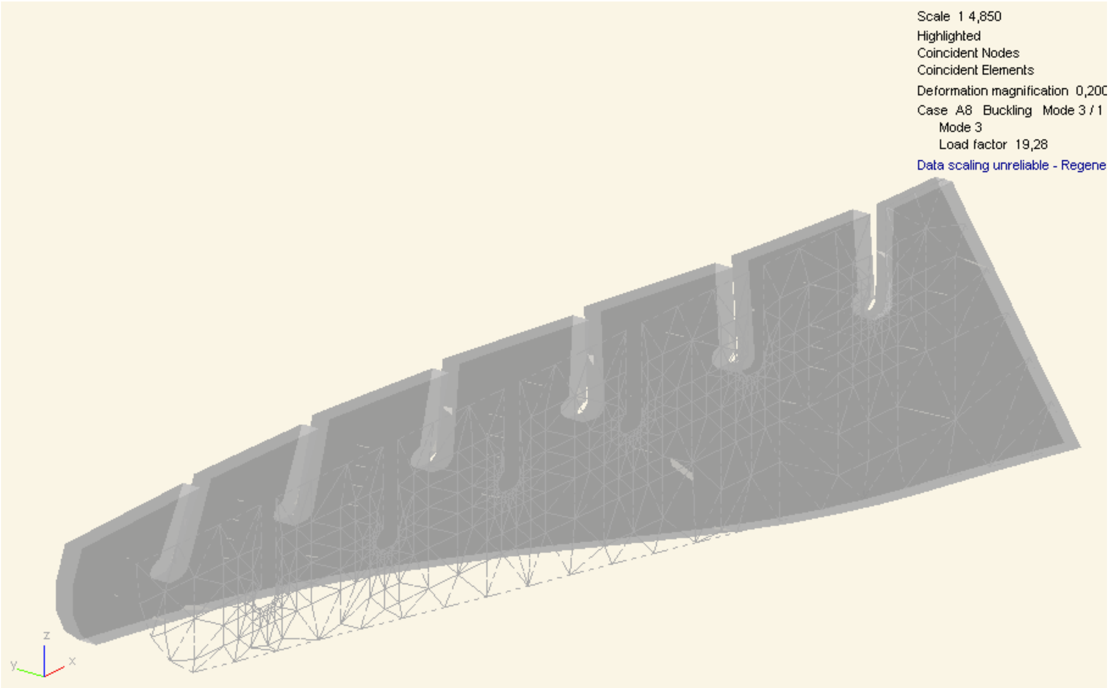
Beam model mesh



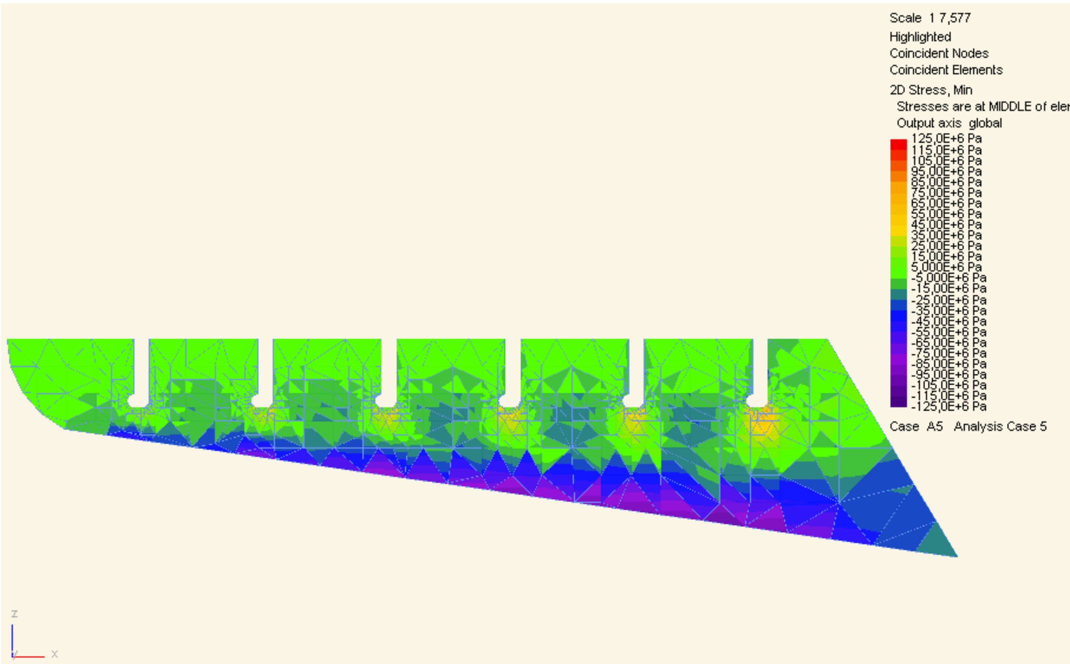
LT-Buckling mode 1 (Load factor = 7.382)



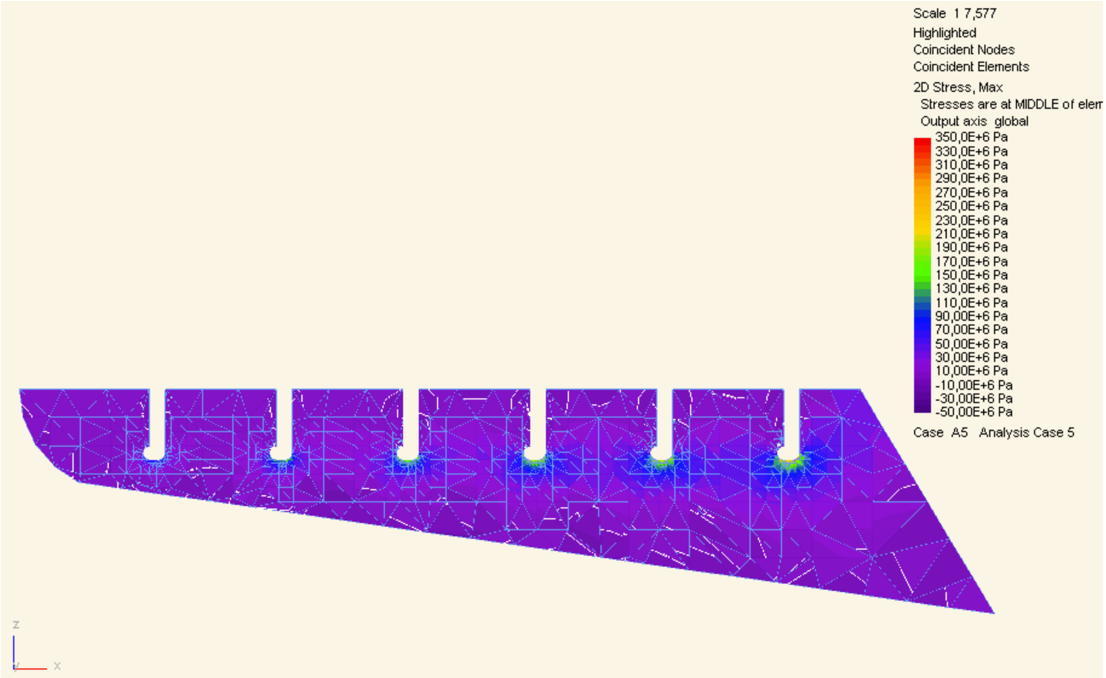
LT-Buckling mode 2 (Load factor = 19.28)



Maximum absolute compression stress $\sigma_{Ed.cmax} := 102.6\ MPa$



Maximum tension stress $\sigma_{Ed.t} := 333\ MPa$ *(peak value at cut-out bottom)*

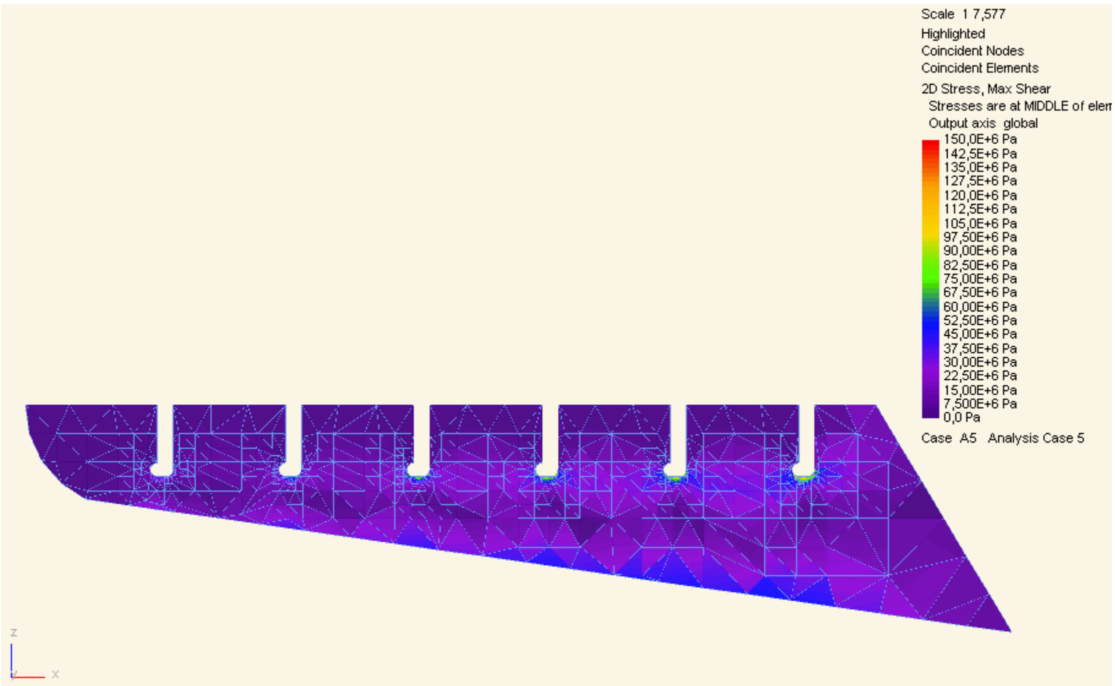


Axial stress verification:

$$\eta_{1.c} := \frac{\sigma_{Ed.cmax}}{\dfrac{f_y}{\gamma_{M0}}} = 0.289$$ < 1 OK

$$\eta_{1.t} := \frac{\sigma_{Ed.t}}{\dfrac{f_y}{\gamma_{M1}}} = 1.032$$ < 1 OK *(it's okay since a conservative load has been applied)*

Maximum absolute shear stress $\tau_{Edmax} := 2\ MPa$



Shear stress verification:

$$\eta_3 := \frac{\sqrt{3} \cdot \tau_{Edmax} \cdot \gamma_{M1}}{f_y \cdot \eta} = 0.012 \qquad < 1 \qquad \text{OK}$$

shear buckling coefficient =1 since: $\dfrac{h_1}{t_w} = 10.939 \quad < \quad 72 \cdot \dfrac{\varepsilon}{\eta} = 67.695$

Axial-shear stress interaction $\sigma_{VonMises} := 307.2\ MPa$

Verification: $\eta := \sigma_{VonMises} \cdot \dfrac{\gamma_{M0}}{f_y} = 0.865 \qquad < 1 \qquad$ OK

F. Mast verification

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0 \qquad \gamma_{M1} := 1.1$

Steel type: S355

Yielding strength: $f_y := 355\ MPa \qquad \varepsilon := \sqrt{\dfrac{235\ MPa}{f_y}} = 0.814$

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa \qquad \nu := 0.3 \qquad G := \dfrac{E}{2 \cdot (1+\nu)} = 80769\ MPa$

MAXIMUM FORCES/MOMENTS

$N_{Ed} := 2982\ kN$

$M_{y.Ed} := 891.1\ kN \cdot m$

$M_{z.Ed} := 143\ kN \cdot m$

$M_{x.Ed} := 0\ kN \cdot m$

MAST GEOMETRY

Length: $L := 23.61\ m$

Top diameter: $d_T := 600\ mm$

Mid-height diameter: $d_M := 600\ mm$

Bottom diameter: $d_B := d_T = 600\ mm$

External diameter (x=longitudinal abscissa):

$$d_e(x) := \begin{cases} \dfrac{(d_M - d_T)}{\frac{L}{2}} \cdot \left(x - \dfrac{L}{2}\right) + d_M & \text{if } x \le \dfrac{L}{2} \\ -\dfrac{(d_M - d_T)}{\frac{L}{2}} \cdot \left(x - \dfrac{L}{2}\right) + d_M & \text{else} \end{cases}$$

Wall thickness: $t := 40\ mm$

Section modulus: $W(x) := \pi \cdot \dfrac{\left(d_e(x)^4 - (d_e(x) - 2 \cdot t)^4\right)}{32 \cdot d_e(x)}$

Cross-section class:

$\frac{d_M}{t} = 15$ < $50 \cdot \varepsilon^2 = 33.099$ Class 1

< $70 \cdot \varepsilon^2 = 46.338$ Class 2

Cross sectional area: $A(x) := \frac{\pi}{4} \cdot \left(d_e(x)^2 - \left(d_e(x) - 2\ t\right)^2\right)$

Resistance verification

Normal stress derived from the design axial loads and bending moments combination.
Verification refers to the most stressed section of the mast (located at 2.47m from the bottom section of the mast - Model element 3439)

Design stresses:

$\sigma_{x.Ed} := 65.92\ MPa$

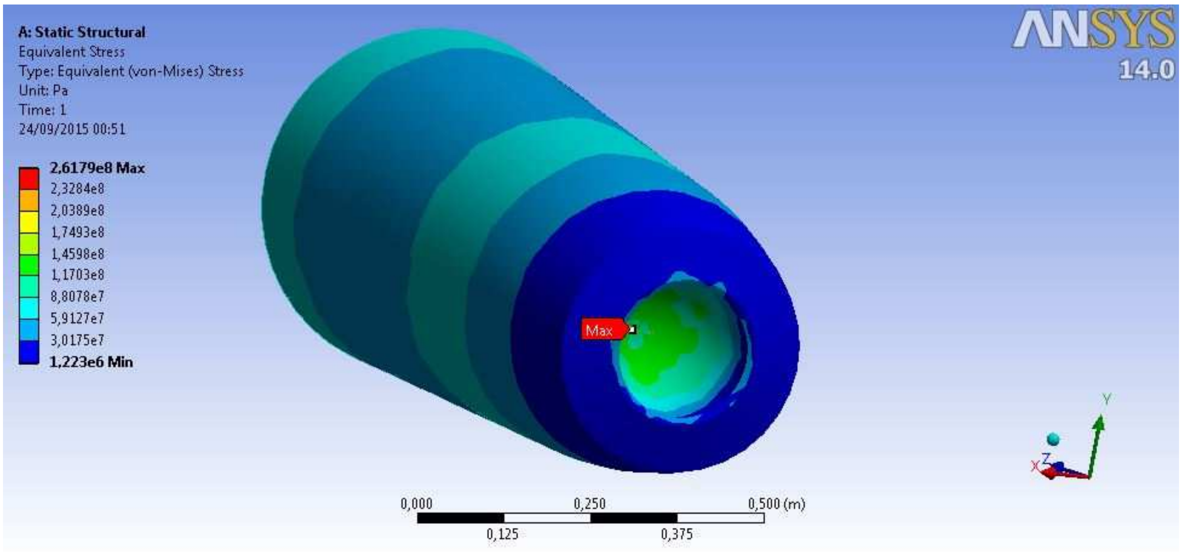
$\tau_{y.Ed} := 345100\ Pa$

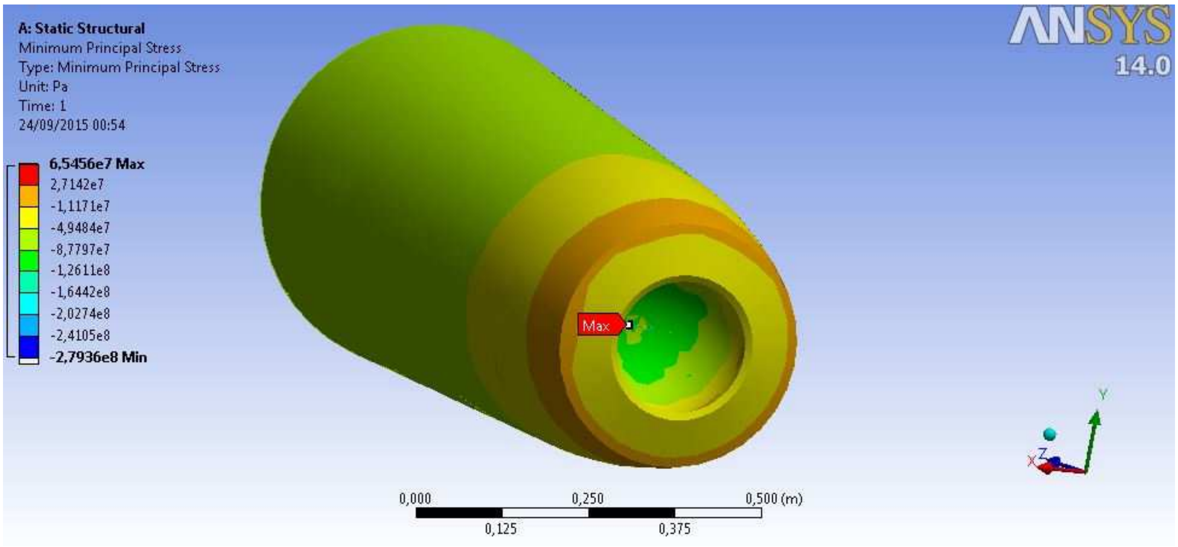
$\tau_{z.Ed} := 1.93\ MPa$

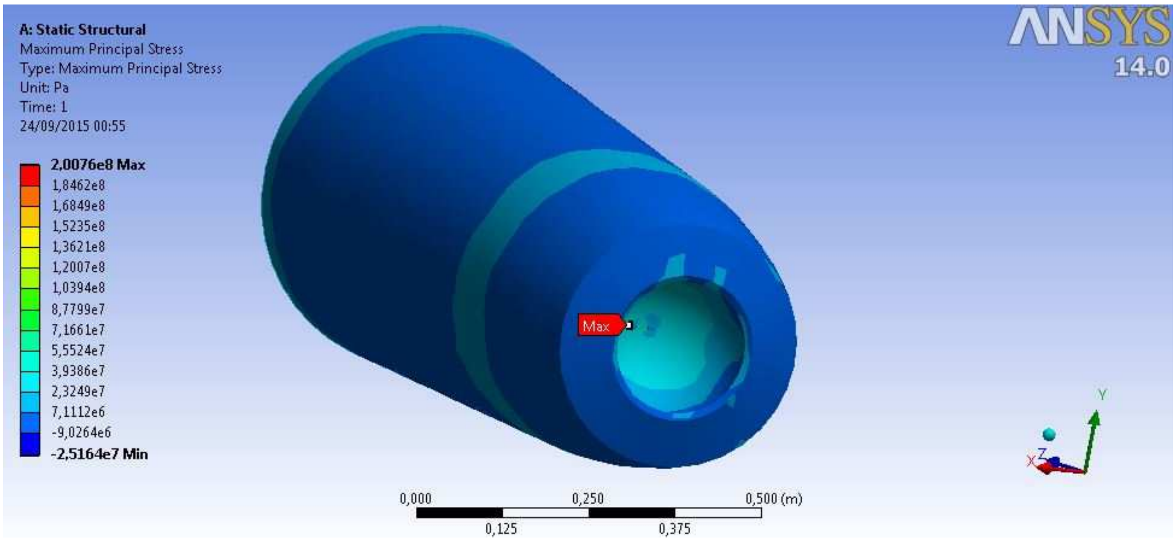
Verification

$\left(\sigma_{x.Ed} \cdot \frac{\gamma_{M0}}{f_y}\right)^2 + 3 \cdot \left(\left(\tau_{y.Ed} + \tau_{z.Ed}\right) \cdot \frac{\gamma_{M0}}{f_y}\right)^2 = 0.035$ < 1 OK

Local effects verification







Buckling verification

Minimum amplifier of the design load to reach characteristic resistance:

$$\alpha_{ult.k} := \frac{f_y}{\sigma_{x.Ed}} = 5.385$$

Imperfection factor for relevant buckling curve (a): $\alpha := 0.21$

Minimum amplifier of the design load to reach elastic critical resistance:

$$\alpha_{cr.op} := 3.17$$

Non-dimensional slenderness:

$$\lambda_{op} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr.op}}} = 1.303$$

$$\Phi := 0.5 \cdot \left(1 + \alpha \cdot \left(\lambda_{op} - 0.2\right) + {\lambda_{op}}^2\right)$$

Reduction factor for the non-dimensional slenderness:

$$\chi_{op} := \frac{1}{\Phi + \sqrt{\Phi^2 + {\lambda_{op}}^2}} = 0.292$$

Verification

$$\chi_{op} \cdot \frac{\alpha_{ult.k}}{\gamma_{M1}} = 1.429 \qquad > \qquad 1 \qquad \text{OK}$$

<u>Pre-design buckling resistance check</u> (simplified model where the mast is treated like a pinned column at both ends)

Second moment of area:

$$J := \frac{\pi}{64} \cdot \left({d_T}^4 - \left(d_T - 2\ t\right)^4\right) = 0.003\ m^4$$

Elastic critical load:

$$P_{cr} := \pi^2 \cdot E \cdot \frac{J}{L^2} = \left(1.031 \cdot 10^4\right)\ kN$$

Verification

$$\frac{N_{Ed}}{P_{cr}} = 0.289 \qquad < \qquad 1 \qquad \text{OK}$$

FEM model incremental load analysis results (uniform circular cross section, CHS 600x40mm)

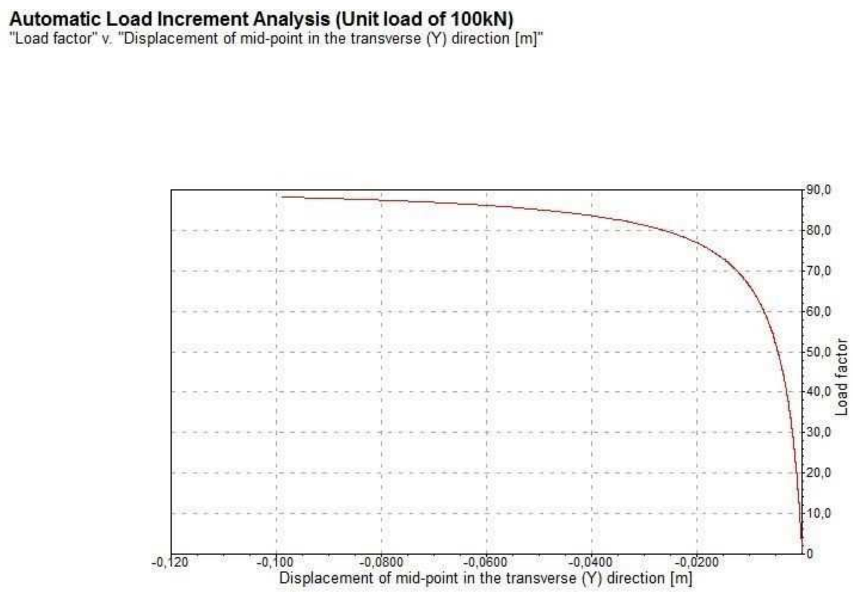
Being on the safe side, we should set a Load factor limit up to 800 so that the corresponding displacement would be less than 20mm. Even by doing so we get an elastic critical buckling load of:

$LF := 800$ $\quad UnitLoad := 10\ kN$ $\quad P_{cr} := LF \cdot UnitLoad = \left(8 \cdot 10^{3}\right)\ kN$

Verification of buckling resistance (FEM simplified model)

$\frac{N_{Ed}}{P_{cr}} = 0.373$ < 1 OK

FEM model incremental load analysis results (uniform circular cross section, CHS 600x40mm)



Being on the safe side, we should set a Load factor limit up to 75 so that the corresponding displacement would be less than 20mm. Even by doing so we get an elastic critical buckling load of:

$LF := 75$ $\qquad$ $UnitLoad := 100\ kN$ $\qquad$ $P_{cr} := LF \cdot UnitLoad = \left(7.5 \cdot 10^{3}\right)\ kN$

Verification of buckling resistance (FEM simplified model)

$\frac{N_{Ed}}{P_{cr}} = 0.398$ $\qquad$ < $\qquad$ 1 $\qquad$ OK

G1. Deck-to-stay cable connection plate verification

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\qquad$ $\gamma_{M1} := 1.1$ $\qquad$ $\gamma_{M2} := 1.25$ $\qquad$ $\gamma_{M6.ser} := 1.0$

Steel type: S355

Yielding strength: $f_y := 355\ MPa$ $\qquad$ $\varepsilon := \sqrt{\frac{235\ MPa}{f_y}} = 0.814$

$f_{y.red} := 335\ MPa$ $\qquad$ (40mm < t < 80mm)

Rupture strength: $f_u := 510\ MPa$

Elastic moduli: $E := 210000\ MPa$ $\qquad$ $\nu := 0.3$ $\qquad$ $G := \frac{E}{2 \cdot (1 + \nu)} = 80769\ MPa$

Pin steel type: 39NiCrMo3

Pin yielding strength: $f_{yp} := 635\ MPa$

Pin rupture strength: $f_{up} := 830\ MPa$

CABLE DESIGN FORCE

ULS: $F_{Ed} := 1265\ kN$

SLS: $F_{Ed.ser} := 1012\ kN$

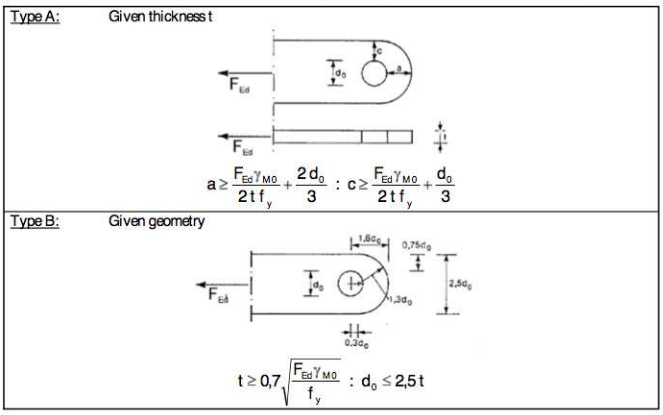
CONNECTION GEOMETRIC DATA

Pin diameter: $d := 155\ mm$

Hole diameter: $d_0 := d + 3\ mm = 158\ mm$

Plate thickness: $t := 50\ mm$

Table 3.9: Geometrical requirements for pin ended members



Minimum plate thickness: $a_{min} := F_{Ed} \cdot \dfrac{\gamma_{M0}}{2 \cdot t \cdot f_{y.red}} + 2 \cdot \dfrac{d_0}{3} = 143.095\ mm$

$c_{min} := F_{Ed} \cdot \dfrac{\gamma_{M0}}{2 \cdot t \cdot f_{y.red}} + \dfrac{d_0}{3} = 90.428\ mm$

Plate dimensions:

$a := 144\ mm$ $\quad c := 144\ mm$ $\quad t = 50\ mm$

Socket width (Bridon catalogue): $L_3 := 348\ mm$

Fork thickness: $t_f := 70\ mm$

Space b/w plate and fork $c := \dfrac{L_3}{2} - \dfrac{t}{2} - t_f = 79\ mm$

ULS pin shear resistance verification

$F_{v.Rd} := 0.6 \cdot \left(\pi \cdot \dfrac{d^2}{4}\right) \cdot \dfrac{f_{up}}{\gamma_{M2}} = \left(7.517 \cdot 10^3\right)\ kN$ > $F_{v.Ed} := \dfrac{F_{Ed}}{2} = 632.5\ kN$ OK

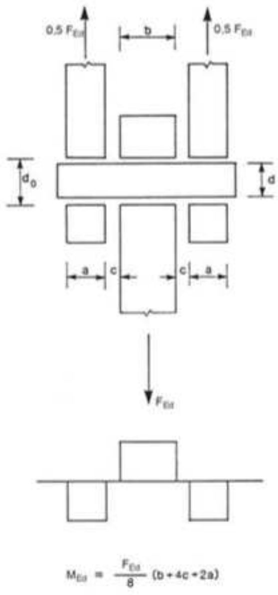
ULS bearing resistance of the plate

$F_{b.p.Rd} := 1.5 \cdot t \cdot d \cdot \dfrac{f_{y.red}}{\gamma_{M0}} = \left(3.894 \cdot 10^3\right)\ kN$ > $F_{Ed} = \left(1.265 \cdot 10^3\right)\ kN$ OK

ULS bearing resistance of the forks

$F_{b.f.Rd} := 1.5 \cdot 2 \cdot t_f \cdot d \cdot \dfrac{f_{y.red}}{\gamma_{M0}} = \left(1.09 \cdot 10^4\right)\ kN$ > $F_{Ed} = \left(1.265 \cdot 10^3\right)\ kN$ OK

ULS bending resistance of the pin



$M_{Rd} := 1.5 \cdot \pi \cdot \dfrac{d^3}{32} \cdot \dfrac{f_{yp}}{\gamma_{M0}} = 348.225\ kN \cdot m$

$M_{Ed} := \dfrac{F_{Ed}}{8} \cdot \left(t + 2 \cdot t_f + 4 \cdot c\right) = 80.011\ (kN \cdot m)$

$\dfrac{M_{Ed}}{M_{Rd}} = 0.23$ < 1 OK

ULS combined shear and bending resistance of the pin

$\left(\dfrac{M_{Ed}}{M_{Rd}}\right)^2 + \left(\dfrac{F_{v.Ed}}{F_{v.Rd}}\right)^2 = 0.06$ < 1 OK

SLS pin replaceability requirements

$F_{b.Rd.ser} := min\left(0.6 \cdot t \cdot d \cdot \dfrac{f_{y.red}}{\gamma_{M6.ser}}, 0.6 \cdot 2 \cdot t_f \cdot d \cdot \dfrac{f_{y.red}}{\gamma_{M6.ser}}\right) = \left(1.558 \cdot 10^3\right)\ kN$

$F_{Ed.ser} = \left(1.012 \cdot 10^3\right)\ kN$

$\dfrac{F_{Ed.ser}}{F_{b.Rd.ser}} = 0.65$ < 1 OK

$$M_{Rd.ser} := 0.8 \cdot \pi \cdot \frac{d^3}{32} \cdot \frac{f_{yp}}{\gamma_{M6.ser}} = 185.72\ kN \cdot m$$

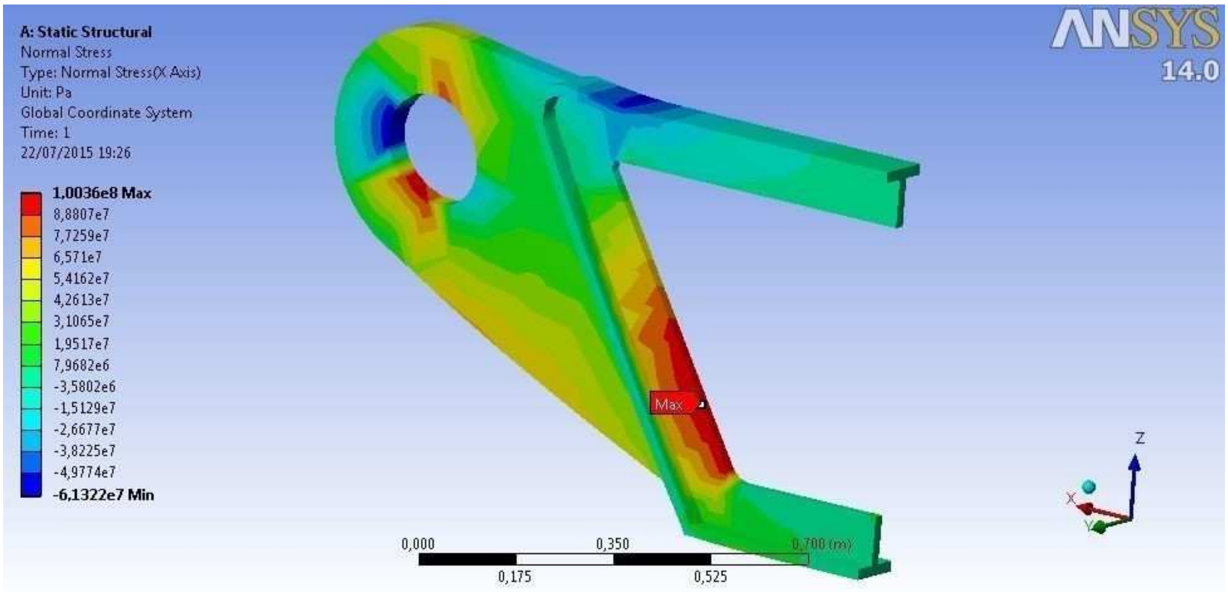
$$M_{Ed.ser} := \frac{F_{Ed.ser}}{8} \cdot (t + 2 \cdot t_f + 4 \cdot c) = 64.009\ kN \cdot m$$

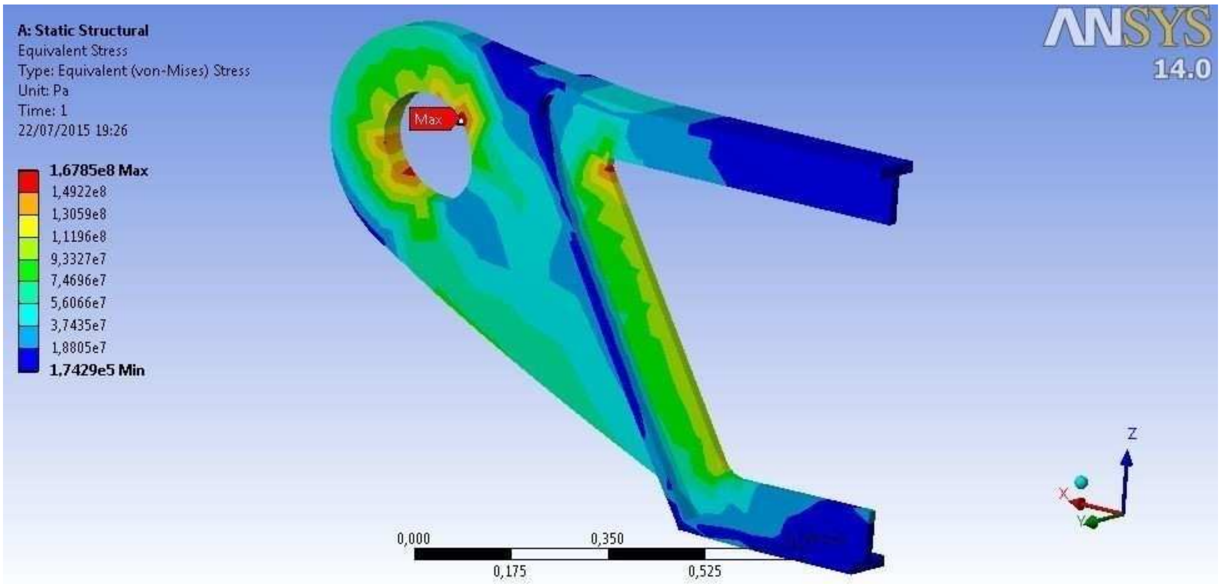
$$\frac{M_{Ed.ser}}{M_{Rd.ser}} = 0.345 \qquad < 1 \qquad \text{OK}$$

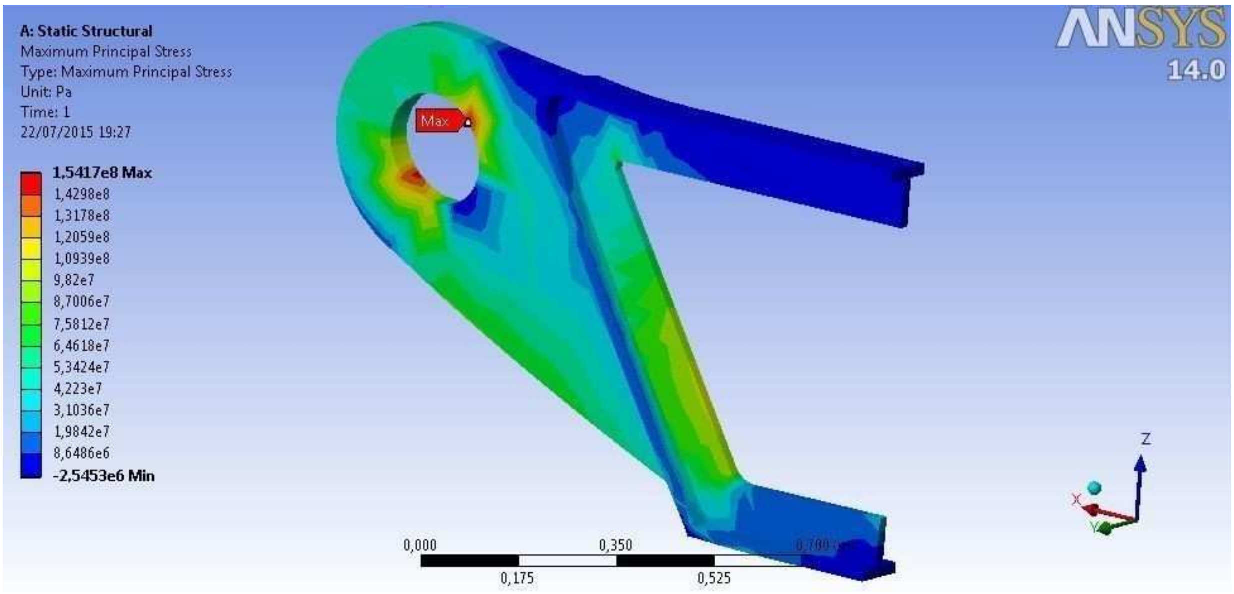
$$\sigma_{h.Ed} := 0.591 \cdot \sqrt{\frac{E \cdot F_{Ed.ser} \cdot (d_0 - d)}{d^2 \cdot t}} = \left(4.306 \cdot 10^8\right)\ Pa$$

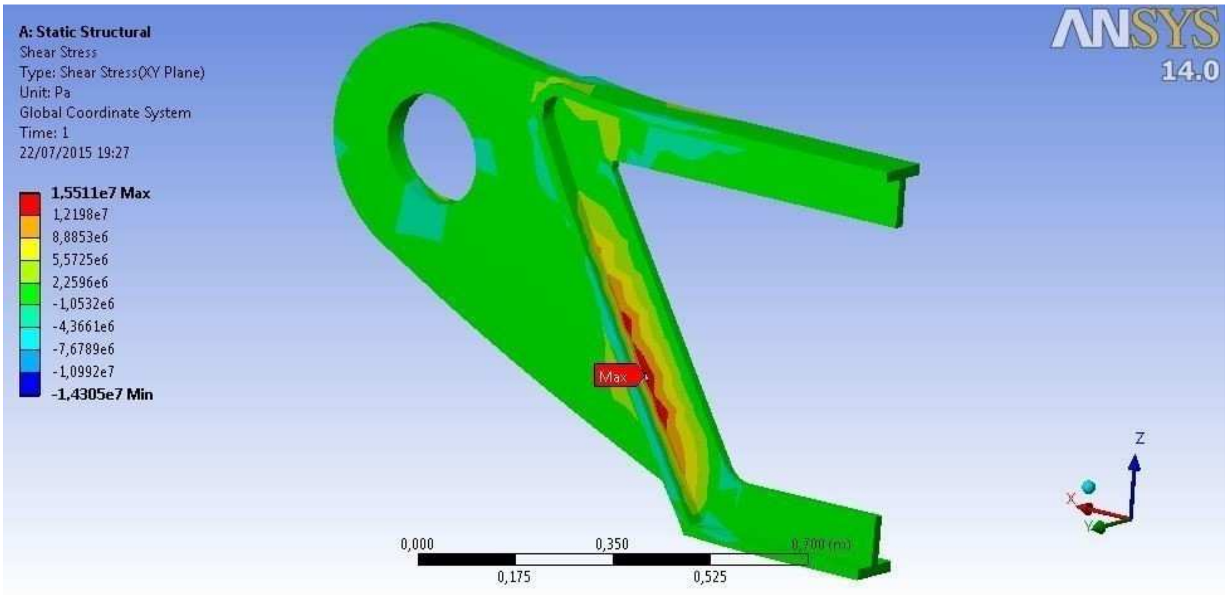
$$f_{h.Ed} := 2.5 \cdot \frac{f_{y.red}}{\gamma_{M6.ser}} = 837.5\ MPa$$

FEM analysis to assess local behaviour of the connection









<u>ULS local peak stress verification</u>

Max normal stress:	$\sigma_{max} := 100.36\ MPa$	<	$\dfrac{f_y}{\gamma_{M1}} = 322.727\ MPa$	OK
Min normal stress:	$\sigma_{min} := \lvert -61.32\ MPa \rvert$	<	$\dfrac{f_y}{\gamma_{M0}} = 355\ MPa$	OK
Max shear stress:	$\tau_{max} := 155.11\ MPa$	<	$\dfrac{f_y}{\sqrt{3}\ \gamma_{M1}} = 195.421\ MPa$	OK
Von-Mises stress:	$\sigma_{VM} := 167.85\ MPa$			

Note: no local buckling is experienced

G2. Mast-stay cable joint verification

GENERAL DATA

Safety factors: $\gamma_{M0} := 1.0$ $\gamma_{M1} := 1.1$ $\gamma_{M2} := 1.25$ $\gamma_{M6.ser} := 1.0$

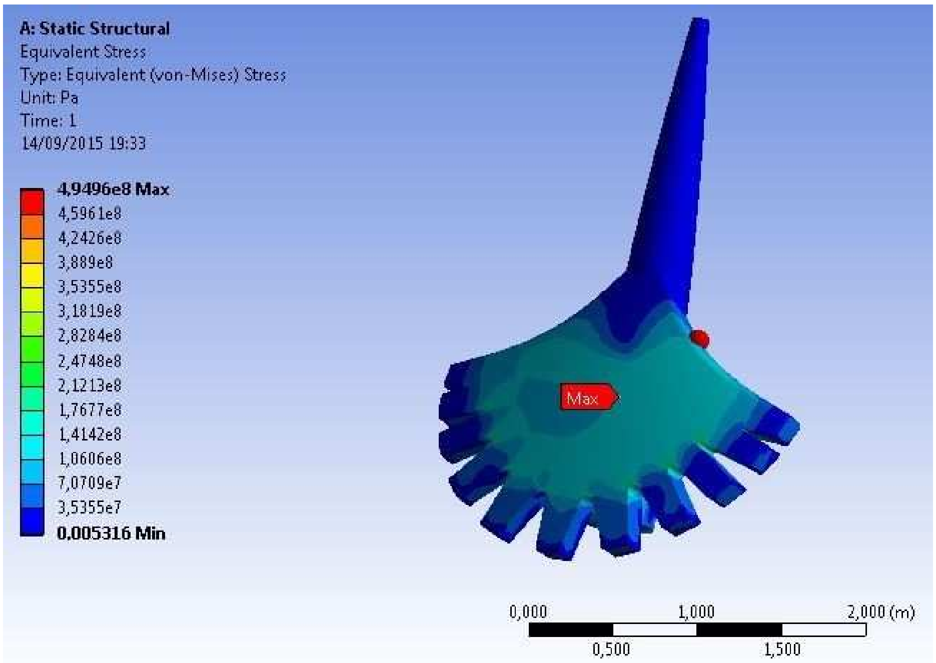
Steel type: S355

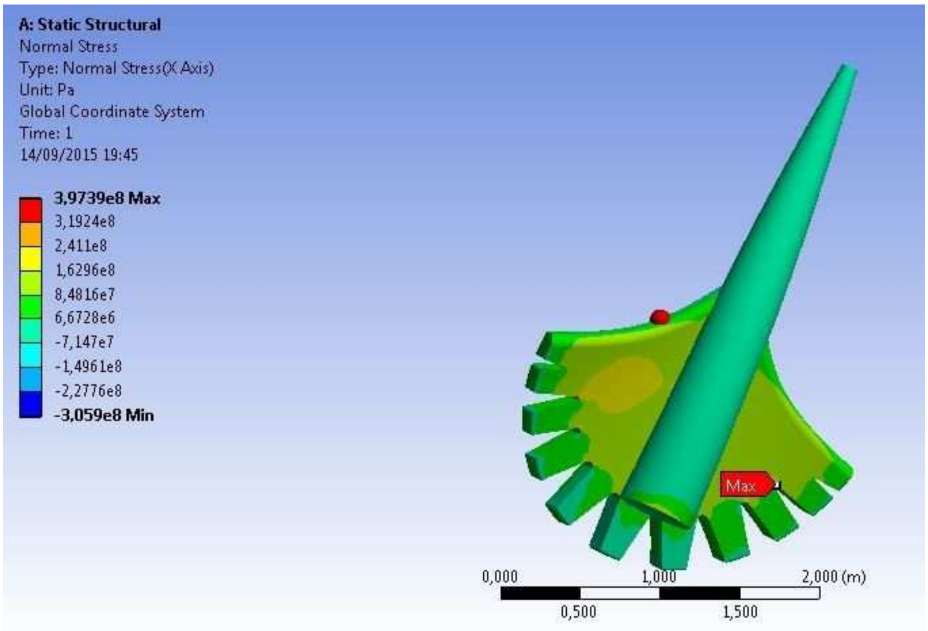
Elastic moduli: $E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2 \cdot (1+\nu)} = 80769\ MPa$

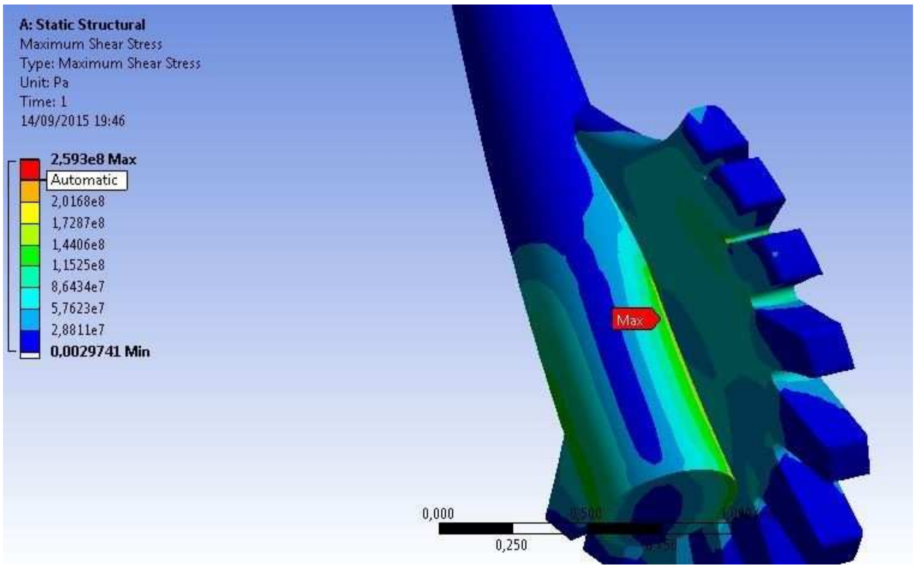
Solid piece steel: 39NiCrMo3

Yielding strength: $f_y := 635\ MPa$

Rupture strength: $f_u := 830\ MPa$







	EUROPE		ITALY	GERMANY		FRANCE	UK	USA
	UNI 10083-3: 2006 UNI 10277-5: 2008		(UNI 7845-78)	(DIN 17200-86)		(NF A 35-552-86)	(BS 970 pt.3-96)	ASTM A 29
	Grade	N°		Werkstoff	N°			
RB2	39NiCrMo3	1.6510	39NiCrMo3	-	-	-	817M40	-

CHEMICAL COMPOSITION (CAST ANALYSIS) (%)

	Europe	C	Si	Mn	P / max	S	Cr	Mo	Ni	Al	Pb
RB2	39NiCrMo3	0,35÷0,43	0,15÷0,40	0,50÷0,80	0,025	0,020÷0,035	0,60÷1,00	0,15÷0,25	0,70÷1,00	0,020÷0,050	-
RB2Pb	39NiCrMo3Pb										0,15÷0,30

MECHANICAL PROPERTIES - AS ROLLED CONDITION

Size mm	HB max to condition:		Quenched and tempered (+QT)			
	Treated to improve shearability (+S)	Soft annealing (+A)	$R_{p0,2}$ (MPa) min	R_m (MPa)	A_5 (%) min	KV (J) min
≤ 16	where the shearability is of importance, this steel should be ordered in the "soft annealed" condition	240	785	980÷1180	11	30
> 16 ≤ 40		240	735	930÷1130	11	30
> 40 ≤ 100		240	685	880÷1080	12	30
> 100≤ 160		240	635	830÷980	12	30
> 160 ≤ 250		240	540	740÷880	13	30

ULS local peak stress verification

Max normal stress: $\sigma_{max} := 397.4\ MPa$ < $\dfrac{f_y}{\gamma_{M1}} = 577.273\ MPa$ OK

Min normal stress: $\sigma_{min} := |-305.9\ MPa|$ < $\dfrac{f_y}{\gamma_{M0}} = 635\ MPa$ OK

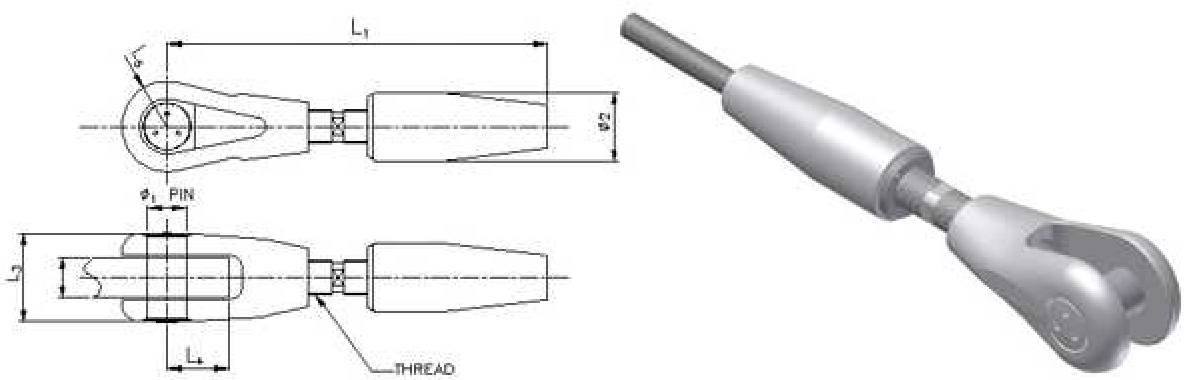
Max shear stress: $\tau_{max} := 259.3\ MPa$ < $\dfrac{f_y}{\sqrt{3}\ \gamma_{M1}} = 349.556\ MPa$ OK

Von-Mises stress: $\sigma_{VM} := 495\ MPa$ < $\dfrac{f_y}{\gamma_{M0}} = 635\ MPa$ OK

H. Cable verification

Cable-to-deck connection socket geometric data (refer to Strand diameter = 85mm)

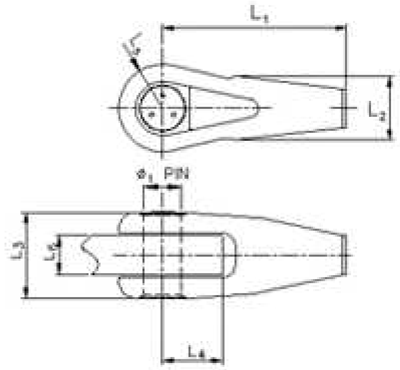
Stylite Adjustable Fork Sockets (ST-AF)



Product Code / Strand Diameter	L1		ADJT +/-	L3	L4	L5	L6		ø1	ø2	Thread Size	Weight
	Max	Min					Max	Min	Pin			Assy
mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	mm	Metric	kg
ST-AF 25	631	531	50	121	73	58	57	55	45	87	M42 x 4.5	20
ST-AF 30	701	601	50	154	86	70	71	68	55	108	M52 x 5.0	36
ST-AF 35	815	695	60	175	100	82.5	85	79	65	120	M60 x 5.5	49
ST-AF 40	865	745	60	187	120	95	90	82	75	135	M68 x 6.0	69
ST-AF 45	894	774	60	210	124	100	95	90	80	145	M76 x 6.0	87
ST-AF 50	974	834	70	221	144	115	100	93	90	155	M80 x 6.0	112
ST-AF 55	1030	890	70	233	155	115	110	100	100	170	M90 x 6.0	131
ST-AF 60	1088	948	70	250	168	125	120	110	110	185	M95 x 6.0	163
ST-AF 65	1161	1021	70	267	187	137	129	115	120	205	M105 x 6.0	210
ST-AF 70	1247	1087	80	285	202	148	137	125	130	220	M105 x 6.0	260
ST-AF 75	1316	1156	80	307	211	154	147	135	135	235	M115 x 6.0	309
ST-AF 80	1416	1236	90	322	226	166	157	145	145	245	M120 x 6.0	365
ST-AF 85	1480	1300	90	348	240	177	167	155	155	265	M125 x 6.0	447

Cable-to-mast connection socket geometric data (refer to Strand diameter = 85mm)

Stylite Fork Sockets (ST-F)



Product Code / Strand Diameter	L1	L2	L3	L4	L5	L6		ø1	Weight	
						Max	Min	Pin	Socket	Pin+Caps
mm	mm	mm	mm	mm	mm	mm	mm	mm	kg	kg
ST-F 25	275	88	121	73	58	57	55	45	8	1.7
ST-F 30	330	105	154	86	70	71	68	55	15.5	2.9
ST-F 35	385	120	175	100	82.5	85	79	65	22.5	4.7
ST-F 40	410	130	187	120	95	90	82	75	31.5	6.6
ST-F 45	420	145	210	124	100	95	90	80	43	8.4
ST-F 50	440	155	221	144	115	100	93	90	56	11
ST-F 55	477	168	233	155	115	110	100	100	60	14
ST-F 60	520	180	250	168	125	120	110	110	74	18
ST-F 65	565	195	267	187	137	129	115	120	94	24
ST-F 70	610	210	285	202	148	137	125	130	117	30
ST-F 75	645	225	307	211	154	147	135	135	141	34
ST-F 80	690	240	322	226	166	157	145	145	167	42
ST-F 85	735	255	348	240	177	167	155	155	209	51

Product Code / Strand Diameter	Minimum Breaking Load	Design Load GR,d = MBL / 1,5 / 1,1	Nominal Metallic Cross Section	Nominal Axial Stiff-ness	Nominal Metallic Mass
d	MBL	GR,d	A	EA	Mass
mm	kN	kN	mm2	MN	kg/m
LC 20	368	223	254	42	2.04
LC 25	574	348	398	66	3.20
LC 30	858	520	594	98	4.77
LC 35	1170	709	808	133	6.49
LC 40	1580	958	1090	180	8.76
LC 45	2000	1212	1390	229	11.1
LC 50	2470	1497	1710	282	13.7
LC 55	3020	1830	2090	345	16.8
LC 60	3590	2176	2490	411	20.0
LC 65	4220	2558	2920	482	23.5
LC 70	4890	2964	3390	559	27.2
LC 75	5620	3406	3890	642	31.3
LC 80	6390	3873	4420	729	35.5
LC 85	7220	4376	5000	824	40.1

Cable maximum tensile stress verification

$F_{Rd} := 4376\ kN$ > $F_{Ed} = (1.265 \cdot 10^3)\ kN$ OK

I - MAST FOUNDATION

SOIL GENERAL DATA:

Soil type: SAND

Friction angle: $\phi := 38°$

Over-consolidation ratio: $OCR := 3$

Specific weights: $\gamma := 20\ \frac{kN}{m^3}$ $\gamma_w := 10\ \frac{kN}{m^3}$ $\gamma' := \gamma - \gamma_w = 10\ \frac{kN}{m^3}$

Elastic modulii: $E := 55500\ \frac{kN}{m^2}$ $\nu := 0.2$ $G := \frac{E}{2 \cdot (1+\nu)} = 23.125\ MPa$

Passive pressure coefficient: $K_p := \frac{1+\sin(\phi)}{1-\sin(\phi)} = 4.204$

N_{spt} : $N_{spt} := 30$ (constant with depth)

All the parameters related to the soil strength are amplified according to the values in EN 1997-1 (Approach 2 (A1+M1+R3)).

***Mast foundation* - Geotechnical design**

Pile cap

GENERAL DATA:

Concrete class: $f_{ck} := 30\ MPa$ (C30/37)

Specific weight: $\gamma_{conc} := 25\ \dfrac{kN}{m^3}$

Thickness: $H := 1.5\ m$

Embedment depth: $H_b := H + 0.5\ m$

Dimensions: $B_x := 3\ m$ $B_y := 3\ m$ $A := B_x \cdot B_y = 9\ m^2$

Volume: $V_{cap} := A \cdot H = 13.5\ m^3$

Pile cap weight: $W_{cap} := V_{cap} \cdot \gamma_{conc} = 337.5\ kN$

Pile

GENERAL DATA:

Length: $L := 10\ m$

Angle: $i_{pile} := 15\ deg$

Projected length: $L_v := L \cdot \cos\left(i_{pile}\right)$

Diameter: $D := 40\ cm$

Number of piles: $n := 4$

Rebars: $n_s := 10$ $d := 3.2\ cm$

Pile cross-sectional area: $A_{tot} := \pi \cdot \dfrac{D^2}{4}$ $A_s := n_s \cdot \pi \cdot \dfrac{d^2}{4} = 0.008\ m^2$ $A_c := A_{tot} - A_s$

Elastic modulii: $E_c := 30000\ MPa$ $E_s := 210000\ MPa$ $E_{eq} := \dfrac{E_c \cdot A_c + E_s \cdot A_s}{A_{tot}}$

Distance b/w piles: $s := 1.5\ m$

Modifiers for pile material type and installation:

MODIFIER TERMS FOR PILE MATERIAL TYPE (C_M) AND INSTALLATION EFFECTS (C_K)

Pile Installation Effects Modifier C_K	Jetted pile	$C_K = 0.5$ to 0.6
	Drilled and bored piles	$C_K = 0.9$ to 1.0
	Low-displacement driven piles (e.g., H-piles; open-ended pipe)	$C_K = 1.0$ to 1.1
	High-displacement driven piles (e.g., closed-ended pipe; precast)	$C_K = 1.1$ to 1.2
Pile Material Effects Modifier C_M	Soil/rough concrete (drilled shafts)	$C_M = 1.0$
	Soil/smooth concrete (precast)	$C_M = 0.9$
	Soil/timber (wood pilings)	$C_M = 0.8$
	Soil/rough steel (normal H- and pipe pilings)	$C_M = 0.7$
	Soil/smooth steel (cone penetrometer)	$C_M = 0.6$
	Soil/stainless steel (flat dilatometer)	$C_M = 0.5$

Adapted after Kulhawy et al. 1983.

$C_K := 0.9$ *(Bored pile)*

$C_M := 1.0$ *(Rough concrete)*

Partial safety factors (Approach 2, A1+M1+R3):

$\gamma_M := 2.3$ *(mixed foundation, bearing capacity)*

$\gamma_{M2} := 1.3$ *(mixed foundation, resistance to horizontal loads)*

$\xi := 1.6$ *(average value on three vertical tests)*

ULS loads

Note: mast vertical plane belongs to the vertical plane through the pile cap's mid-section. Py component is taken as the maximum reaction along y-axis experienced when maximum rotation of mast occurs. Px and Pz components belong to the pile cap's mid-section plane.

$x_1 := 26.08\ m$ $y_1 := 11.49\ m$ $z_1 := -6.127\ m$

Initial position of top node: $x_2 := 17.68\ m$ $y_2 := 17.55\ m$ $z_2 := 15.09\ m$

Maximum displacements: $u_{x1} := 14.58\ cm$ $u_{x2} := -16.27\ cm$

$u_{y1} := 31.48\ cm$ $u_{y2} := -27.70\ cm$

Final coordinates of top node: $x_2' := x_2 + u_{x1}$ $y_2' := y_2 + u_{y1}$ $z_2' := z_2$

$x_2'' := x_2 + u_{x1}$ $y_2'' := y_2 + u_{y2}$ $z_2'' := z_2$

$x_2''' := x_2 + u_{x2}$ $y_2''' := y_2 + u_{y1}$ $z_2''' := z_2$

$x_2'''' := x_2 + u_{x2}$ $y_2'''' := y_2 + u_{y2}$ $z_2'''' := z_2$

Mast final angles about x-axis in the horizontal plane:

$$\beta' := \operatorname{atan}\left(\frac{|y_2' - y_1|}{|x_2' - x_1|}\right) = 0.658 \qquad \beta'' := \operatorname{atan}\left(\frac{|y_2'' - y_1|}{|x_2'' - x_1|}\right) = 0.611$$

$$\beta''' := \operatorname{atan}\left(\frac{|y_2''' - y_1|}{|x_2''' - x_1|}\right) = 0.64 \qquad \beta'''' := \operatorname{atan}\left(\frac{|y_2'''' - y_1|}{|x_2'''' - x_1|}\right) = 0.594$$

Maximum mast final angles about x-axis in the horizontal plane:

$$\beta_1 := \max\left(\beta', \beta'', \beta''', \beta''''\right) = 0.658 \qquad \beta_1 \cdot \frac{180}{\pi} = 37.679 \text{ deg}$$

Initial angle about x-axis of mast in the horizontal plane:

$$\beta_0 := \operatorname{atan}\left(\frac{|y_2 - y_1|}{|x_2 - x_1|}\right) = 0.625 \qquad \beta_0 \cdot \frac{180}{\pi} = 35.808 \text{ deg}$$

Maximum rotation of mast in the horizontal plane:

$$\beta := \beta_1 - \beta_0 = 0.033 \qquad \beta \cdot \frac{180}{\pi} = 1.872 \quad \text{deg}$$

Note: maximizing β leads to an estimation of the maximum horizontal force in the X direction corresponding to final top mast node location at (X2', Y2',Z2')

Mast final angles in the vertical plane:

$$\gamma' := \operatorname{atan}\left(\frac{(z_2' - z_1)}{\sqrt{|x_1 - x_2'|^2 + |y_1 - y_2'|^2}}\right) = 1.114 \qquad \gamma'' := \operatorname{atan}\left(\frac{(z_2'' - z_1)}{\sqrt{|x_1 - x_2''|^2 + |y_1 - y_2''|^2}}\right) = 1.127$$

$$\gamma''' := \operatorname{atan}\left(\frac{(z_2''' - z_1)}{\sqrt{|x_1 - x_2'''|^2 + |y_1 - y_2'''|^2}}\right) = 1.105 \qquad \gamma'''' := \operatorname{atan}\left(\frac{(z_2'''' - z_1)}{\sqrt{|x_1 - x_2''''|^2 + |y_1 - y_2''''|^2}}\right) = 1.118$$

Maximum mast final angles in the vertical plane:

$$\gamma_1 := min(\gamma', \gamma'', \gamma''', \gamma'''') = 1.105 \qquad \gamma_1 \cdot \frac{180}{\pi} = 63.291 \quad \text{deg}$$

Note: minimizing γ leads to an estimation of the maximum horizontal resultant, corresponding to final top mast node location at (X2''', Y2''',Z2''')

Final vertical angle(maximize horizontal resultant):

$$\gamma_{mast} := \operatorname{atan}\left(\frac{(z_2''' - z_1)}{\sqrt{|x_1 - x_2'''|^2 + |y_1 - y_2'''|^2}}\right) = 1.105 \qquad \gamma_{mast} \cdot \frac{180}{\pi} = 63.291 \quad \text{deg}$$

Resultant reaction (same angles as mast):

$$R := 4352 \; kN$$

$$R_H := R \cdot \cos(\gamma_{mast}) = (1.956 \cdot 10^3) \; kN$$

Components:

$$P_x := R_H \cdot \sin(\beta) = 63.88 \; kN$$

$$P_y := R_H \cdot \cos(\beta) = (1.955 \cdot 10^3) \; kN$$

$$P_z := R \cdot \sin(\gamma_{mast}) = (3.888 \cdot 10^3) \; kN$$

Moments acting at the embedment depth due to the load horizontal components:

$$M_x := P_y \cdot H = (2.932 \cdot 10^3) \; kN \cdot m$$

$$M_y := P_x \cdot H = 95.82 \; kN \cdot m$$

Total vertical load at the embedment depth:

$$Q_{tot} := P_z + 1.35 \cdot W_{cap} = (4.343 \cdot 10^3) \; kN$$

Total vertical load at the base of piles:

$$Q_d := Q_{tot} + n \cdot \gamma_{conc} \cdot \pi \cdot \frac{D^2}{4} \cdot L = (4.469 \cdot 10^3) \; kN$$

Total horizontal load:

$$H_{tot} := R_H = (1.956 \cdot 10^3) \; kN$$

Eccentricities:

$$e_x := \frac{M_y}{Q_{tot}} = 0.022\ m$$

$$e_y := \frac{M_x}{Q_{tot}} = 0.675\ m$$

Design cap dimensions:

$$B_x' := B_x - 2 \cdot e_x = 2.956\ m$$

$$B_y' := B_y - 2 \cdot e_y = 1.65\ m$$

Mixed foundation (pile cap + piles) ULS resistance verification

Pile cap verification

$$\gamma' := \gamma - \gamma_w = 10\ \frac{kN}{m^3}$$

Bearing capacity factors:

$$N_q := \frac{1 + \sin(\phi)}{1 - \sin(\phi)} \cdot e^{\pi \cdot \tan(\phi)} = 48.933$$

$$N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi) = 78.024$$

Shape factors:

$$s_q := 1 + \frac{B_x'}{B_y'} \cdot \tan(\phi) = 2.4$$

$$s_\gamma := 1 - 0.4 \cdot \frac{B_x'}{B_y'} = 0.283$$

Depth factors:

$$d_q := 1 + 2 \cdot \frac{H_b}{B_x'} \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 = 1.156 \qquad d_\gamma := 1$$

Angle factors:

$$m' := \frac{2 + \frac{B_x'}{B_y'}}{1 + \frac{B_x'}{B_y'}} \qquad i_q := \left(1 - \frac{H_{tot}}{Q_{tot}}\right)^{m'} = 0.444 \qquad i_\gamma := \left(1 - \frac{H_{tot}}{Q_{tot}}\right)^{m'+1} = 0.244$$

Stress at the embedment depth:

$$q := \gamma' \cdot H_b = 0.02\ MPa$$

Bearing capacity:

$$q_{lim} := \frac{1}{2} \gamma \cdot B_x' \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma + q \cdot N_q \cdot s_q \cdot d_q \cdot i_q = 1.364\ MPa$$

$$A_{net} := B_x' \cdot B_y' - n \cdot \pi \cdot \frac{D^2}{4} = 4.373\ m^2$$

$$Q_{r.cap} := q_{lim} \cdot A_{net} = (5.965 \cdot 10^3)\ kN$$

$$Q_{k.cap} := \frac{Q_{r.cap}}{\xi} = (3.728 \cdot 10^3)\ kN$$

Bearing capacity verification

$$Q_{kr.cap} := \frac{Q_{k.cap}}{\gamma_M} = (1.621 \cdot 10^3)\ kN \quad < \quad Q_d = (4.469 \cdot 10^3)\ kN$$

NOT OK: *Piles contribution to the global bearing capacity cannot be neglected*

Pile cap + piles verification (global behaviour)

Lateral capacity of a single pile

$$f_s := (1-\sin(\phi)) \cdot OCR^{\sin(\phi)} \cdot \gamma' \cdot \left(H_b + \frac{L_v}{2}\right) \cdot \tan(\phi) \cdot C_M \cdot C_K = 0.036\ MPa$$

$$Q_{lat} := f_s \cdot \pi \cdot D \cdot L = 456.161\ kN$$

$$Q_{lat.v} := Q_{lat} \cdot \cos\left(i_{pile}\right) = 440.618\ kN$$

$$Q_{klat.v} := \frac{Q_{lat.v}}{\xi} = 275.386\ kN$$

End bearing capacity of a single pile

Bearing capacity factor:

$$I_R := \frac{E}{2 \cdot (1+\nu) \cdot \left(\gamma' \cdot \left(H_b + L\right) \cdot \tan(\phi)\right)} = 246.655$$

$N_\sigma := 200$ (safe-sided value obtained from Table)

$$q_E' := \gamma' \cdot \left(H_b + L_v\right) \cdot N_\sigma = 23.319\ MPa$$

$$Q_E := q_E' \cdot \pi \cdot \frac{D^2}{4} = \left(2.93 \cdot 10^3\right)\ kN$$

$$Q_{E.v} := Q_E \cdot \cos\left(i_{pile}\right) = \left(2.83 \cdot 10^3\right)\ kN$$

$$Q_{kE.v} := \frac{Q_{E.v}}{\xi} = \left(1.769 \cdot 10^3\right)\ kN$$

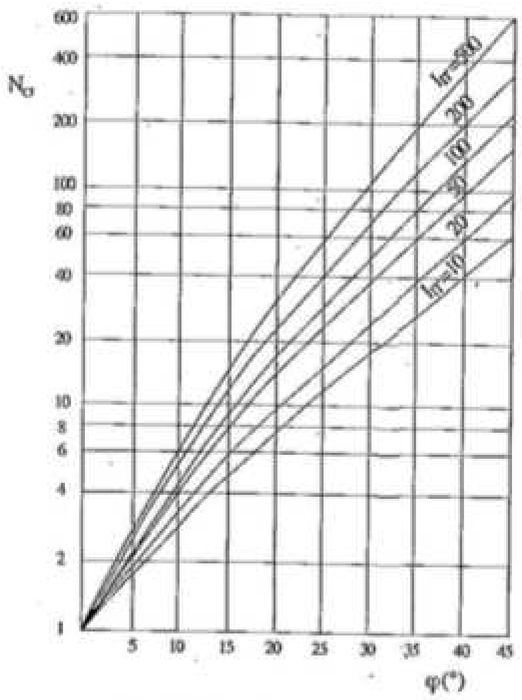


Fig. 13.4. Coefficienti Nσ (eq. 13.5)

Mixed foundation bearing capacity verification

$$Q_{kr} := Q_{kr.cap} + \frac{n \cdot \left(Q_{klat.v} + Q_{kE.v}\right)}{\gamma_M} = \left(5.176 \cdot 10^3\right)\ kN \qquad > \qquad Q_d = \left(4.469 \cdot 10^3\right)\ kN \qquad \text{OK}$$

Mixed foundation lateral bearing capacity verification

Pile plastic moment: $M_{pl} := 748\ kN \cdot m$

Short pile failure mode:

$$H_{r1} := \frac{3}{2} \cdot \left(\frac{L_v}{D}\right)^3 \cdot K_p \cdot \gamma' \cdot \frac{D^4}{L_v} = \left(2.353 \cdot 10^3\right)\ kN$$

Intermediate pile failure mode:

$$H_{r2} := \left(\frac{1}{2}\left(\frac{L_v}{D}\right)^2 + M_{pl} \cdot \frac{D}{K_p \cdot \gamma' \cdot D^4 \cdot L_v}\right) \cdot K_p \cdot \gamma' \cdot D^3 = 861.868\ kN$$

Long pile failure mode:

$$H_{r3} := K_p \cdot \gamma' \cdot D^3 \cdot \sqrt[3]{\left(3.676 \cdot \frac{M_{pl}}{K_p \cdot \gamma' \cdot D^4}\right)^2} = 502.825\ kN$$

Horizontal pile capacity:

$$H_r := min\left(H_{r1}, H_{r2}, H_{r3}\right) = 502.825\ kN$$

$$H_{rk} := \frac{H_r}{1.7} = 295.779\ kN$$

Contribution from lateral capacity:

$$Q_{lat.h} := Q_{lat} \cdot \sin\left(i_{pile}\right) = 118.063\ kN$$

$$Q_{klat.h} := \frac{Q_{lat.h}}{\xi} = 73.79\ kN$$

Contribution from end bearing capacity:

$$Q_{E.h} := Q_E \cdot \sin\left(i_{pile}\right) = 758.415\ kN$$

$$Q_{kE.h} := \frac{Q_{E.h}}{\xi} = 474.009\ kN$$

Reduction factor (group effect):

$\eta := 0.7$ *(safe-sided value, Viggiani)*

Pile cap passive lateral resistance:

$$Q_{cap.h} := K_p \cdot \gamma' \cdot \left(H_b + 0.5\ m\right) \cdot \frac{H}{2} \cdot B_x = 236.461\ kN$$

Total lateral capacity:

$$Q_{htot} := n \cdot \eta \cdot \left(Q_{klat.h} + Q_{kE.h} + H_{rk}\right) + Q_{cap.h} = \left(2.598 \cdot 10^3\right)\ kN$$

Verification

$$Q_{htot} = \left(2.598 \cdot 10^3\right)\ kN \qquad > \qquad H_{tot} = \left(1.956 \cdot 10^3\right)\ kN \qquad \text{OK}$$

Mixed foundation (pile cap + piles) SLS resistance verification

SLS loads

$$Q_{tot}$$

Settlement calculation

PILE CAP SETTLEMENT (Burland and Burbridge method)

Influence zone: $H := B_x^{\,0.7}$ $\quad Z_I := B_x^{\,0.7}$

Depth factor: $k_{HZ} := min\left(1, \dfrac{H}{Z_I}\right)$ $\quad f_h := \dfrac{k_{HZ}}{2 - k_{HZ}}$

Shape factor: $f_s := \left(\dfrac{1.25}{1 + 0.25 \cdot \dfrac{B_x}{B_y}}\right)^2$

Nominal design life: $t := 100$

Time factor: $f_t := 1.3 + 0.2 \cdot \log\left(\dfrac{t}{3}\right)$

Compressibility index: $N_C := 15 + \dfrac{N_{spt} - 15}{2}$ $\quad I_C := \dfrac{1.71}{N_C^{\,1.4}}$

Effective pressure on gross area:

$$q_{tot} := \frac{Q_{tot}}{B_x \cdot B_y}$$

Pre-consolidation stress: $\sigma'_{v0} := \gamma' \cdot H_b$ $\quad \sigma'_p := OCR \cdot \sigma'_{v0}$

$$\sigma_a := min\left(\sigma'_p, q_{tot}\right)$$

$$\sigma_b := \max\left(0, q_{tot} - \sigma_a\right)$$

Settlement

$$w := f_s \cdot f_h \cdot f_t \cdot Z_I \cdot m^{\frac{-7}{10}} \cdot I_C \cdot \left(\frac{\sigma_a}{3} + \sigma_b\right) \cdot kPa^{-1} \cdot mm = 33.518\ mm$$

MIXED FOUNDATION STIFFNESS (Fleming et al.)

Load on a single pile: $Q_{pile} := \dfrac{Q_{tot}}{n}$ *(rigid pile cap hyp.)*

$$k := \frac{E_{eq}}{E} = 748.108$$

$$\frac{L_v}{D} = 24.148$$

--------> $\quad I_w := 3$

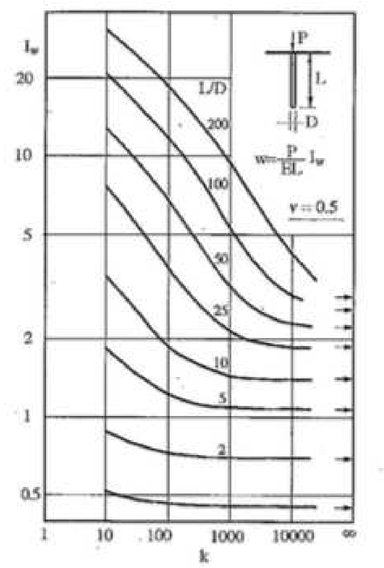


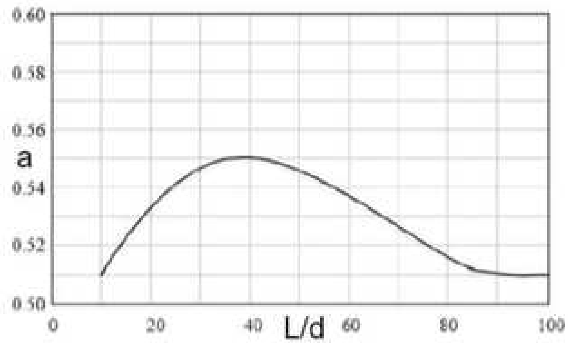
Fig. 14.5. Coefficienti di influenza I_w per il calcolo del cedimento di un palo isolato in un semispazio elastico

Settlement of a single pile

$$w_s := \frac{I_w \cdot Q_{pile}}{E \cdot L_v} = 6.076\ mm$$

Stiffness of a single pile

$$K_S := \frac{Q_{pile}}{w_s}$$



Stiffness correction factors

$\frac{L_v}{D} = 24.148$ --------> $a_{standard} := 0.53$

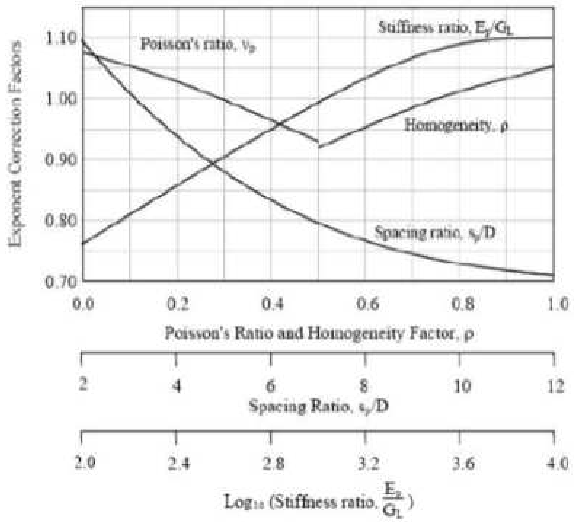
Spacing ratio factor: $\frac{s}{D} = 3.75$ ---> $a_{sd} := 0.95$

Poisson's ratio factor: $\nu = 0.2$ ---> $a_{\nu} := 1.03$

Homogeneity factor: $\rho := 1$ ---> $a_p := 1.05$

Stiffness ratio factor:

$\log\left(\frac{E_{eq}}{G}\right) = 3.254$ ---> $a_{EP} := 1.03$



Global correction factor:

$$a := a_{standard} \cdot a_{sd} \cdot a_p \cdot a_{\nu} \cdot a_{EP} = 0.561$$

MIXED FOUNDATION SETTLEMENT (Poulos and Davis)

Piles group stiffness: $K_{piles} := K_S \cdot n^{1-a} = \left(3.285 \cdot 10^5\right) \frac{kN}{m}$

Pile cap stiffness: $K_{cap} := \frac{Q_{tot}}{w} = \left(1.296 \cdot 10^5\right) \frac{kN}{m}$

Global stiffness: $$K_{mixed} := K_{piles} \cdot \frac{1 - 0.6 \cdot \frac{K_{cap}}{K_{piles}}}{1 - 0.64 \cdot \frac{K_{cap}}{K_{piles}}} = \left(3.354 \cdot 10^5\right) \frac{kN}{m}$$

Global settlement

$$w_f := \frac{Q_{tot}}{K_{mixed}} = 12.949\ mm$$

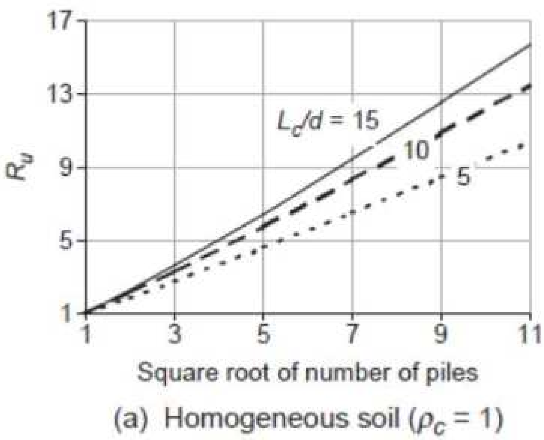
MIXED FOUNDATION LATERAL SETTLEMENT (Randolph)

Note: for the purpose of the lateral settlement assessment the piles have been conservatively considered to be vertical.

Critical length: $L_c := D \cdot \left(2 \frac{E_{eq}}{G \cdot (1 + 0.75\,\nu)}\right)^{\frac{2}{7}} = 3.986\ m$

Horizontal load on a single pile: $P_{xpile} := \frac{H_{tot}}{n}$

Horizontal displacement (rotation inhibited): $y := \frac{\left(\frac{E_{eq}}{G}\right)^{\frac{1}{7}}}{\rho \cdot G} \cdot \left(0.27 - \frac{0.11}{\sqrt{\rho}}\right) \cdot \frac{P_{xpile}}{\frac{L_c}{2}} = 0.005\ m$



$\frac{L_c}{D} = 9.964$

$\sqrt{n} = 2$ ------> $R_u := 2$

<u>Horizontal settlement</u> $y_g := y \cdot R_u = 9.904\ mm$

<u>Mast foundation</u> - Structural design

Mast base circular column

GENERAL DATA:

Concrete class: $f_{ck} := 30\ MPa$ (C30/37)

Partial safety factors: $\gamma_c := 1.5$ $\alpha_{cc} := 0.85$

Design compressive strength: $f_{cd} := \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 17\ MPa$

Reinforcement steel type: B450C

Steel yielding strength: $f_{yd} := 391.3\ MPa$

Specific weight: $\gamma_{conc} := 25\ \frac{kN}{m^3}$

Maximum aggregate dimension: $d_g := 32\ mm$

Column inclined height: $h_c := 166\ cm$

Column diameter: $d_c := 120\ cm$

Steel cover: $c := 6\ cm$

ULS resistance verification

Initial vertical angle: $\gamma_{mast0} := \operatorname{atan}\left(\dfrac{(z_2 - z_1)}{\sqrt{\left|x_1 - x_2\right|^2 + \left|y_1 - y_2\right|^2}}\right) = 1.117$ $\gamma_{mast0} \cdot \dfrac{180}{\pi} = 63.979$ deg

Final vertical angle(maximize horizontal components): $\gamma_{mast} = 1.105$ $\gamma_{mast} \cdot \dfrac{180}{\pi} = 63.291$ deg

Angle b/w Pz and initial position of mast: $\gamma_{Pz} := \pi - \dfrac{\pi}{2} - \gamma_{mast0} = 0.454$

Horizontal load components: $V := \left|P_y \cdot \cos\left(\dfrac{\pi}{2} - \gamma_{mast0}\right) - P_z \cdot \sin\left(\gamma_{Pz}\right)\right| = 51.313\ kN$

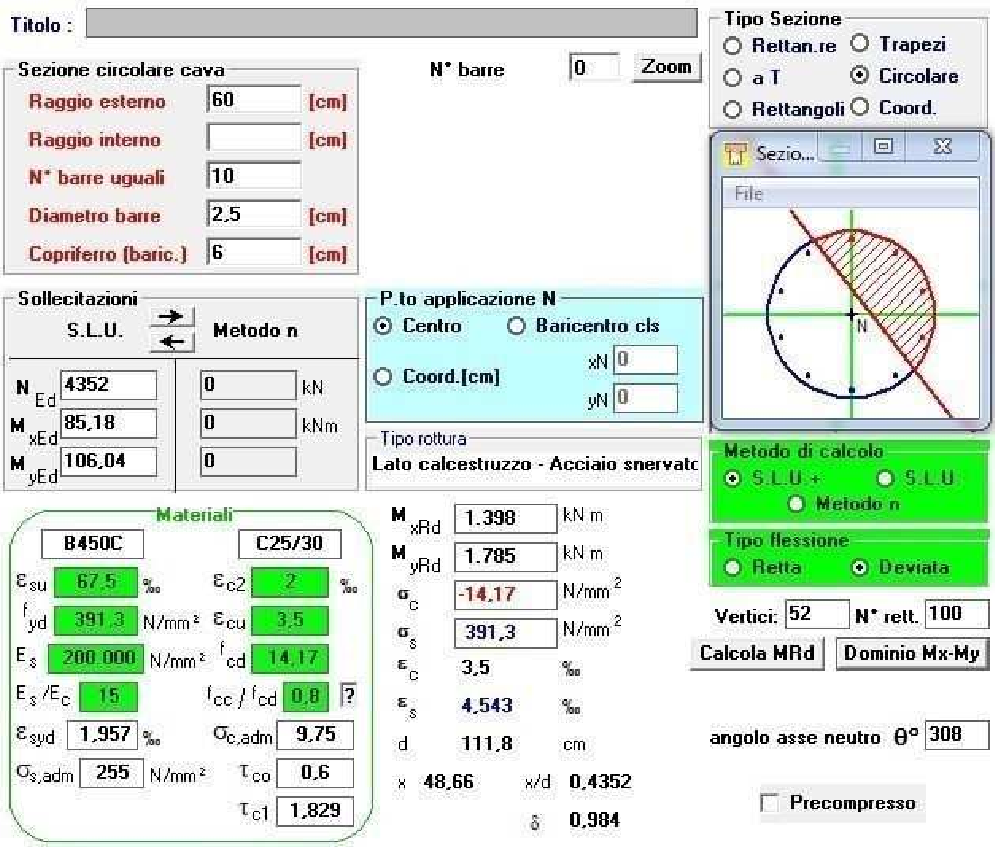
Axial load: $N := R = \left(4.352 \cdot 10^3\right)\ kN$

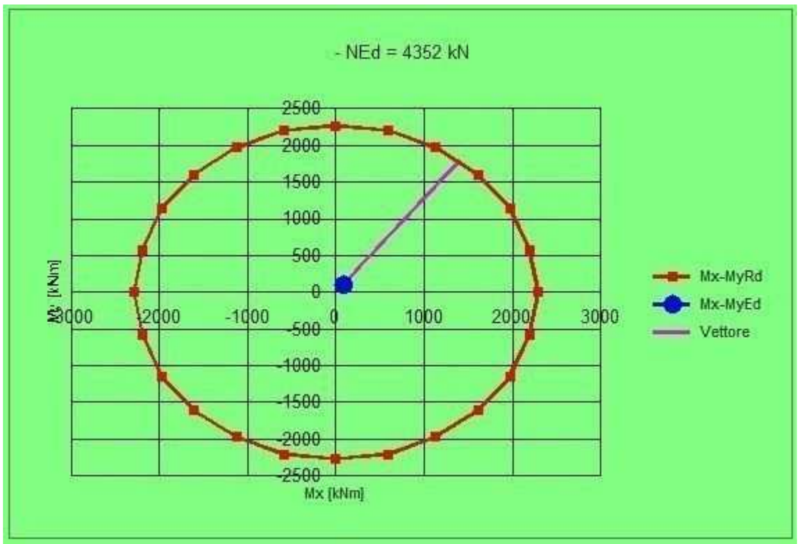
Moment about x axis: $M_x := V \cdot h_c = 85.18\ kN \cdot m$

Moment about y axis: $M_y := P_x \cdot h_c = 106.041\ kN \cdot m$

Minimum longitudinal reinforcement:

$$A_{s.min} := \max\left(0.1 \cdot \frac{N}{f_{yd}}, 0.003 \cdot \pi \cdot \frac{d_c^{\ 2}}{4}\right) = 33.929\ cm^2$$





$A_s := 10 \cdot \pi \cdot \frac{(2.5\ cm)^2}{4} = 49.087\ cm^2$ > $A_{s.min} = 33.929\ cm^2$ OK

Reinforcement geometric ratio:

$\rho_{min} := 0.003$ < $\rho := \frac{A_s}{\pi \cdot \frac{d_c^2}{4}} = 0.004$ < $\rho_{max} := 0.04$ OK

Axial force and biaxial bending verification OK

Longitudinal reinforcement diameter: $d_{sl} := 2.5\ cm$

Shear verification (*EC2 + Orr, Darby, Ibell paper*)

Efficiency factor for circular section: $\lambda_1 := 0.85$

Radius for shear steel: $r_{sv} := \frac{d_c}{2} - c = 54\ cm$

Spiral pitch: $p := 0\ cm$ (closed links)

$$\lambda_2 := \left(\left(\frac{p}{2 \cdot \pi \cdot r_{sv}}\right)^2 + 1\right)^{-0.5} = 1$$

Minimum stirrup diameter: $\max\left(6\ mm, \frac{1}{4}\ d_{sl}\right) = 0.625\ cm$

Stirrup diameter: $d_s := 1.4\ cm$ (>1/4 longitudinal rebar diam.)

Stirrup area: $A_{sw} := 2 \cdot \pi \cdot \frac{d_s^2}{4}$

Shear reinforcement yielding strength: $f_{ywd} := 391.3\ MPa$

Equivalent rectangular section (Clarke & Birjandi) $h := 1119\ mm$ $b := 1011\ mm$ (Bartolomeo Ravera.it)

Stirrup maximum spacing: $s_{max} := min\left(12 \cdot d_{sl}, 300\ mm, b\right) = 30\ cm$

Stirrup spacing: $s := 20\ cm$

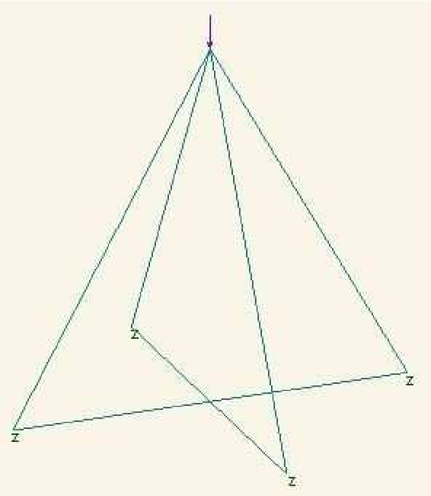
Truss angle: $cot\left(\theta\right) := 1$

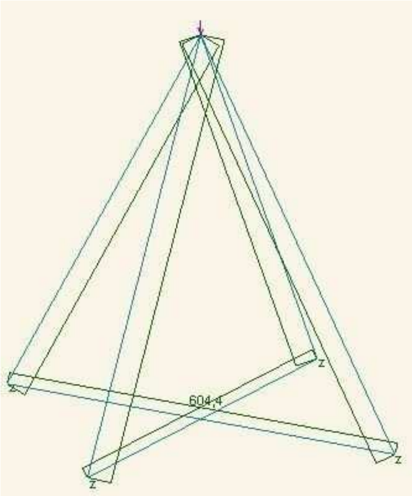
Lever arm: $z := 0.9 \cdot \left(\frac{d_c}{2} + 2 \cdot \frac{d_c}{2 \cdot \pi}\right)$

Shear resistance:

$V_{Rd.s} := \lambda_1 \cdot \lambda_2 \cdot \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot cot\left(\theta\right) = 452.498\ kN$ > $V_{Ed} := \sqrt{V^2 + P_x^{\,2}} = 81.937\ kN$ OK

Shear verification (*S&T simplified model to assess confinement effect stresses in the shear reinforcement*)





$T_{Ed} := 605\ kN$

Note: we assume that only the sections of the ties that are normal to the tension direction contribute to the concrete cylinder confinement (safe-sided choice)

Number of ties involved in the confinement effect at base of cylinder (6 ties at 10cm o.c.): $n_t := 6$

$T_{Rd} := f_{yd} \cdot A_{sw} \cdot n_t = 722.831\ kN$ > T_{Ed} OK

Note: design of transverse reinforcement is controlled by shear at the top of the cylinder while at its base tension due to confinement effect controls.

Piles

FEM analysis results

Note: for elastic design analyses, the reinforcement has been neglected in the finite element modeling since the stiffness contribution of the concrete is much greater than the reinforcement. The reinforcement can safely be assumed to keep the composite structure intact, and thus act (for analysis purposes) as a homogenous elastic body. Forces and moments have been then extracted from the FEA solution and used as a basis to size the reinforcement needed to carry the net tensile forces in the section.

Height of pile cap: $H := 150\ cm$

Pile diameter: $D := 40\ cm$

Steel cover: $c := 6\ cm$

Vertical stiffness: $K_v := \dfrac{Q_d}{A \cdot w_f} = \left(3.835 \cdot 10^{7}\right) \dfrac{N}{m^{3}}$

Horizontal stiffness: $K_h := \dfrac{R_H}{\left(n \cdot \pi \cdot D \cdot L + B_x \cdot H\right) \cdot y_g} = \left(3.606 \cdot 10^{6}\right) \dfrac{N}{m^{3}}$

Discrete vertical stiffness (pile cap): $k_{v.cap} := K_v \cdot \dfrac{A}{72} = \left(4.793 \cdot 10^{3}\right) \dfrac{kN}{m}$

Discrete vertical stiffness (piles): $k_{v.piles} := K_v \cdot n \cdot \pi \cdot \dfrac{\frac{D^2}{4}}{4} = \left(4.819 \cdot 10^{3}\right) \dfrac{kN}{m}$

Discrete horizontal stiffness (passive pressure): $k_{h.cap} := K_h \cdot B_x \cdot \dfrac{H}{9} = \left(1.803 \cdot 10^{3}\right) \dfrac{kN}{m}$

Discrete horizontal stiffness (piles): $k_{h.piles} := K_h \cdot n \cdot \pi \cdot D \cdot \dfrac{L}{n \cdot (10)} = \left(4.532 \cdot 10^{6}\right) \dfrac{N}{m}$

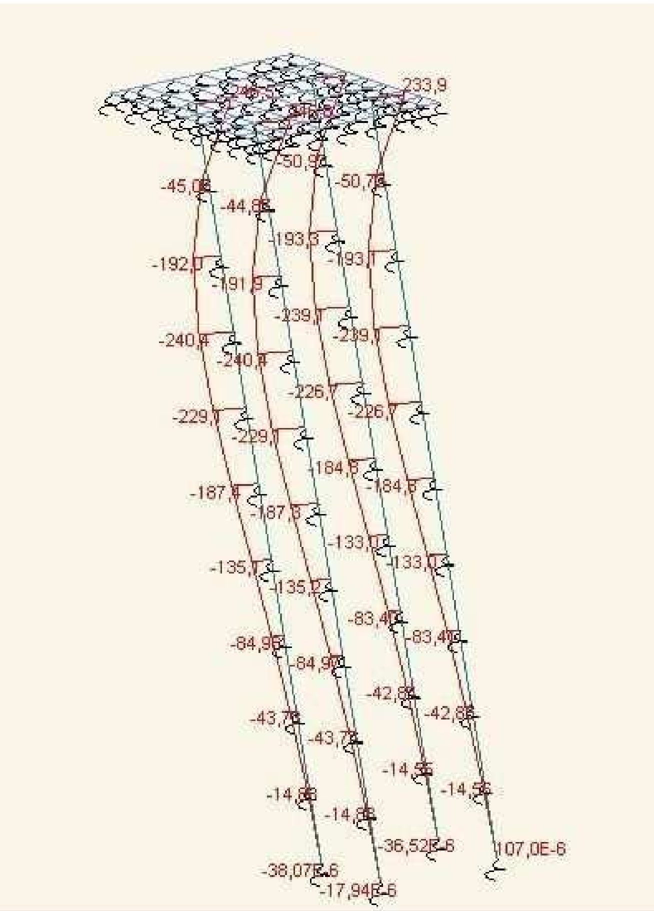
ULS Loads on pile

$N := -79.01\ kN$

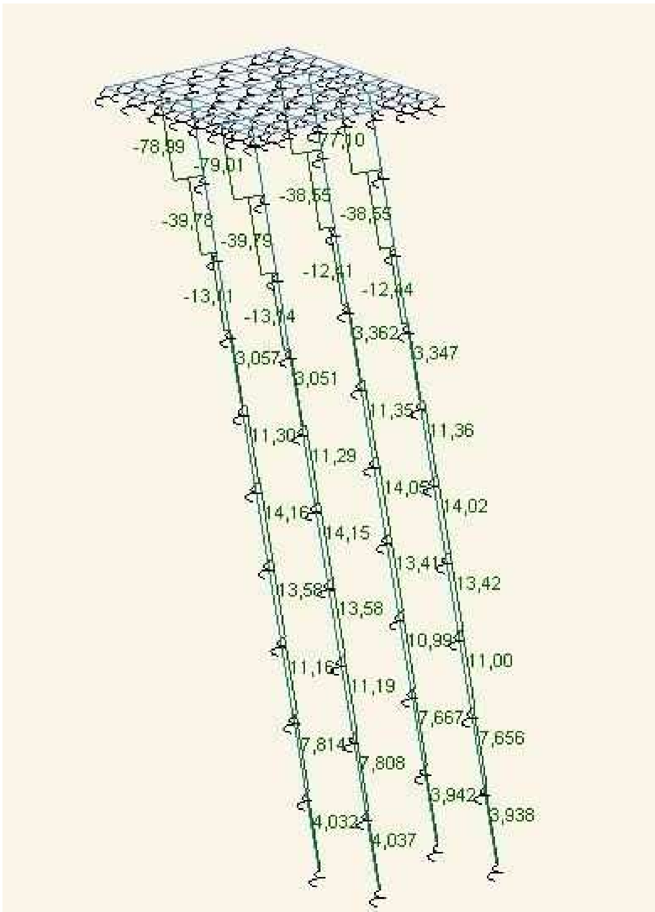
$M_x := 246.9\ kN \cdot m$

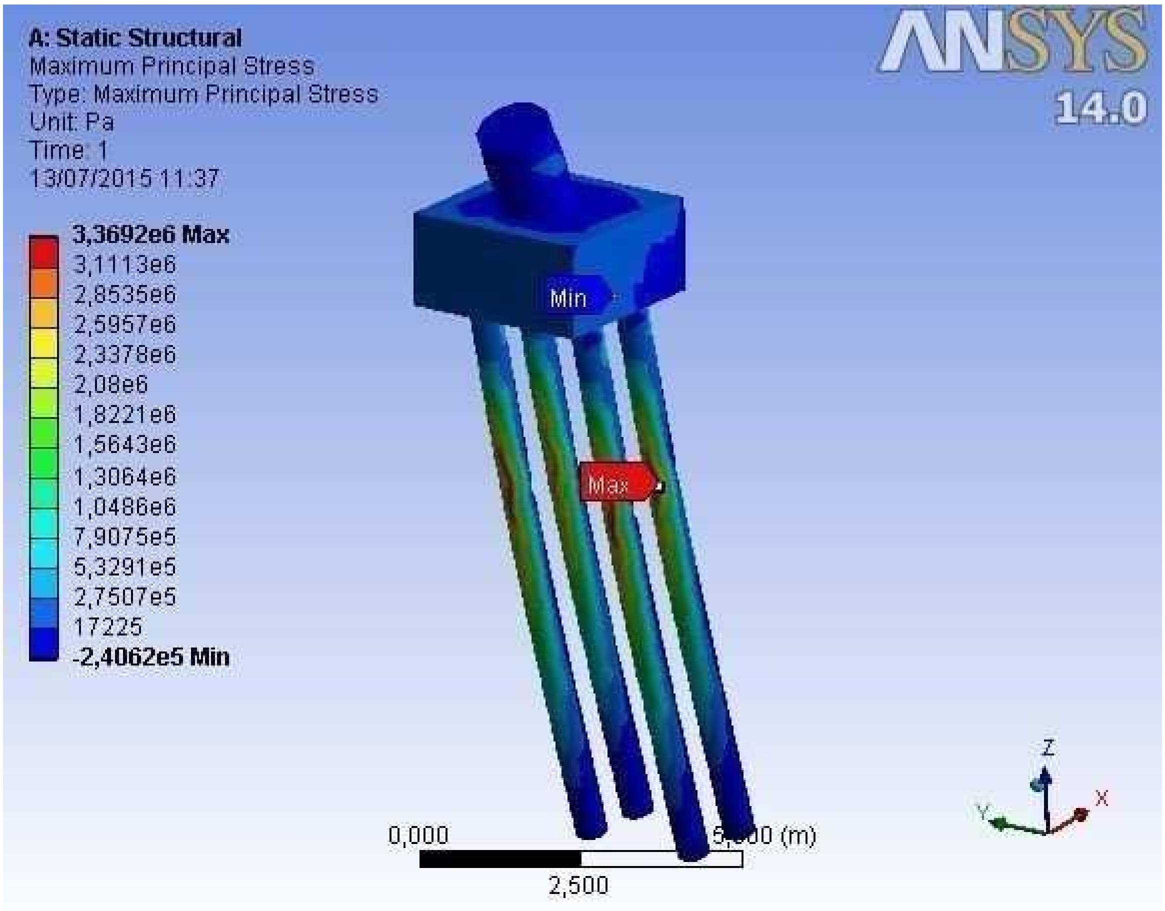
$M_y := 18.49\ kN \cdot m$

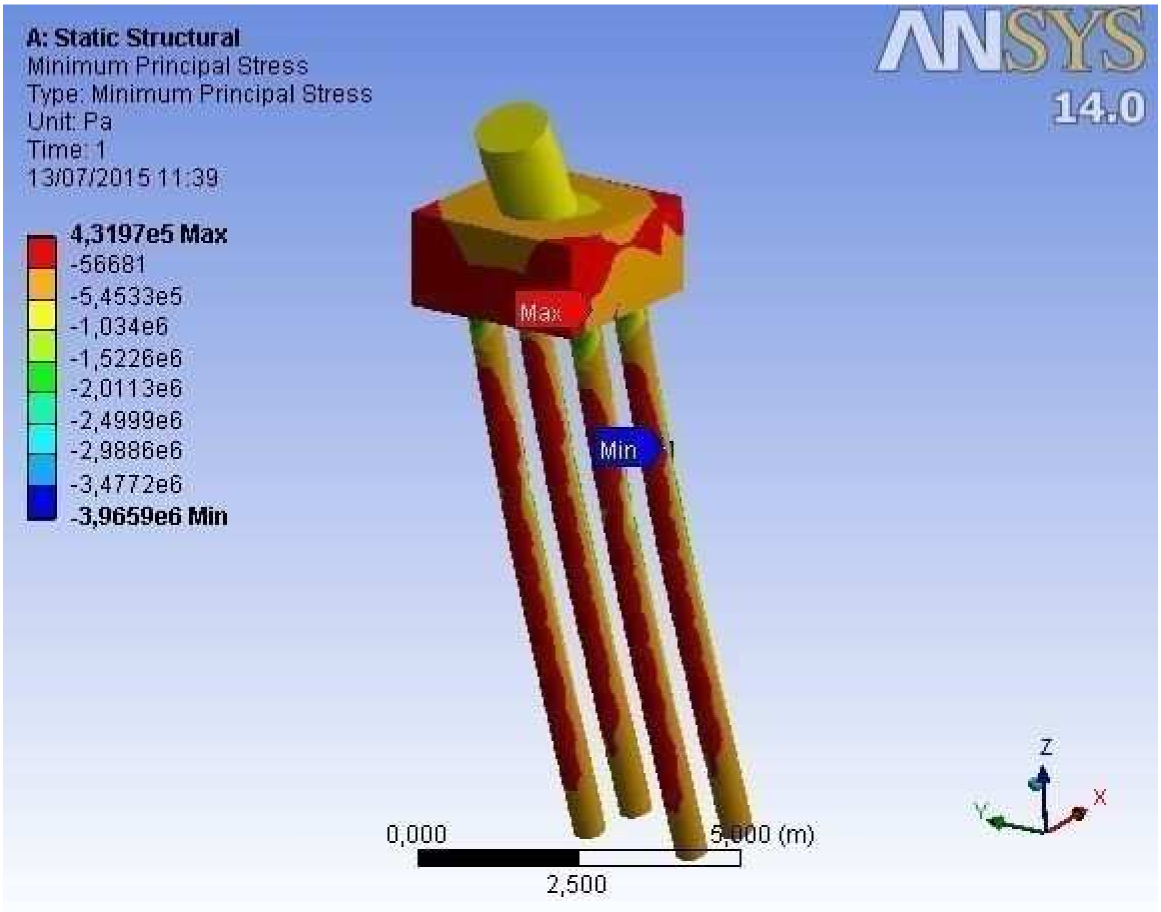
Bending moment

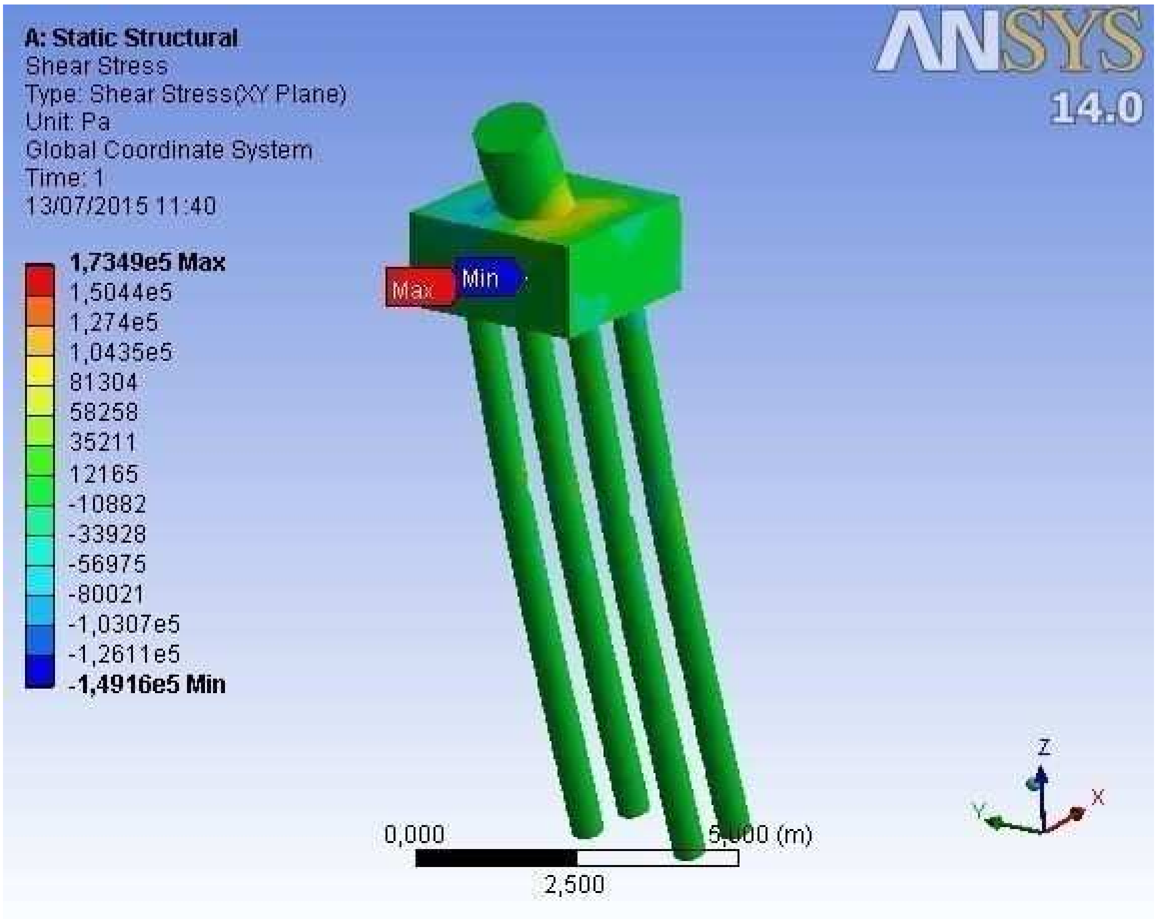


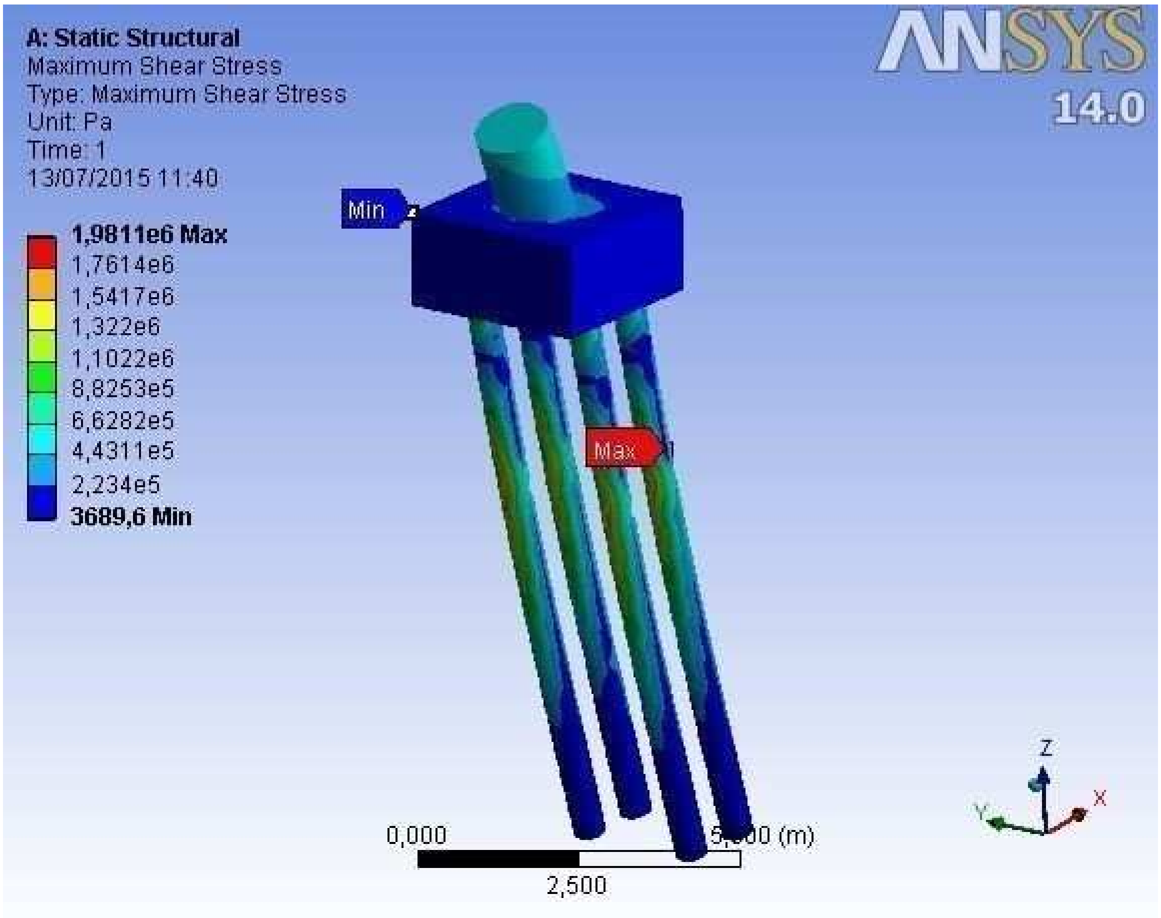
Axial force



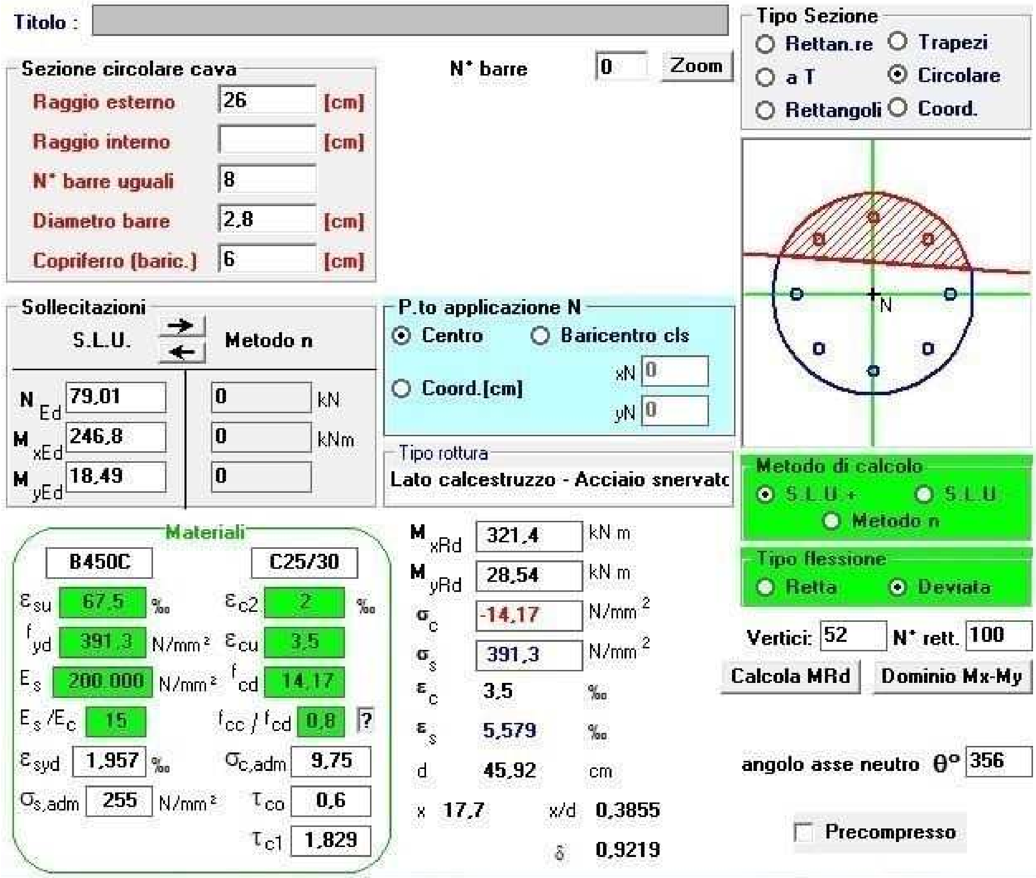

A: Static Structural
Maximum Principal Stress
Type: Maximum Principal Stress
Unit: Pa
Time: 1
13/07/2015 11:37
3,3692e6 Max
3,1113e6
2,8535e6
2,5957e6
2,3378e6
2,08e6
1,8221e6
1,5643e6
1,3064e6
1,0486e6
7,9075e5
5,3291e5
2,7507e5
17225
-2,4062e5 Min
ANSYS
14.0
Min
Max
0,000
5,000 (m)
2,500
Z
Y
X

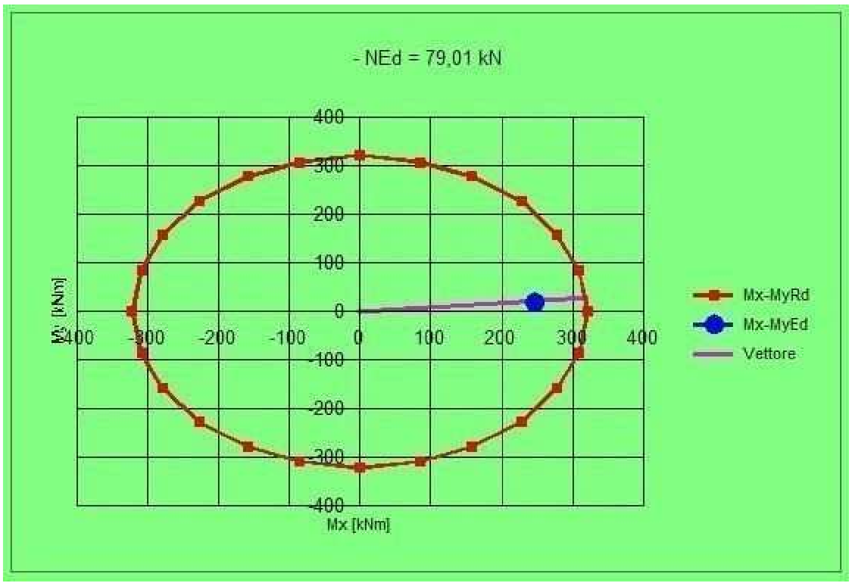

A: Static Structural
Minimum Principal Stress
Type: Minimum Principal Stress
Unit: Pa
Time: 1
13/07/2015 11:39
4,3197e5 Max
-56681
-5,4533e5
-1,034e6
-1,5226e6
-2,0113e6
-2,4999e6
-2,9886e6
-3,4772e6
-3,9659e6 Min
ANSYS
14.0
Max
Min
0,000
5,000 (m)
2,500
Z
Y
X


A: Static Structural
Shear Stress
Type: Shear Stress(XY Plane)
Unit: Pa
Global Coordinate System
Time: 1
13/07/2015 11:40
1,7349e5 Max
1,5044e5
1,274e5
1,0435e5
81304
58258
35211
12165
-10882
-33928
-56975
-80021
-1,0307e5
-1,2611e5
-1,4916e5 Min
Max
Min
ANSYS
14.0
0,000
2,500
(m)
Z
Y
X


A: Static Structural
Maximum Shear Stress
Type: Maximum Shear Stress
Unit: Pa
Time: 1
13/07/2015 11:40
1,9811e6 Max
1,7614e6
1,5417e6
1,322e6
1,1022e6
8,8253e5
6,6282e5
4,4311e5
2,234e5
3689,6 Min
Min
Max
ANSYS
14.0
0,000
2,500
(m)
Z
Y
X

Piles cross section





Axial force and biaxial bending verification OK

Longitudinal reinforcement diameter: $d_{sl} := 2.5\ cm$

Shear verification (EC2 + Orr, Darby, Ibell paper)

Efficiency factor for circular section: $\lambda_1 := 0.85$

Radius for shear steel: $r_{sv} := \frac{D}{2} - c = 14\ cm$

Spiral pitch: $p := 10\ cm$ (spiral)

$$\lambda_2 := \left(\left(\frac{p}{2 \cdot \pi \cdot r_{sv}}\right)^2 + 1\right)^{-0.5} = 0.994$$

Minimum stirrup diameter: $\max\left(6\ mm, \frac{1}{4} d_{sl}\right) = 0.625\ cm$

Stirrup diameter: $d_s := 1.4\ cm$

Stirrup area: $A_{sw} := 2 \cdot \pi \cdot \frac{d_s^{\ 2}}{4}$

Shear reinforcement yielding strength: $f_{ywd} := 391.3\ MPa$

Equivalent rectangular section (Clarke & Birjandi) $h := 374\ mm$ $b := 336\ mm$ (Bartolomeo Ravera.it)

Stirrup maximum spacing: $s_{max} := min\left(12 \cdot d_{sl}, 300\ mm, b\right) = 30\ cm$

Stirrup spacing: $s := p$

Truss angle: $cot(\theta) := 1$

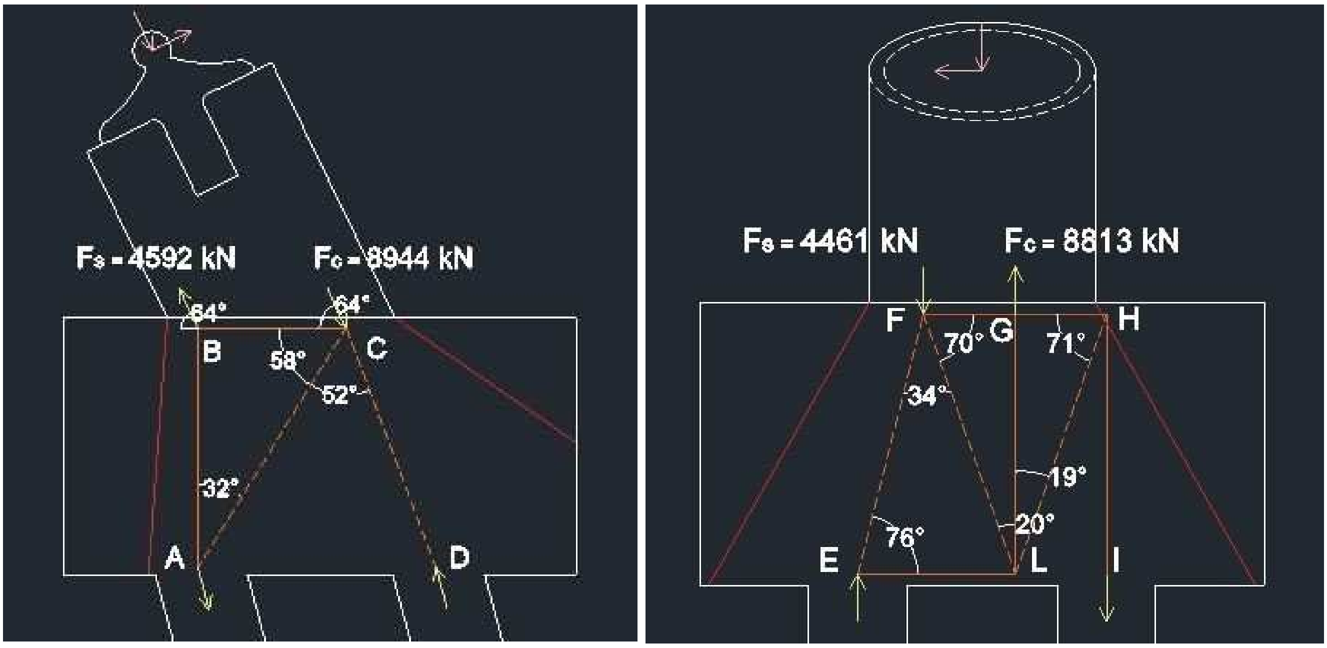
Lever arm: $z := 0.9 \cdot \left(\frac{D}{2} + 2 \cdot \frac{D}{2 \cdot \pi}\right)$

Shear resistance:

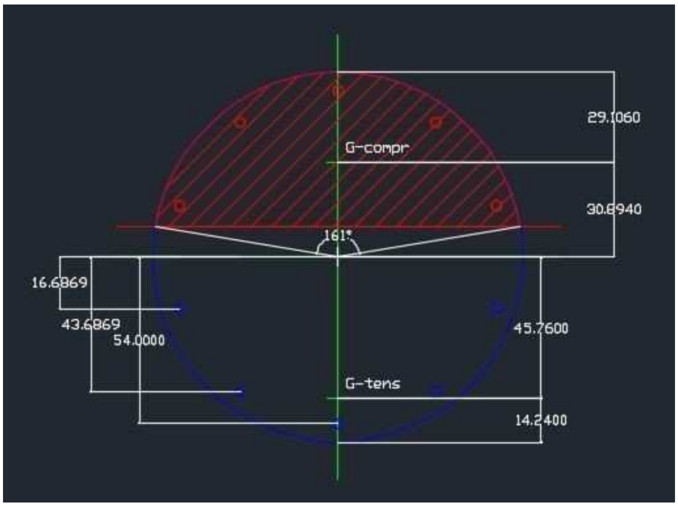
$V_{Rd.s} := \lambda_1 \cdot \lambda_2 \cdot \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot cot(\theta) = 299.734\ kN$ > $V_{Ed} := 291\ kN$ (GSA model) OK

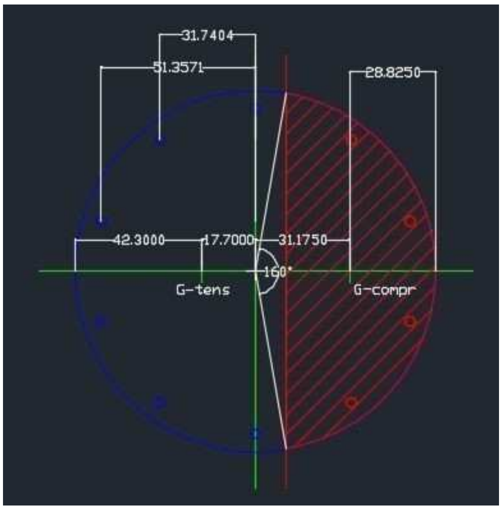
Pile cap

S&T model



Location of compression and tension resultants *(X-Z and Y-Z strut and tie frames have been considered to behave independently, thus simple compression + uniaxial bending stresses have been used)* :





Centers of gravity of compression zone (bending about the X and Y axis respectively):

$$y_{G.Xc} := 2 \cdot 60\ cm \cdot \frac{\sin\left(\frac{161°}{2}\right)^3}{3 \cdot \left(2.81\ \frac{rad}{2} - \sin\left(\frac{161°}{2}\right) \cdot \cos\left(\frac{161°}{2}\right)\right)} = 30.894\ cm$$

$$y_{G.Yc} := 2 \cdot 60\ cm \cdot \frac{\sin\left(\frac{160°}{2}\right)^3}{3 \cdot \left(2.793\ \frac{rad}{2} - \sin\left(\frac{160°}{2}\right) \cdot \cos\left(\frac{160°}{2}\right)\right)} = 31.175\ cm$$

Centers of gravity of tension zone (bending about the X and Y axis respectively):

$$y_{G.Xt} := 2 \cdot A_s \cdot \frac{(16.7\ cm + 43.7\ cm + 54\ cm)}{5 \cdot A_s} = 45.76\ cm$$

$$y_{G.Yt} := 2 \cdot A_s \cdot \frac{(31.74\ cm + 51.36\ cm)}{6 \cdot A_s} = 27.7\ cm$$

<u>X-Z Plane1 S&T model</u>

Circular segment area: $A := \frac{1}{2} \cdot (60\ cm)^2 \cdot (2.81 - \sin(161°)) = 0.447\ m^2$

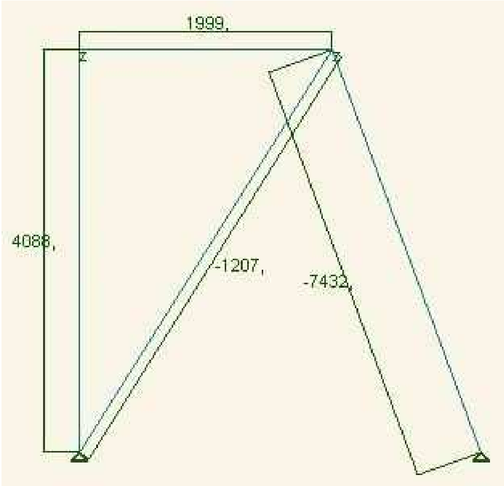
Compression resultant: $F_c := A \cdot f_{cd} = \left(7.602 \cdot 10^3\right)\ kN$

Axial load: $N := 4352\ kN$

Tension resultant: $F_s := F_c - N = \left(3.25 \cdot 10^3\right)\ kN$

Results

Axial loads on strut and tie elements:



AB Reinforcement design: $N_{AB} := 4088\ kN$

$A_{sAB} := \frac{N_{AB}}{f_{yd}} = 104.472\ cm^2$ ---> 10 ϕ26 $\phi := 2.6\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

BC Reinforcement design: $N_{BC} := 2000\ kN$

$A_{sBC} := \frac{N_{BC}}{f_{yd}} = 51.112\ cm^2$ ---> 12 ϕ24 $\phi := 2.4\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $5\ cm$

Y-Z Plane2 S&T model

Circular segment area: $A := \frac{1}{2} \cdot (60\ cm)^2 \cdot (2.79 - \sin(160°)) = 0.441\ m^2$

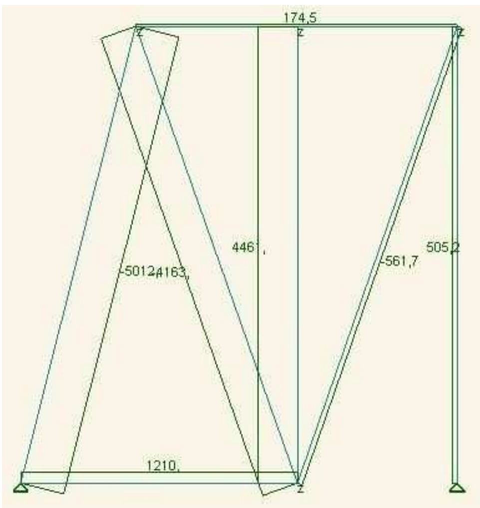
Compression resultant: $F_c := A \cdot f_{cd} = (7.491 \cdot 10^3)\ kN$

Axial load: $N := 4352\ kN$

Tension resultant: $F_s := F_c - N = (3.139 \cdot 10^3)\ kN$

Results

Axial loads on strut and tie elements:



FH Reinforcement design: $N_{FH} := 175\ kN$

$$A_{sFH} := \frac{N_{FH}}{f_{yd}} = 4.472\ cm^2 \quad \text{--->} \quad 3\,\phi 18 \quad \phi := 1.8\ cm$$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $5\ cm$

HI Reinforcement design: $N_{HI} := 506\ kN$

$$A_{sHI} := \frac{N_{HI}}{f_{yd}} = 12.931\ cm^2 \quad \text{--->} \quad 5\,\phi 20 \quad \phi := 2\ cm$$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $5\ cm$

GL Reinforcement design: $N_{GL} := 4461\ kN$

$$A_{sGL} := \frac{N_{GL}}{f_{yd}} = 114.005\ cm^2 \quad \text{--->} \quad 19\,\phi 28 \quad \phi := 2.8\ cm$$

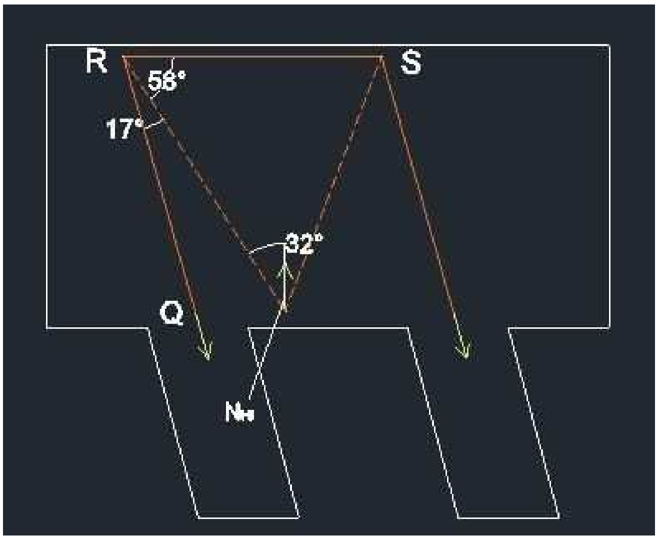
Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $6\ cm$

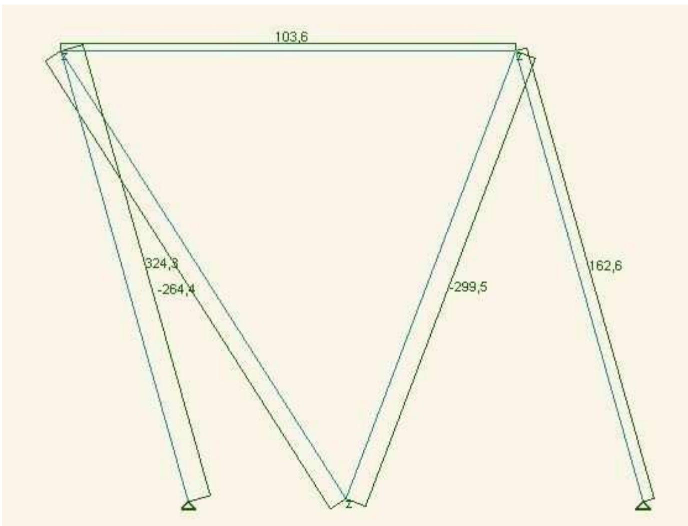
EL Reinforcement design: $N_{EL} := 1210\ kN$

$$A_{sEL} := \frac{N_{EL}}{f_{yd}} = 30.923\ cm^2 \quad \text{--->} \quad 6\,\phi 26 \quad \phi := 2.6\ cm$$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $5\ cm$

X-Z Plane5 S&T model





RQ Reinforcement design: $N_{RQ} := 324\ kN$

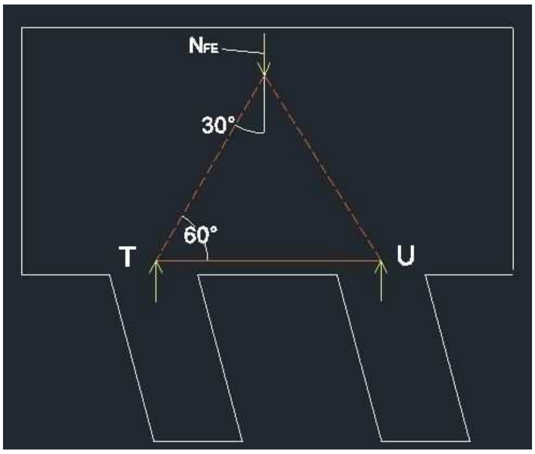
$A_{sRQ} := \frac{N_{RQ}}{f_{yd}} = 8.28\ cm^2$ ---> *(Use pile reinf.)*

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> Pile rebars distance

<u>RS Reinforcement design:</u> $N_{RS} := 103.6\ kN$

$A_{sEL} := \frac{N_{RS}}{f_{yd}} = 2.648\ cm^2$ ---> *(Use skin rebars)*

<u>X-Z Plane6 S&T model</u>



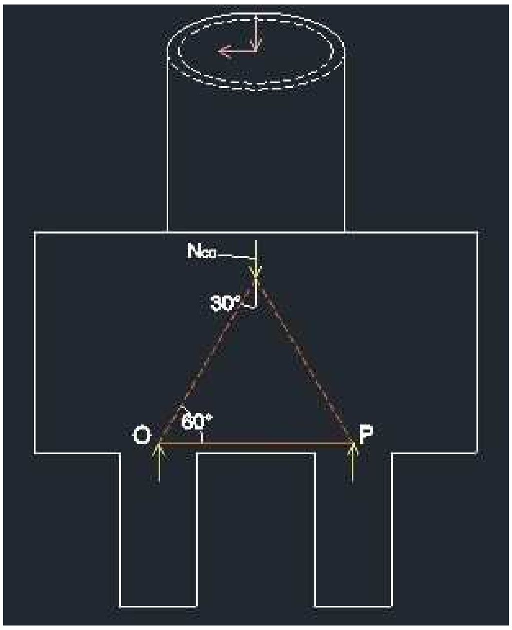
$N_{FE} := |-5012\ kN|$

<u>TU Reinforcement design:</u> $N_{TU} := \frac{N_{FE}}{2 \cdot \tan(60°)} = (1.447 \cdot 10^3)\ kN$

$A_{sTU} := \frac{N_{TU}}{f_{yd}} = 36.975\ cm^2$ ---> $8\,\phi 25$ $\phi := 2.5\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $8\ cm$

<u>Y-Z Plane3 S&T model</u>



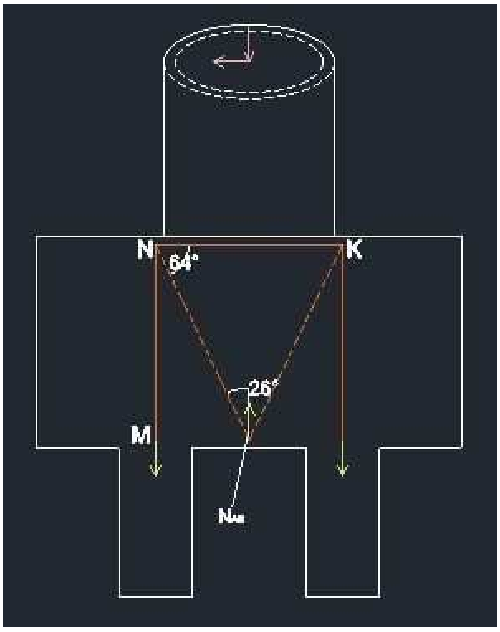
$N_{CD} := |-7043\ kN|$

OP Reinforcement design: $N_{OP} := \dfrac{N_{CD}}{2 \cdot \tan(60°)} = \left(2.033 \cdot 10^{3}\right)\ kN$

$A_{sOP} := \dfrac{N_{OP}}{f_{yd}} = 51.959\ cm^{2}$ ---> $8\,\phi 30$ $\phi := 3\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $8\ cm$

Y-Z Plane4 S&T model



MN Reinforcement design: $N_{MN} := \dfrac{N_{AB}}{2} = \left(2.044 \cdot 10^{3}\right)\ kN$

$A_{sMN} := \dfrac{N_{MN}}{f_{yd}} = 52.236\ cm^{2}$ ---> $8\,\phi 30$ $\phi := 3\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $8\ cm$

NK Reinforcement design: $N_{NK} := \dfrac{N_{AB}}{2 \cdot \cos(26°)} \cdot \cos(64°) = 996.925\ kN$

$A_{sNK} := \dfrac{N_{NK}}{f_{yd}} = 25.477\ cm^{2}$ ---> $7\,\phi 22$ $\phi := 2.2\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $8\ cm$

$r := 92.5\ mm$ $t := 50\ mm$

$1265\ \dfrac{kN}{\pi \cdot r \cdot t} = \left(8.706 \cdot 10^{7}\right)\ Pa$

L - ABUTMENT

SOIL GENERAL DATA:

Soil type: SAND

Friction angle: $\phi := 38°$

Over-consolidation ratio: $OCR := 3$

Specific weights: $\gamma := 20 \, \frac{kN}{m^3}$ $\gamma_w := 10 \, \frac{kN}{m^3}$ $\gamma' := \gamma - \gamma_w = 10 \, \frac{kN}{m^3}$

Elastic modulii: $E := 55500 \, \frac{kN}{m^2}$ $\nu := 0.2$ $G := \frac{E}{2 \cdot (1 + \nu)} = 23.125 \, MPa$

Passive pressure coefficient: $K_p := \frac{1 + \sin(\phi)}{1 - \sin(\phi)} = 4.204$

N_{spt} : $N_{spt} := 30$ (constant with depth)

All the parameters related to the soil strength are amplified according to the values in EN 1997-1 (Approach 2 (A1+M1+R3)).

<u>Abutment foundation</u> - Geotechnical design

Pile cap

GENERAL DATA:

Concrete class: $f_{ck} := 30 \, MPa$ (C30/37)

Specific weight: $\gamma_{conc} := 25 \, \frac{kN}{m^3}$

Thickness: $H := 1 \, m$

Embedment depth: $H_b := H$

Dimensions: $B_x := 2.88 \, m$ $B_y := 6.4 \, m$ $A := B_x \cdot B_y = 18.432 \, m^2$

Volume: $V_{cap} := A \cdot H = 18.432 \, m^3$

Pile cap weight: $W_{cap} := V_{cap} \cdot \gamma_{conc} = 460.8 \, kN$

Pile

GENERAL DATA:

Length: $L := 16 \, m$

Angle: $i_{pile} := 0 \, deg$

Projected length: $L_v := L \cdot \cos(i_{pile})$

Diameter: $D := 60 \, cm$

Number of piles: $n := 6$

Rebars: $n_s := 10$ $\quad d := 2.4\ cm$

Pile cross-sectional area: $A_{tot} := \pi \cdot \dfrac{D^2}{4}$ $\quad A_s := n_s \cdot \pi \cdot \dfrac{d^2}{4} = 0.005\ m^2$ $\quad A_c := A_{tot} - A_s$

Elastic modulii: $E_c := 30000\ MPa$ $\quad E_s := 210000\ MPa$ $\quad E_{eq} := \dfrac{E_c \cdot A_c + E_s \cdot A_s}{A_{tot}}$

Distance b/w piles: $s_x := 1.28\ m$ $\quad s_y := \dfrac{3.2}{2}\ m$

Modifiers for pile material type and installation:

MODIFIER TERMS FOR PILE MATERIAL TYPE (C_M) AND INSTALLATION EFFECTS (C_K)

Pile Installation Effects Modifier C_K	Jetted pile	$C_K = 0.5$ to 0.6
	Drilled and bored piles	$C_K = 0.9$ to 1.0
	Low-displacement driven piles (e.g., H-piles; open-ended pipe)	$C_K = 1.0$ to 1.1
	High-displacement driven piles (e.g., closed-ended pipe; precast)	$C_K = 1.1$ to 1.2
Pile Material Effects Modifier C_M	Soil/rough concrete (drilled shafts)	$C_M = 1.0$
	Soil/smooth concrete (precast)	$C_M = 0.9$
	Soil/timber (wood pilings)	$C_M = 0.8$
	Soil/rough steel (normal H- and pipe pilings)	$C_M = 0.7$
	Soil/smooth steel (cone penetrometer)	$C_M = 0.6$
	Soil/stainless steel (flat dilatometer)	$C_M = 0.5$

Adapted after Kulhawy et al. 1983.

$C_K := 0.9$ *(Bored pile)*

$C_M := 1.0$ *(Rough concrete)*

Partial safety factors (Approach 2, A1+M1 +R3):

$\gamma_M := 2.3$ *(mixed foundation, bearing capacity)*

$\gamma_{M2} := 1.3$ *(mixed foundation, resistance to horizontal loads)*

$\xi := 1.6$ *(average value on three vertical tests)*

<u>ULS loads</u>

Components:

$P_x := 980\ kN$

$P_y := 544\ kN$

$P_{z.tension} := 1225\ kN$

$P_{z.compr} := 1361\ kN$

Moments acting at the embedment depth due to the load horizontal components:

$M_x := 5302\ kN \cdot m$

$M_y := 2098\ kN \cdot m$

Total vertical load at the embedment depth: $Q_{tot} := P_{z.compr} + 1.35 \cdot W_{cap} = \left(1.983 \cdot 10^3\right)\ kN$

Total vertical load at the base of piles: $Q_d := Q_{tot} + n \cdot \gamma_{conc} \cdot \pi \cdot \dfrac{D^2}{4} \cdot L = \left(2.662 \cdot 10^3\right)\ kN$

Total horizontal load: $H_{tot} := \sqrt{P_x^{\ 2} + P_y^{\ 2}} = \left(1.121 \cdot 10^3\right)\ kN$

Eccentricities: $e_x := \dfrac{M_y}{Q_{tot}} = 1.058\ m$

$e_y := \dfrac{M_x}{Q_{tot}} = 2.674\ m$

Design cap dimensions: $B_x' := B_x - 2 \cdot e_x = 0.764\ m$

$B_y' := B_y - 2 \cdot e_y = 1.053\ m$

Mixed foundation (pile cap + piles) ULS resistance verification

Pile cap verification

$$\gamma' := \gamma - \gamma_w = 10\ \frac{kN}{m^3}$$

Bearing capacity factors: $N_q := \dfrac{1 + \sin(\phi)}{1 - \sin(\phi)} \cdot e^{\pi \cdot \tan(\phi)} = 48.933$

$N_\gamma := 2 \cdot (N_q + 1) \cdot \tan(\phi) = 78.024$

Shape factors: $s_q := 1 + \dfrac{B_x'}{B_y'} \cdot \tan(\phi) = 1.567$

$s_\gamma := 1 - 0.4 \cdot \dfrac{B_x'}{B_y'} = 0.71$

Depth factors: $d_q := 1 + 2 \cdot \dfrac{H_b}{B_x'} \cdot \tan(\phi) \cdot (1 - \sin(\phi))^2 = 1.302$ $\qquad d_\gamma := 1$

Angle factors: $m' := \dfrac{2 + \dfrac{B_x'}{B_y'}}{1 + \dfrac{B_x'}{B_y'}}$ $\qquad i_q := \left(1 - \dfrac{H_{tot}}{Q_{tot}}\right)^{m'} = 0.268$ $\qquad i_\gamma := \left(1 - \dfrac{H_{tot}}{Q_{tot}}\right)^{m'+1} = 0.117$

Stress at the embedment depth: $q := \gamma' \cdot H_b = 0.01\ MPa$

Bearing capacity: $q_{lim} := \dfrac{1}{2}\gamma \cdot B_x' \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma + q \cdot N_q \cdot s_q \cdot d_q \cdot i_q = 0.317\ MPa$

$A_{net} := B_x' \cdot B_y' = 0.804\ m^2$

$Q_{r.cap} := q_{lim} \cdot A_{net} = 255.227\ kN$

$Q_{k.cap} := \dfrac{Q_{r.cap}}{\xi} = 159.517\ kN$

Bearing capacity verification

$Q_{kr.cap} := \dfrac{Q_{k.cap}}{\gamma_M} = 69.355\ kN$ $\quad < \quad$ $Q_d = \left(2.662 \cdot 10^3\right)\ kN$ $\quad$ NOT OK: *Piles contribution to the global bearing capacity could not be neglected*

Pile cap + piles verification (global behaviour)

Lateral capacity of a single pile

$$f_s := (1 - \sin(\phi)) \cdot OCR^{\sin(\phi)} \cdot \gamma' \cdot \left(H_b + \frac{L_v}{2}\right) \cdot \tan(\phi) \cdot C_M \cdot C_K = 0.048\ MPa$$

$$Q_{lat} := f_s \cdot \pi \cdot D \cdot L = \left(1.443 \cdot 10^3\right)\ kN$$

$$Q_{lat.v} := Q_{lat} \cdot \cos\left(i_{pile}\right) = \left(1.443 \cdot 10^3\right)\ kN$$

$$Q_{klat.v} := \frac{Q_{lat.v}}{\xi} = 901.685\ kN$$

End bearing capacity of a single pile

Bearing capacity factor:

$$I_R := \frac{E}{2 \cdot (1 + \nu) \cdot \left(\gamma' \cdot \left(H_b + L\right) \cdot \tan(\phi)\right)} = 174.11$$

$N_\sigma := 200$ (safe-sided value obtained from Table)

$$q_E' := \gamma' \cdot \left(H_b + L_v\right) \cdot N_\sigma = 34\ MPa$$

$$Q_E := q_E' \cdot \pi \cdot \frac{D^2}{4} = \left(9.613 \cdot 10^3\right)\ kN$$

$$Q_{E.v} := Q_E \cdot \cos\left(i_{pile}\right) = \left(9.613 \cdot 10^3\right)\ kN$$

$$Q_{kE.v} := \frac{Q_{E.v}}{\xi} = \left(6.008 \cdot 10^3\right)\ kN$$

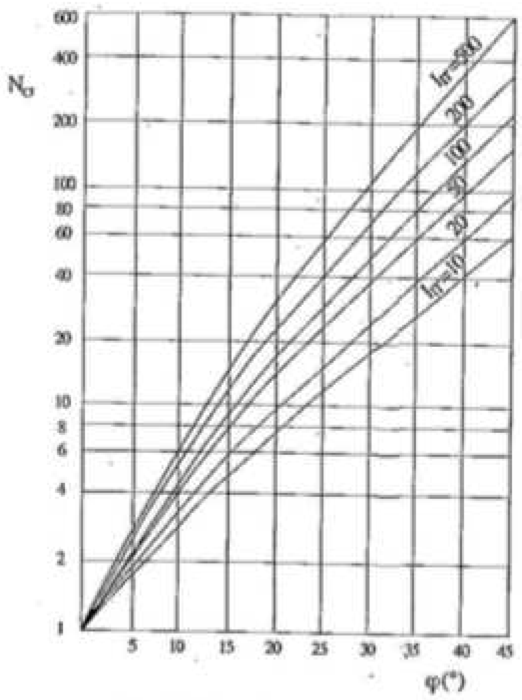


Fig. 13.4. Coefficienti Nσ (eq. 13.5)

Mixed foundation bearing capacity verification

$$Q_{kr} := Q_{kr.cap} + \frac{n \cdot (Q_{klat.v} + Q_{kE.v})}{\gamma_M} = (1.81 \cdot 10^4)\ kN \quad > \quad Q_d = (2.662 \cdot 10^3)\ kN \quad \text{OK}$$

Mixed foundation lateral bearing capacity verification

Pile plastic moment: $M_{pl} := 588\ kN \cdot m$

Short pile failure mode: $H_{r1} := \frac{3}{2} \cdot \left(\frac{L_v}{D}\right)^3 \cdot K_p \cdot \gamma' \cdot \frac{D^4}{L_v} = (9.685 \cdot 10^3)\ kN$

Intermediate pile failure mode:

$$H_{r2} := \left(\frac{1}{2}\left(\frac{L_v}{D}\right)^2 + M_{pl} \cdot \frac{D}{K_p \cdot \gamma' \cdot D^4 \cdot L_v}\right) \cdot K_p \cdot \gamma' \cdot D^3 = (3.265 \cdot 10^3)\ kN$$

Long pile failure mode: $H_{r3} := K_p \cdot \gamma' \cdot D^3 \cdot \sqrt[3]{\left(3.676 \cdot \frac{M_{pl}}{K_p \cdot \gamma' \cdot D^4}\right)^2} = 490.265\ kN$

Horizontal pile capacity: $H_r := min\,(H_{r1}, H_{r2}, H_{r3}) = 490.265\ kN$

$$H_{rk} := \frac{H_r}{1.7} = 288.391\ kN$$

Contribution from lateral capacity:

$$Q_{lat.h} := Q_{lat} \cdot \sin(i_{pile}) = 0\ kN$$

$$Q_{klat.h} := \frac{Q_{lat.h}}{\xi} = 0\ kN$$

Contribution from end bearing capacity:

$$Q_{E.h} := Q_E \cdot \sin(i_{pile}) = 0\ kN$$

$$Q_{kE.h} := \frac{Q_{E.h}}{\xi} = 0\ kN$$

Reduction factor (group effect):

$\eta := 0.7$ *(safe-sided value, Viggiani)*

Pile cap passive lateral resistance: $Q_{cap.h} := K_p \cdot \gamma' \cdot (H_b + 0.5\ m) \cdot \frac{H}{2} \cdot B_x = 90.801\ kN$

Total lateral capacity: $Q_{htot} := n \cdot \eta \cdot (Q_{klat.h} + Q_{kE.h} + H_{rk}) + Q_{cap.h} = (1.302 \cdot 10^3)\ kN$

Verification

$$Q_{htot} = (1.302 \cdot 10^3)\ kN \quad > \quad H_{tot} = (1.121 \cdot 10^3)\ kN \quad \text{OK}$$

Mixed foundation (pile cap + piles) SLS resistance verification

SLS loads

$P_x := 635\ kN$

$P_y := 453\ kN$

$P_{z.tension} := 1088\ kN$

$P_{z.compr} := 1188\ kN$

$$Q_{tot} := \frac{P_{z.compr}}{n} + \left(P_{z.compr} + P_{z.tension}\right) \cdot \frac{4.8}{2 \cdot 2} + P_x \cdot \frac{2.14}{3} = \left(3.382 \cdot 10^3\right)\ kN$$

Settlement calculation

PILE CAP SETTLEMENT (Burland and Burbridge method)

Influence zone: $H := B_x^{\ 0.7}$ $\quad Z_I := B_x^{\ 0.7}$

Depth factor: $k_{HZ} := min\left(1, \frac{H}{Z_I}\right)$ $\quad f_h := \frac{k_{HZ}}{2 - k_{HZ}}$

Shape factor: $f_s := \left(\frac{1.25}{1 + 0.25 \cdot \frac{B_x}{B_y}}\right)^2$

Nominal design life: $t := 100$

Time factor: $f_t := 1.3 + 0.2 \cdot \log\left(\frac{t}{3}\right)$

Compressibility index: $N_C := 15 + \frac{N_{spt} - 15}{2}$ $\quad I_C := \frac{1.71}{N_C^{\ 1.4}}$

Effective pressure on gross area: $q_{tot} := \frac{Q_{tot}}{B_x \cdot B_y}$

Pre-consolidation stress: $\sigma'_{v0} := \gamma' \cdot H_b$ $\quad \sigma'_p := OCR \cdot \sigma'_{v0}$

$\sigma_a := min\left(\sigma'_p, q_{tot}\right)$

$\sigma_b := \max\left(0, q_{tot} - \sigma_a\right)$

Settlement

$$w := f_s \cdot f_h \cdot f_t \cdot Z_I \cdot m^{\frac{-7}{10}} \cdot I_C \cdot \left(\frac{\sigma_a}{3} + \sigma_b\right) \cdot kPa^{-1} \cdot mm = 15.191\ mm$$

MIXED FOUNDATION STIFFNESS (Fleming et al.)

Load on a single pile: $Q_{pile} := \frac{Q_{tot}}{n}$ *(rigid pile cap hyp.)*

$k := \frac{E_{eq}}{E} = 592.432$

$\frac{L_v}{D} = 26.667$ --------> $I_w := 3$

Settlement of a single pile

$$w_s := \frac{I_w \cdot Q_{pile}}{E \cdot L_v} = 1.904\ mm$$

Stiffness of a single pile

$$K_S := \frac{Q_{pile}}{w_s}$$

Stiffness correction factors

$\frac{L_v}{D} = 26.667$ --------> $a_{standard} := 0.53$

Spacing ratio factor: $\frac{s}{D} = 0.167$ ---> $a_{sd} := 0.95$

Poisson's ratio factor: $\nu = 0.2$ ---> $a_\nu := 1.03$

Homogeneity factor: $\rho := 1$ ---> $a_p := 1.05$

Stiffness ratio factor:

$\log\left(\frac{E_{eq}}{G}\right) = 3.153$ ---> $a_{EP} := 1.03$

Global correction factor:

$$a := a_{standard} \cdot a_{sd} \cdot a_p \cdot a_\nu \cdot a_{EP} = 0.561$$

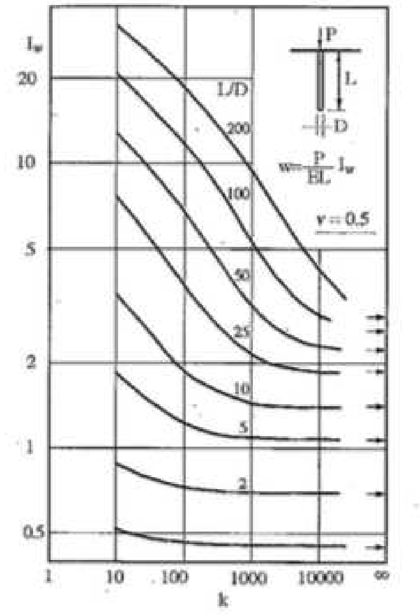
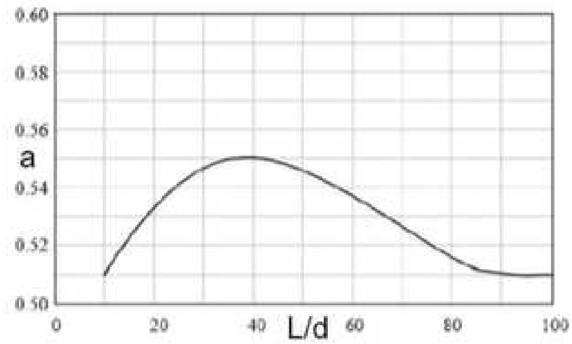
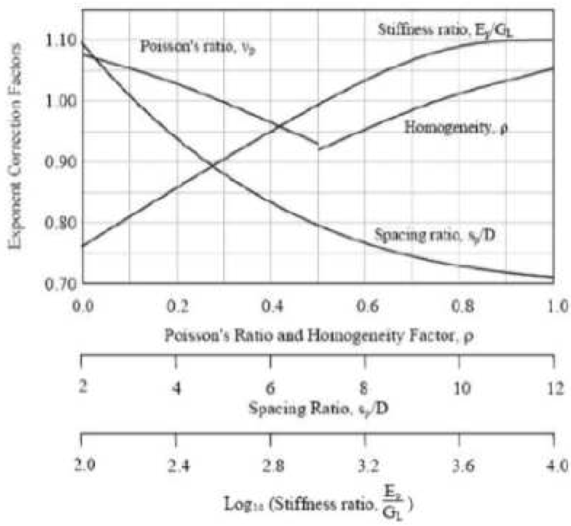


Fig. 14.5. Coefficienti di influenza I_w per il calcolo del cedimento di un palo isolato in un semispazio elastico





MIXED FOUNDATION SETTLEMENT (Poulos and Davis)

Piles group stiffness: $K_{piles} := K_S \cdot n^{1-a} = (6.501 \cdot 10^5)\ \frac{kN}{m}$

Pile cap stiffness: $K_{cap} := \frac{Q_{tot}}{w} = (2.226 \cdot 10^5)\ \frac{kN}{m}$

Global stiffness: $K_{mixed} := K_{piles} \cdot \frac{1 - 0.6 \cdot \frac{K_{cap}}{K_{piles}}}{1 - 0.64 \cdot \frac{K_{cap}}{K_{piles}}} = (6.615 \cdot 10^5)\ \frac{kN}{m}$

Global settlement

$$w_f := \frac{Q_{tot}}{K_{mixed}} = 5.113\ mm$$

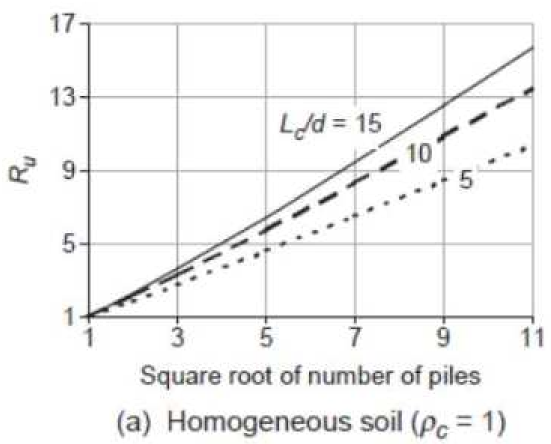
MIXED FOUNDATION LATERAL SETTLEMENT (Randolph)

Note: for the purpose of the lateral settlement assessment the piles have been conservatively considered to be vertical.

Critical length: $L_c := D \cdot \left(2 \frac{E_{eq}}{G \cdot (1 + 0.75\ \nu)}\right)^{\frac{2}{7}} = 5.593\ m$

Horizontal load on a single pile: $P_{xpile} := \frac{H_{tot}}{n}$

Horizontal displacement (rotation inhibited): $y := \frac{\left(\frac{E_{eq}}{G}\right)^{\frac{1}{7}}}{\rho \cdot G} \cdot \left(0.27 - \frac{0.11}{\sqrt{\rho}}\right) \cdot \frac{P_{xpile}}{\frac{L_c}{2}} = 0.001\ m$



$\frac{L_c}{D} = 9.321$

$\sqrt{n} = 2.449$ ------> $R_u := 2$

Horizontal settlement $y_g := y \cdot R_u = 2.608\ mm$

***Abutment foundation* - Structural design**

Piles

FEM analysis results

Note: for elastic design analyses, the reinforcement has been neglected in the finite element modeling since the stiffness contribution of the concrete is much greater than the reinforcement. The reinforcement can safely be assumed to keep the composite structure intact, and thus act (for analysis purposes) as a homogenous elastic body. Forces and moments have been then extracted from the FEA solution and used as a basis to size the reinforcement needed to carry the net tensile forces in the section.

Height of pile cap: $H := 100\ cm$

Pile diameter: $D := 60\ cm$

Steel cover: $c := 6\ cm$

Vertical stiffness: $K_v := \dfrac{Q_d}{A \cdot w_f} = \left(2.824 \cdot 10^7\right) \dfrac{N}{m^3}$

Horizontal stiffness: $K_h := \dfrac{H_{tot}}{(n \cdot \pi \cdot D \cdot L + B_x \cdot H) \cdot y_g} = \left(2.338 \cdot 10^6\right) \dfrac{N}{m^3}$

Discrete vertical stiffness (pile cap): $k_{v.cap} := K_v \cdot \dfrac{A}{35} = \left(1.487 \cdot 10^4\right) \dfrac{kN}{m}$

Discrete vertical stiffness (piles): $k_{v.piles} := K_v \cdot n \cdot \pi \cdot \dfrac{\frac{D^2}{4}}{6} = \left(7.986 \cdot 10^3\right) \dfrac{kN}{m}$

Discrete horizontal stiffness (piles): $k_{h.piles} := K_h \cdot n \cdot \pi \cdot D \cdot \dfrac{L}{n \cdot (16)} = \left(4.407 \cdot 10^3\right) \dfrac{kN}{m}$

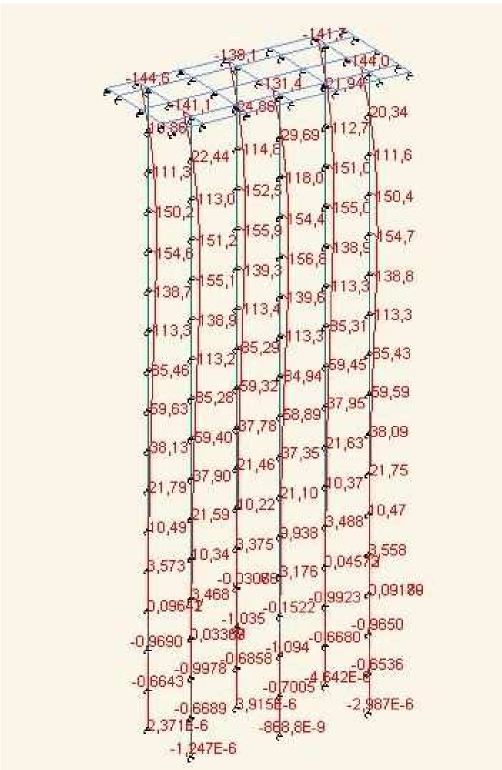
ULS Loads on pile

$N := -78\ kN$ $V := 189\ kN$

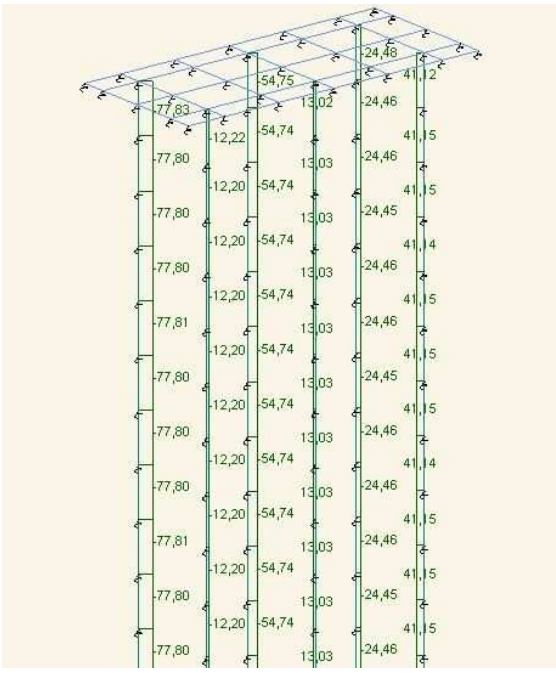
$M_x := 258\ kN \cdot m$

$M_y := 157\ kN \cdot m$

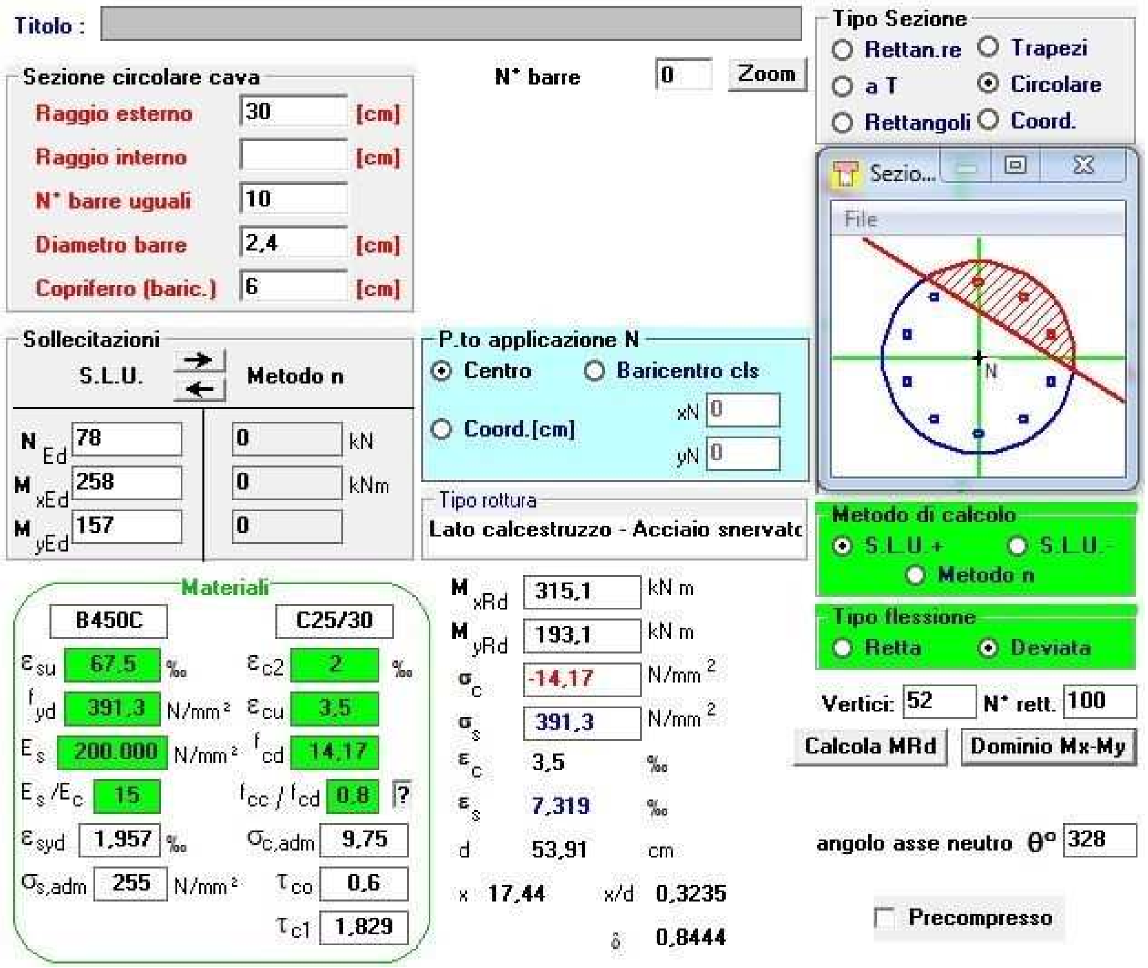
Bending moment

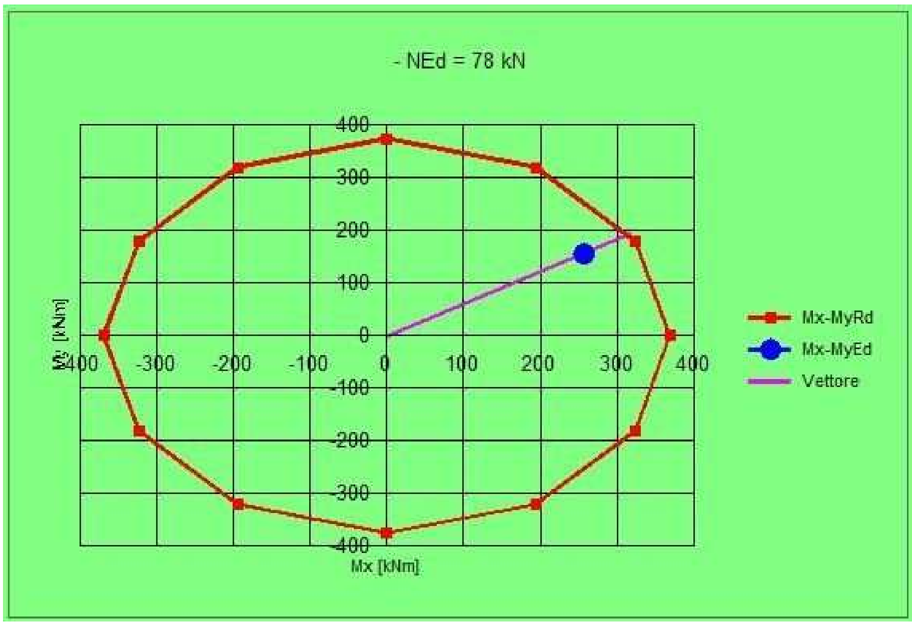


Axial force



Piles cross section





Axial force and biaxial bending verification OK

Longitudinal reinforcement diameter: $d_{sl} := 2.2\ cm$

Shear verification (EC2 + Orr, Darby, Ibell paper)

Efficiency factor for circular section: $\lambda_1 := 0.85$

Radius for shear steel: $r_{sv} := \frac{D}{2} - c = 24\ cm$

Spiral pitch: $p := 10\ cm$ (spiral)

$$\lambda_2 := \left(\left(\frac{p}{2 \cdot \pi \cdot r_{sv}}\right)^2 + 1\right)^{-0.5} = 0.998$$

Minimum stirrup diameter: $\max\left(6\ mm, \frac{1}{4}\ d_{sl}\right) = 0.6\ cm$

Stirrup diameter: $d_s := 1.4\ cm$

Stirrup area: $A_{sw} := 2 \cdot \pi \cdot \frac{{d_s}^2}{4}$

Shear reinforcement yielding strength: $f_{ywd} := 391.3\ MPa$

Equivalent rectangular section (Clarke & Birjandi) $h := 374\ mm$ $b := 336\ mm$ (Bartolomeo Ravera.it)

Stirrup maximum spacing: $s_{max} := min\left(12 \cdot d_{sl}, 300\ mm, b\right) = 26.4\ cm$

Stirrup spacing: $s := p$

Truss angle: 10 $cot(\theta) := 1$

Lever arm: $z := 0.9 \cdot \left(\frac{D}{2} + 2 \cdot \frac{D}{2 \cdot \pi}\right)$

Shear resistance:

$V_{Rd.s} := \lambda_1 \cdot \lambda_2 \cdot \frac{A_{sw}}{s} \cdot z \cdot f_{ywd} \cdot 1 = 451.506\ kN$ > $V = 189\ kN$ (GSA model) OK

Pile cap

S&T plane 1-1 model

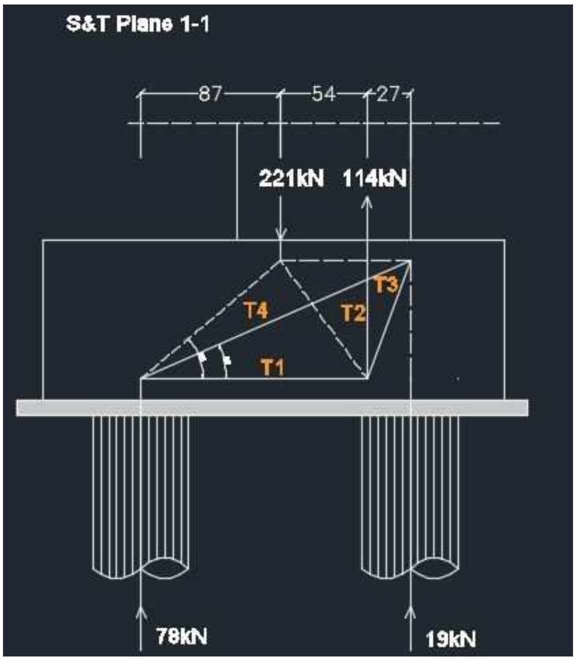
Partial safety factors: $\gamma_c := 1.5$ $\alpha_{cc} := 0.85$

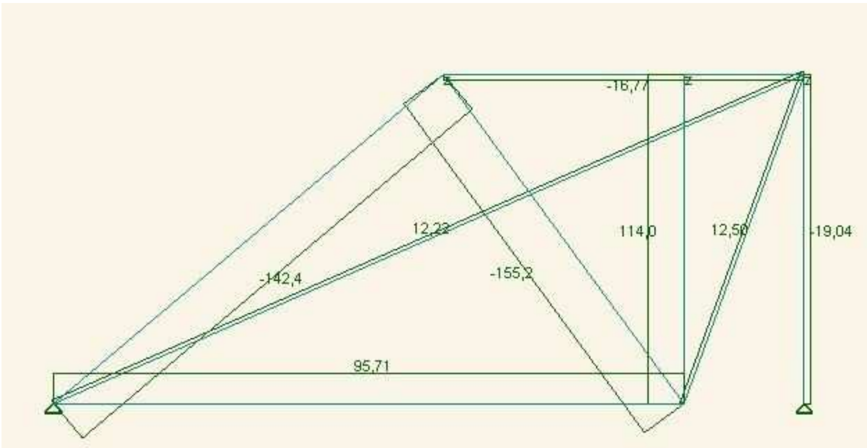
Design compressive strength: $f_{cd} := \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 17\ MPa$

Reinforcement steel type: B450C

Steel yielding strength: $f_{yd} := 391.3\ MPa$

General reinforcement from preliminary design: $A_{s.linear} := 2\ cm^2$





<u>Bottom chord reinforcement design:</u>

$T_1 := 96\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s1} := \dfrac{T_1}{f_{yd}} = 2.453\ cm^2$ ---> $4\,\phi 12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max\left(\phi, 20\ mm, d_g + 5\ mm\right) = 3.7\ cm$ --> $4\ cm$

<u>Vertical chord reinforcement design:</u> $T_2 := 114\ kN$

$A_{s2} := \dfrac{T_2}{f_{yd}} = 2.913\ cm^2$ ---> $4\,\phi 12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max\left(\phi, 20\ mm, d_g + 5\ mm\right) = 3.7\ cm$ --> $4\ cm$

Diagonal (1) chord reinforcement design: $T_3 := 13\ kN$

$A_{s3} := \frac{T_3}{f_{yd}} = 0.332\ cm^2$ ---> Use general reinf.

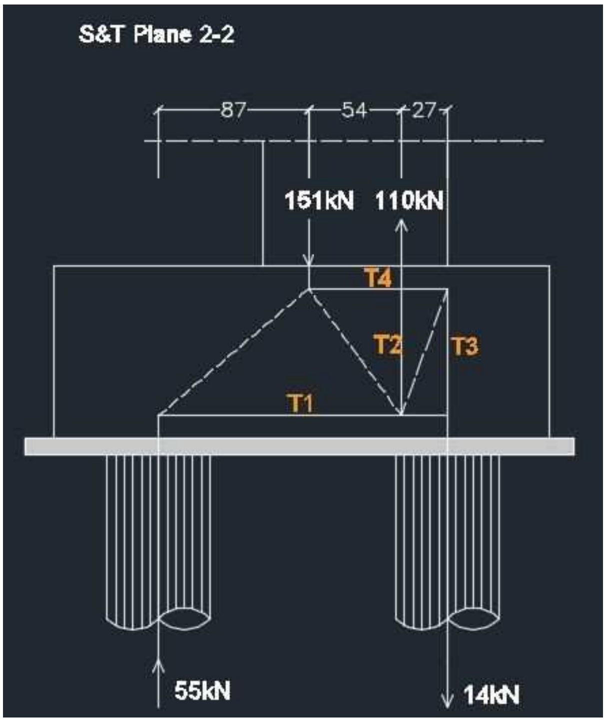
Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Diagonal (3) chord reinforcement design: $T_4 := 13\ kN$

$A_{s4} := \frac{T_4}{f_{yd}} = 0.332\ cm^2$ ---> Use general reinf.

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

S&T plane 2-2 model



Bottom chord reinforcement design:

$T_1 := 79\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s1} := \frac{T_1}{f_{yd}} = 2.019\ cm^2$ ---> $4\,\phi 12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Vertical chord (1) reinforcement design: $T_2 := 110\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s2} := \frac{T_2}{f_{yd}} = 2.811\ cm^2$ ---> $4\phi 12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Vertical chord (3) reinforcement design: $T_3 := 14\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s3} := \frac{T_3}{f_{yd}} = 0.358\ cm^2$ ---> Use general reinf.

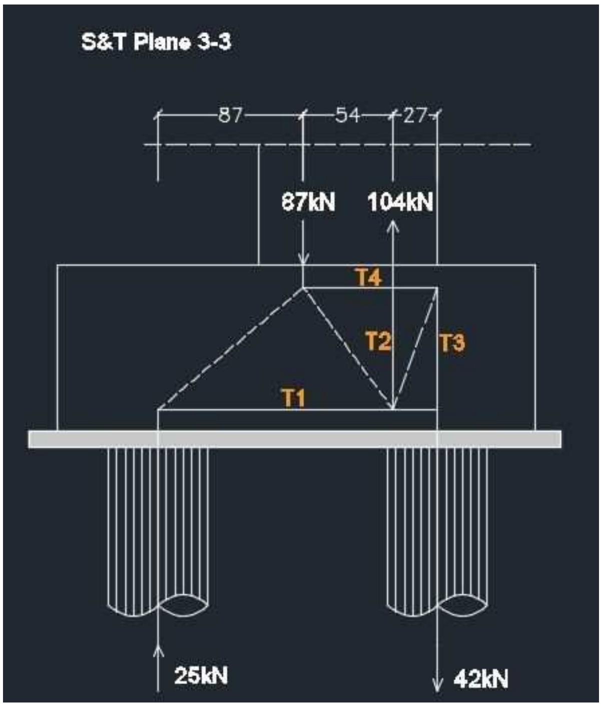
Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Top chord reinforcement design: $T_4 := 6\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s4} := \frac{T_4}{f_{yd}} = 0.153\ cm^2$ ---> Use general reinf.

S&T plane 3-3 model



Bottom chord reinforcement design: $T_1 := 15\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s1} := \frac{T_1}{f_{yd}} = 0.383\ cm^2$ ---> Use general reinf.

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Vertical chord (1) reinforcement design: $T_2 := 104\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s2} := \frac{T_2}{f_{yd}} = 2.658\ cm^2$ ---> $4\phi12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Vertical chord (3) reinforcement design: $T_3 := 42\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s3} := \frac{T_3}{f_{yd}} = 1.073\ cm^2$ ---> $4\phi12$ $\phi := 1.2\ cm$

Minimum distance b/w rebars: $\max(\phi, 20\ mm, d_g + 5\ mm) = 3.7\ cm$ --> $4\ cm$

Top chord reinforcement design: $T_4 := 16\ kN$

Maximum aggregate dimension: $d_g := 32\ mm$

$A_{s1} := \frac{T_1}{f_{yd}} = 0.383\ cm^2$ ---> Use general reinf.

Cantilever beam model (1-m strip beam fixed at the pile cap-abutment stem intersection)

$d := 1\ m - 0.06\ m = 0.94\ m$

Design out-of-plane bending moment from ULS combination: $M_{y.Ed} := 50\ kN \cdot m$ (per linear m)

$A_s := \frac{M_{y.Ed}}{0.9 \cdot d \cdot f_{yd}} = 1.51\ cm^2$ -----> $4\phi20$ at 25cm o.c

Stem

Cantilever beam model (1-m strip beam fixed at the pile cap-abutment stem intersection)

$d := 1.08\ m - 0.06\ m = 1.02\ m$

Design out-of-plane bending moment from ULS combination: $M_{y.Ed} := 700\ kN \cdot m$ (per linear m)

Reinforcement required:

$A_s := \frac{M_{y.Ed}}{0.9 \cdot d \cdot f_{yd}} = 19.487\ cm^2$ -----> $5\phi24$ at 20cm o.c.

Multi-directional bearing support

Design axial compression from ULS combination: $N_{Ed} := 1361\ kN$

Bottom chord tension from S&T model:

$$T := \frac{\frac{N_{Ed}}{2}}{\tan(59°)} = 408.886\ kN$$

Reinforcement required:

$$A_s := \frac{T}{f_{yd}} = 10.449\ cm^2$$ -----> $5\,\phi16$ at 20cm o.c.

Multi-directional bearing support

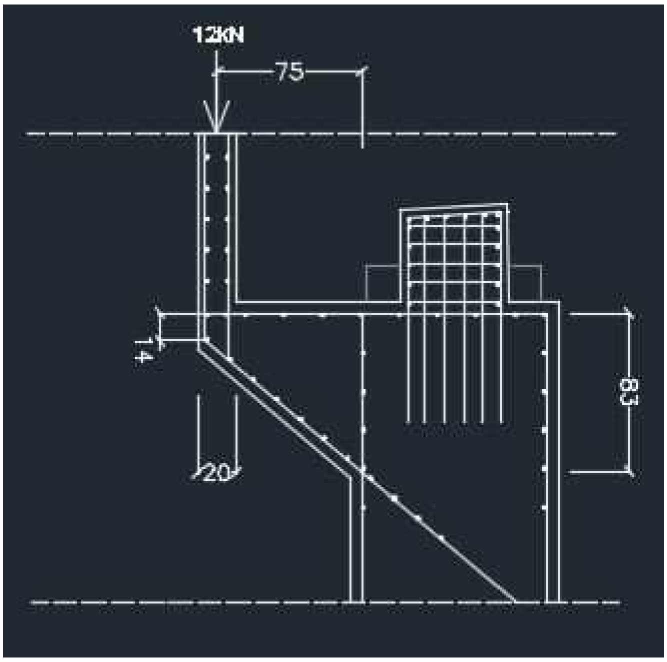
Design axial tension from ULS combination: $N_{Ed} := 1225\ kN$

Reinforcement required:

$$A_s := \frac{N_{Ed}}{f_{yd}} = 31.306\ cm^2$$ -----> $27\,\phi14$

1m-strip cantilever beam (back of end-diaphragm)

Design load from ULS combination: $N_{Ed} := 12\ kN$



Top chord tension: $T := 13\ kN$

Reinforcement required:

$$A_s := \frac{T}{f_{yd}} = 0.332\ cm^2$$

-----> $\phi18$ at 30cm o.c.

Loads definition

Traffic Loads

Gr1: UDL + Fh

Gr2: Qsv + Fh

UDL

$$q_{fk} := 5 \frac{kN}{m^2}$$

Service vehicle (axle loads)

$Q_{sv1} := 80\ kN$ $\qquad$ $Q_{sv2} := 40\ kN$

Horizontal force

$$A_{sur} := 386.5146\ m^2$$

$$Q_{flk} := \max\left(0.1 \cdot q_{fk} \cdot A_{sur}, 0.6 \cdot \left(2 \cdot Q_{sv1} + 2 \cdot Q_{sv2}\right)\right) = 193.257\ kN$$

Wind Loads

Structure parameters

$b := 4\ m$ $\qquad$ $L := 110\ m$ $\qquad$ $d := 0.5\ m$ $\qquad$ $f_B := 0.4673\ Hz$

Wind parameters

$\rho := 1.25 \frac{kg}{m^3}$ $\qquad$ $V_r := 27 \frac{m}{s}$

$$M := \frac{\left(2002 \cdot kN \cdot \dfrac{1}{9.81 \dfrac{m}{s^2}}\right)}{L} = \left(1.855 \cdot 10^3\right) \frac{kg}{m}$$

A.10 **ITALIA**

(1) La velocità di riferimento del vento è definita da:

$v_{ref} = v_{ref,0}$ per $a_s \leq a_0$;

$v_{ref} = v_{ref,0} + k_a (a_s - a_0)$ per $a_s > a_0$;

dove:

a_s è l'altitudine sopra il livello del mare in [m] del sito della struttura.

Zona	Descrizione	$v_{ref,0}$ [m/s]	a_0 [m]	k_a [l/s]
1	Valle d'Aosta, Piemonte, Lombardia, Trentino Alto Adige, Veneto, Friuli Venezia Giulia (eccetto la provincia di Trieste)	25	1 000	0,012
2	Emilia Romagna	25	750	0,024
3	Toscana, Marche, Umbria, Lazio, Abruzzo, Molise, Campania, Puglia, Basilicata, Calabria (eccetto la provincia di Reggio Calabria)	27	500	0,030
4	Sicilia e provincia di Reggio Calabria	28	500	0,030
5	Sardegna (lato est della linea congiungente capo Teulada con l'isola della Maddalena)	28	750	0,024
6	Sardegna (lato ovest della linea congiungente capo Teulada con l'isola della Maddalena	28	500	0,030
7	Liguria	29	1 000	0,024
8	Provincia di Trieste	31	1 500	0,012
9	Isole (eccetto Sicilia e Sardegna) e mare aperto	31	500	0,030

Vr

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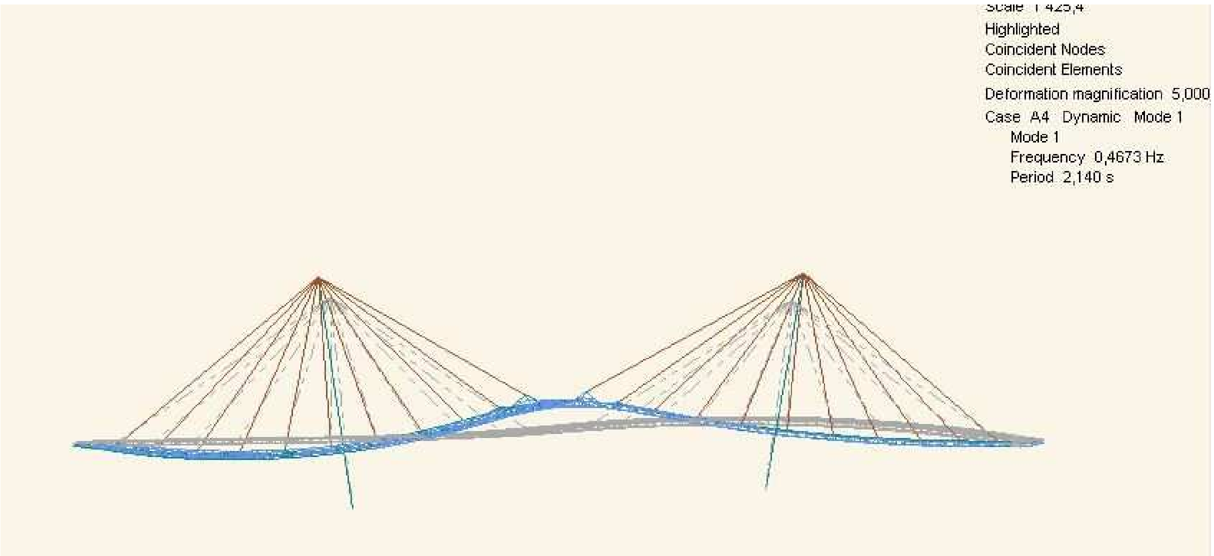


fb

$P_b := \rho \cdot \frac{b^2}{M} \cdot 16 \cdot \frac{V_r^{\ 2}}{b \cdot L \cdot f_B^{\ 2}} = 1.309$

> 1 bridge is very susceptible to aerodynamic excitation according to BD 49-Part 3, 2.1 (c), thus its stability shall be verified by means of specific studies, or through wind tunnel tests on scale models.

Wind force on the deck

Simplified method

$d_{tot} := d + 0.3\ m = 0.8\ m$ $\quad \frac{b}{d_{tot}} = 5$ $\quad A_{ref.x} := L \cdot d_{tot} = 88\ m^2$ $\quad z := 4\ m$

$z_0 := 0.05\ m$ $\quad z_{0.II} := 0.05\ m$ $\quad z_{min} := 2\ m$ $\quad z_{max} := 200\ m$ $\quad k_r := 0.19 \cdot \left(\frac{z_0}{z_{0.II}}\right)^{0.07}$ $\quad k_l := 1$

$v_{b.0} := V_r = 27\ \frac{m}{s}$ $\quad c_{dir} := 1$ $\quad c_{season} := 1$ $\quad c_0 := 1$ $\quad c_r := k_r \cdot \ln\left(\frac{z}{z_0}\right)$

$v_b := v_{b.0} \cdot c_{dir} \cdot c_{season} = 27\ \frac{m}{s}$ $\quad v_m := c_r \cdot c_0 \cdot v_b = 22.48\ \frac{m}{s}$

$\sigma_v := k_r \cdot v_b \cdot k_l = 5.13\ \frac{m}{s}$ $\quad I_v := \frac{k_l}{c_0 \cdot \ln\left(\frac{z}{z_0}\right)} = 0.228$

$q_p := (1 + 7 \cdot I_v) \cdot \frac{1}{2}\rho \cdot v_m^{\ 2} = 820.369\ Pa$ $\quad q_b := \frac{1}{2} \cdot \rho \cdot v_b^{\ 2} = 455.625\ Pa$ $\quad c_e := \frac{q_p}{q_b} = 1.801$

$c_{f.x0} := 1.3$ $\quad c_{f.x} := c_{f.x0} = 1.3$ $\quad C_x := c_{f.x} \cdot c_e = 2.341$ $\quad$ Use recommended value: $\quad C_x := 3.6$

$$F_{W.x} := \frac{1}{2} \cdot \rho \cdot v_b^{\ 2} \cdot C_x \cdot A_{ref.x} = 144.342\ kN$$ Wind force along x

+/- $\quad c_{f.z} := 0.9$ $\quad C_z := c_{f.z} \cdot c_e = 1.62$ $\quad A_{ref.z} := b \cdot L = 440\ m^2$

+/- $$F_{W.z} := \frac{1}{2} \cdot \rho \cdot v_b^{\ 2} \cdot C_z \cdot A_{ref.z} = 324.866\ kN$$ Wind force along z (+/-)

$F_{W.y} := 0.25 \cdot F_{W.x} = 36.086\ kN$ Wind force along y

Table 4.1 — Terrain categories and terrain parameters

Terrain category		z_0 m	z_{min} m
0	Sea or coastal area exposed to the open sea	0,003	1
I	Lakes or flat and horizontal area with negligible vegetation and without obstacles	0,01	1
II	Area with low vegetation such as grass and isolated obstacles (trees, buildings) with separations of at least 20 obstacle heights	0,05	2
III	Area with regular cover of vegetation or buildings or with isolated obstacles with separations of maximum 20 obstacle heights (such as villages, suburban terrain, permanent forest)	0,3	5
IV	Area in which at least 15 % of the surface is covered with buildings and their average height exceeds 15 m	1,0	10
NOTE: The terrain categories are illustrated in A.1.			

Table 8.2 — Recommended values of the force factor *C* for bridges

b/d_{tot}	$z_e \leq 20$ m	$z_e = 50$ m
$\leq 0,5$	6,7	8,3
$\geq 4,0$	3,6	4,5

This table is based on the following assumptions :

– terrain category II according to Table 4.1

– force coefficient $c_{f,x}$ according to 8.3.1 (1)

– c_0=1,0

– k_l=1,0

For intermediate values of b/d_{tot}, and of z_e linear interpolation may be used

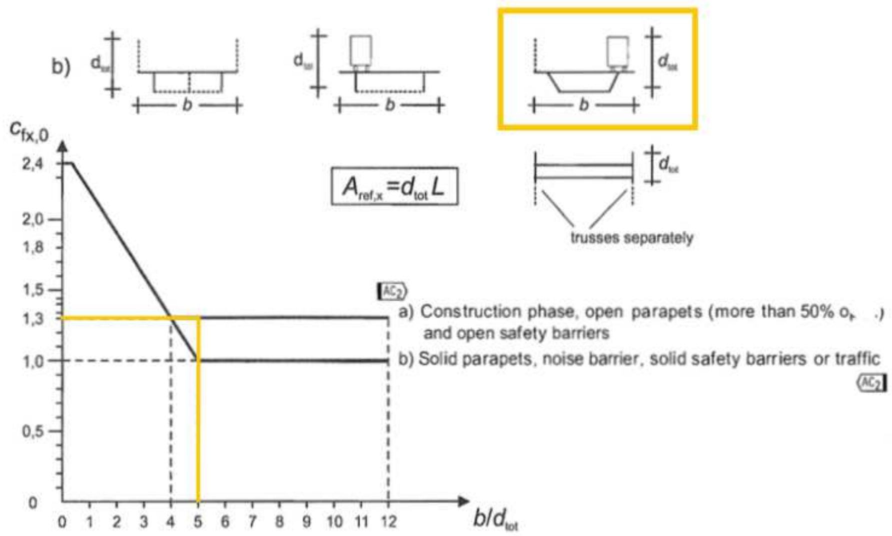


Figure 8.3 — Force coefficient for bridges, $c_{fx,0}$

Wind force on other structural elements

5.3 Wind forces

(1) The wind forces for the whole structure or a structural component should be determined:

by calculating forces using force coefficients (see (2)) or

by calculating forces from surface pressures (see (3))

(2) The wind force F_w acting on a structure or a structural component may be determined directly by using Expression (5.3)

$$F_w = c_s c_d \cdot c_f \cdot q_p(z_e) \cdot A_{ref} \qquad (5.3)$$

or by vectorial summation over the individual structural elements (as shown in 7.2.2) by using Expression (5.4)

25

Masts

7.9.2 Force coefficients

(1) The force coefficient c_f for a finite circular cylinder should be determined from Expression (7.19).

$$c_f = c_{f,0} \cdot \psi_\lambda \qquad (7.19)$$

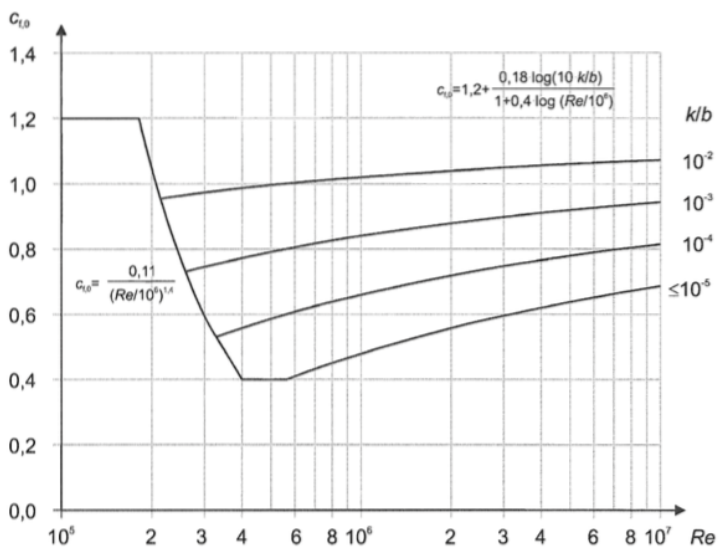
where:

$c_{f,0}$ is the force coefficient of cylinders without free-end flow (see Figure 7.28)

ψ_λ is the end-effect factor (see 7.13)

$c_s := 1$ $\quad$ $c_d := 1$ $\quad$ $diam := 0.6\ m$ $\quad$ $\nu := 15 \cdot 10^{-6}\ \dfrac{m^2}{s}$ $\quad$ $z := 22\ m$

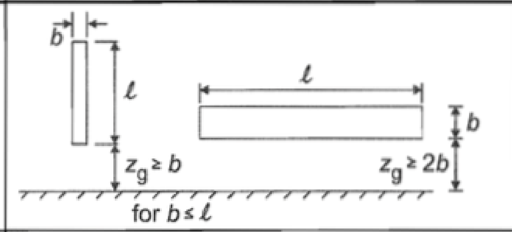
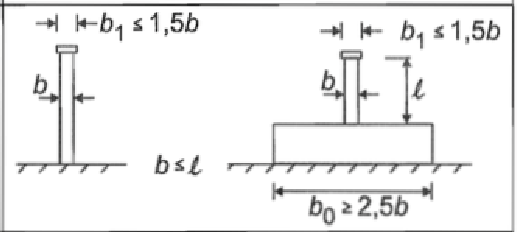
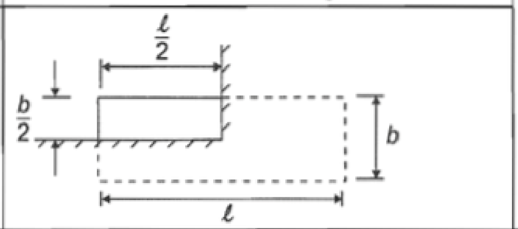
$$I_v := \frac{k_l}{c_0 \cdot \ln\left(\dfrac{z}{z_0}\right)} = 0.164 \qquad q_p := (1 + 7 \cdot I_v) \cdot \frac{1}{2}\,\rho \cdot v_m^{\ 2} = 679.063\ Pa \qquad Re := diam \cdot \frac{\sqrt{2 \cdot \dfrac{q_p}{\rho}}}{\nu} = 1.318 \cdot 10^6$$



----> $\quad c_{f.0} := 1.2$

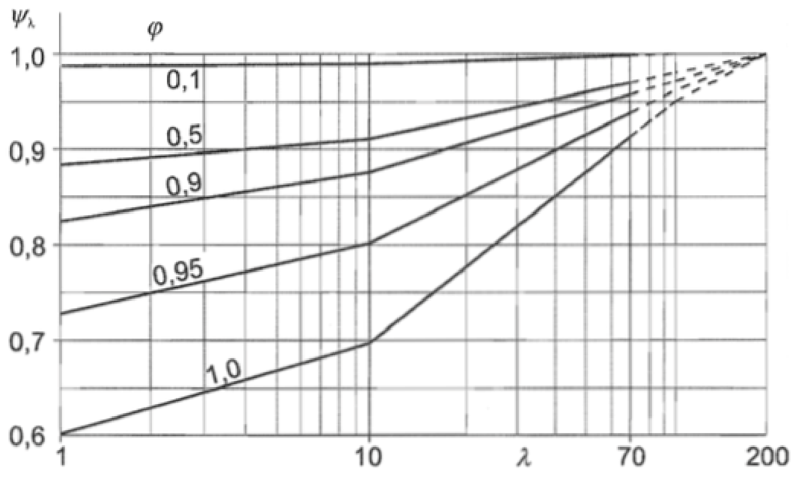
Figure 7.28 — Force coefficient $c_{f,0}$ for circular cylinders without free-end flow and for different equivalent roughness k/b

Table 7.16 — Recommended values of λ for cylinders, polygonal sections, rectangular sections, sharp edged structural sections and lattice structures

No.	Position of the structure, wind normal to the plane of the page	Effective slenderness λ
1		For polygonal, rectangular and sharp edged sections and lattice structures: for $\ell \geq 50$ m, λ =1,4 ℓ/b or λ = 70, whichever is smaller for ℓ <15 m, λ =2 ℓ/b or λ = 70, whichever is smaller For circular cylinders: for $\ell \geq 50$, λ =0,7 ℓ/b or λ =70, whichever is smaller for ℓ <15 m, λ =ℓ/b or λ =70, whichever is smaller For intermediate values of ℓ, linear interpolation should be used
2		
3		
		for $\ell \geq 50$ m, λ =0,7 ℓ/b or λ = 70,

We derive λ by interpolating λ_1 (15m) and λ_2 (50m), since the length of the mast is 30m.

$L_m := 30\ m$ $\quad \lambda_1 := min\left(2 \cdot \dfrac{L_m}{diam}, 70\right) = 70$ $\quad \lambda_2 := min\left(1.4 \cdot \dfrac{L_m}{diam}, 70\right) = 70$ ----> $\lambda := 70$



$\phi := 1$

$\psi_\lambda := 0.925$

Figure 7.36 — Indicative values of the end-effect factor ψ_λ as a function of solidity ratio φ versus slenderness λ

$$c_f := c_{f.0} \cdot \psi_\lambda = 1.11$$

$A_{ref} := diam \cdot \pi \cdot L_m = 56.549\ m^2$

$F_{W.mast} := c_s \cdot c_d \cdot c_f \cdot q_p \cdot A_{ref} = 42.624\ kN$ --------> per unità di lunghezza: $\dfrac{F_{W.mast}}{L_m} = 1.421\ \dfrac{kN}{m}$

Cables

7.9.2 Force coefficients

(1) The force coefficient c_f for a finite circular cylinder should be determined from Expression (7.19).

$$c_f = c_{f,0} \cdot \psi_\lambda \qquad (7.19)$$

where:

$c_{f,0}$ is the force coefficient of cylinders without free-end flow (see Figure 7.28)

ψ_λ is the end-effect factor (see 7.13)

$c_s := 1$ $c_d := 1$ $diam := 0.1\ m$ $\nu := 15 \cdot 10^{-6}\ \dfrac{m^2}{s}$ $z := 15\ m$

$I_v := \dfrac{k_l}{c_0 \cdot \ln\left(\dfrac{z}{z_0}\right)} = 0.175$ $q_p := (1 + 7 \cdot I_v) \cdot \dfrac{1}{2}\ \rho \cdot v_m^{\ 2} = 703.453\ Pa$ $Re := diam \cdot \dfrac{\sqrt{2 \cdot \dfrac{q_p}{\rho}}}{\nu} = 2.237 \cdot 10^5$

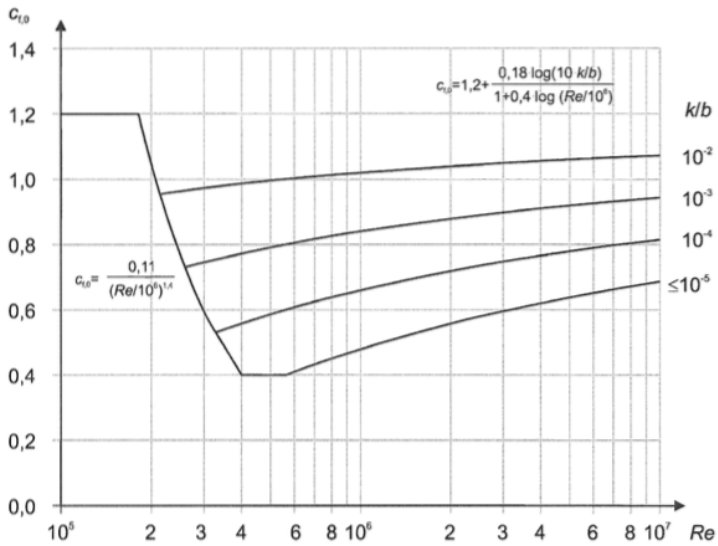
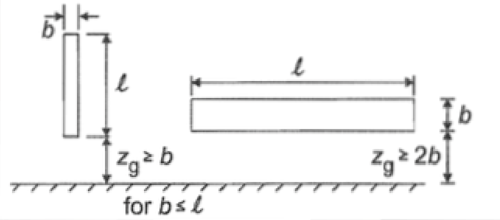
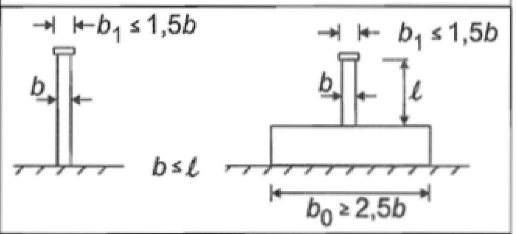
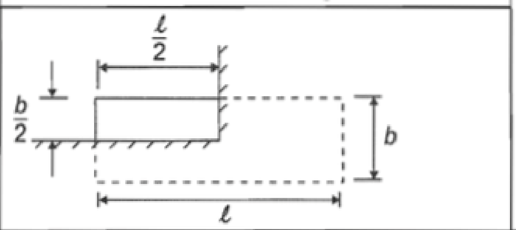


Figure 7.28 — Force coefficient $c_{f,0}$ for circular cylinders without free-end flow and for different equivalent roughness k/b

----> $c_{f.0} := 1.2$

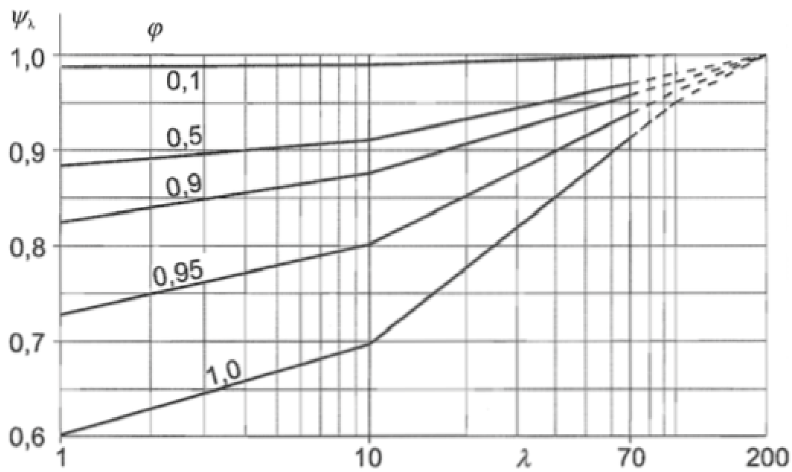
Table 7.16 — Recommended values of λ for cylinders, polygonal sections, rectangular sections

Table 7.16 — Recommended values of λ for cylinders, polygonal sections, rectangular sections, sharp edged structural sections and lattice structures

No.	Position of the structure, wind normal to the plane of the page	Effective slenderness λ
1		For polygonal, rectangular and sharp edged sections and lattice structures: for $\ell \geq 50$ m, $\lambda = 1{,}4\ \ell/b$ or $\lambda = 70$, whichever is smaller
2		for $\ell < 15$ m, $\lambda = 2\ \ell/b$ or $\lambda = 70$, whichever is smaller For circular cylinders: for $\ell \geq 50$, $\lambda = 0{,}7\ \ell/b$ or $\lambda = 70$, whichever is smaller for $\ell < 15$ m, $\lambda = \ell/b$ or $\lambda = 70$, whichever is smaller
3		For intermediate values of ℓ, linear interpolation should be used
		for $\ell \geq 50$ m, $\lambda = 0{,}7\ \ell/b$ or $\lambda = 70$,

We derive λ by interpolating λ_1 (15m) and λ_2 (50m), since the length of the cables lies between those values.

$L_c := 27\ m$ $\quad$ $\lambda_1 := min\left(2 \cdot \frac{L_c}{diam}, 70\right) = 70$ $\quad$ $\lambda_2 := min\left(1.4 \cdot \frac{L_c}{diam}, 70\right) = 70$ $\quad$ ----> $\quad$ $\lambda := 70$



$\phi := 1$

$\psi_\lambda := 0.925$

Figure 7.36 — Indicative values of the end-effect factor ψ_λ as a function of solidity ratio φ versus slenderness λ

$c_f := c_{f.0} \cdot \psi_\lambda = 1.11$

$A_{ref} := diam \cdot \pi \cdot L_c = 8.482\ m^2$

$F_{W.cable} := c_s \cdot c_d \cdot c_f \cdot q_p \cdot A_{ref} = 6.623\ kN$ --------> per linear m: $\frac{F_{W.cable}}{L_c} = 0.245\ \frac{kN}{m}$

Snow loads

A.10.2 **Zona nazionale II**

Regioni: Liguria, Toscana, Umbria, Lazio, Campania (solo le provincie di Caserta, Benevento, Avellino), Puglia (solo la provincia di Foggia):

$s_k = 1{,}15$	kN/m^2	$A \leq 200$ m
$s_k = 1{,}15 + 2{,}6\,(A - 200)/1\,000$	kN/m^2	$200 < A \leq 750$ m
$s_k = 2{,}58 + 8{,}5\,(A - 750)/1\,000$	kN/m^2	$A > 750$ m

$\mu := 0.8$ $C_e := 1$ $C_t := 1$ $s_k := 1.15\ \frac{kN}{m^2}$ $s := \mu \cdot C_e \cdot C_t \cdot s_k = 0.92\ \frac{kN}{m^2}$

A. Deck geometry optimization

Girder restraints: encastres at both ends

Girder radius = 25 m

$$M_\theta := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi \rightarrow M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta - \theta_0) - 1)$$

$$T_\theta := -M_C \cdot \sin(\theta) + T_C \cdot \cos(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi \rightarrow T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta - \theta_0)$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot \cos(\theta) - T_\theta \cdot \sin(\theta)\, d\theta$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot \sin(\theta) + T_\theta \cdot \cos(\theta)\, d\theta$$

$E := 210000\ MPa$ $\quad \nu := 0.3$ $\quad G := \dfrac{E}{2(1+\nu)} = 80769.231\ MPa$ $\quad b := 4\ m$ $\quad h := 2\ m$

$I := b \cdot \dfrac{h^3}{12} = 2.667\ m^4$ $\quad J := \dfrac{1}{3} \cdot (b - 0.53 \cdot h) \cdot h^3 = 7.84\ m^4$ $\quad r := 2500\ cm$ $\quad \theta_0 := 0\ deg$ $\quad \theta_1 := 180\ deg$

$m := \dfrac{(G \cdot J)}{(E \cdot I)} = 1.131$ $\quad t := 10\ kN \cdot \dfrac{m}{m}$

$$C_1 := \frac{(2 \cdot \theta_1 - 2 \cdot \theta_0 + \sin(2 \cdot \theta_0) - \sin(2 \cdot \theta_1))}{4} - m \cdot \left(\frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0)}{4}\right) + m \cdot \left(\frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_1)}{4}\right) = 3.347$$ multiplier of Mc (1 eq.)

$$C_2 := \frac{\left(\sin(\theta_0)^2 - \sin(\theta_1)^2\right)}{2} + \frac{m \cdot \left(\cos(\theta_0)^2 - \cos(\theta_1)^2\right)}{2} = 0$$ multiplier of Tc (1 eq.)

$$C_3 := \left(\frac{\theta_0}{2} - \frac{\sin(2 \cdot \theta_0)}{4}\right) - \left(\frac{\theta_1}{2} - \frac{\sin(2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\sin(\theta_0)}{4} + \frac{\theta_0 \cdot \cos(\theta_0)}{2}\right) - m \cdot (\sin(\theta_0) - \sin(\theta_1)) + \sin(\theta_0) \cdot (\cos(\theta_0) - \cos(\theta_1)) + m \cdot \left(\frac{\sin(\theta_0 - 2 \cdot \theta_1)}{4} - \frac{\theta_1 \cdot \cos(\theta_0)}{2}\right) = -3.347$$
multiplier of tr (1 eq.)

$$C_4 := \frac{-\left(\cos(\theta_0)^2 - \cos(\theta_1)^2\right)}{2} + \frac{m \cdot \left(\cos(\theta_0)^2 - \cos(\theta_1)^2\right)}{2} = 0$$ multiplier of Mc (2 eq.)

$$C_5 := \left(\frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_1)}{4}\right) - \left(\frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0)}{4}\right) + \frac{m \cdot (2 \cdot \theta_1 - 2 \cdot \theta_0 + \sin(2 \cdot \theta_0) - \sin(2 \cdot \theta_1))}{4} = 3.347$$ multiplier of Tc (2 eq.)

$$C_6 := \frac{\left(\cos(\theta_0)^2 - \cos(\theta_1)^2\right)}{2} + \frac{m \cdot (\cos(\theta_0 - 2 \cdot \theta_1) - \cos(\theta_0) + 2 \cdot \theta_0 \cdot \sin(\theta_0) - 2 \cdot \theta_1 \cdot \sin(\theta_0))}{4} + m \cdot (\cos(\theta_0) - \cos(\theta_1)) + \sin(\theta_0) \cdot (\sin(\theta_0) - \sin(\theta_1)) = 2.262$$
multiplier of tr (2 eq.)

Valori ipotizzati

$M_C := 1$ $T_C := 1$

Vincoli

$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 25 \cdot C_3 = 0$

$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 25 \cdot C_6 = 0$

Solutore

$\textbf{Find}\left(M_C, T_C\right) = \begin{bmatrix} 250 \\ -168.923 \end{bmatrix}$

$M_C := 250\ (kN \cdot m)$ $T_C := -168.923\ (kN \cdot m)$

$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot \left(\cos\left(\theta - \theta_0\right) - 1\right)$

$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin\left(\theta - \theta_0\right)$

$M_\theta(0) = 250\ kN \cdot m$ $T_\theta(0) = -168.923\ kN \cdot m$

$M_\theta(10°) = 220.667\ kN \cdot m$ $T_\theta(10°) = -166.357\ kN \cdot m$

$M_\theta(20°) = 192.225\ kN \cdot m$ $T_\theta(20°) = -158.736\ kN \cdot m$

$M_\theta(30°) = 165.539\ kN \cdot m$ $T_\theta(30°) = -146.292\ kN \cdot m$

$M_\theta(40°) = 141.418\ kN \cdot m$ $T_\theta(40°) = -129.403\ kN \cdot m$

$M_\theta(50°) = 120.597\ kN \cdot m$ $T_\theta(50°) = -108.582\ kN \cdot m$

$M_\theta(60°) = 103.708\ kN \cdot m$ $T_\theta(60°) = -84.462\ kN \cdot m$

$M_\theta(70°) = 91.264\ kN \cdot m$ $T_\theta(70°) = -57.775\ kN \cdot m$

$M_\theta(80°) = 83.643\ kN \cdot m$ $T_\theta(80°) = -29.333\ kN \cdot m$

$M_\theta(90°) = 81.077\ kN \cdot m$ $T_\theta(90°) = 0\ kN \cdot m$

$M_\theta(100°) = 83.643\ kN \cdot m$ $T_\theta(100°) = 29.333\ kN \cdot m$

$M_\theta(110°) = 91.264\ kN \cdot m$ $T_\theta(110°) = 57.775\ kN \cdot m$

$M_\theta(120°) = 103.708\ kN \cdot m$ $T_\theta(120°) = 84.461\ kN \cdot m$

$M_\theta(130°) = 120.597\ kN \cdot m$ $T_\theta(130°) = 108.582\ kN \cdot m$

$M_\theta(140°) = 141.418\ kN \cdot m$ $T_\theta(140°) = 129.403\ kN \cdot m$

$M_\theta(150°) = 165.539\ kN \cdot m$ $T_\theta(150°) = 146.292\ kN \cdot m$

$M_\theta\langle 160°\rangle = 192.225\ kN \cdot m$ $T_\theta\langle 160°\rangle = 158.736\ kN \cdot m$

$M_\theta\langle 170°\rangle = 220.667\ kN \cdot m$ $T_\theta\langle 170°\rangle = 166.357\ kN \cdot m$

$M_\theta\langle 180°\rangle = 250\ kN \cdot m$ $T_\theta\langle 180°\rangle = 168.923\ kN \cdot m$

Girder radius = 26 m

$$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) + \int_0^\theta t \cdot r \cdot \sin(\theta - \phi)\, d\phi$$

$\theta_0 := 16\ deg$ $\theta_1 := 164\ deg$

$$T_\theta(\theta) := -M_C \cdot \sin(\theta) + T_C \cdot \cos(\theta) + \int_0^\theta t \cdot r \cdot \cos(\theta - \phi)\, d\phi$$

$$\int_0^{\theta_1 - \theta_0} \left(m \cdot M_\theta \cdot \cos(\theta) - T_\theta \cdot \sin(\theta)\right) d\theta$$

$$\int_0^{\theta_1 - \theta_0} \left(m \cdot M_\theta \cdot \sin(\theta) + T_\theta \cdot \cos(\theta)\right) d\theta$$

$E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2(1+\nu)} = 80769.231\ MPa$ $b := 4\ m$ $h := 2\ m$

$I := b \cdot \frac{h^3}{12} = 4.36$ $J := \frac{1}{3} \cdot (b - 0.53 \cdot h) \cdot h^3 = 12.818$ $r := 2600\ cm$

$m := \frac{(G \cdot J)}{(E \cdot I)} = 1.131$ $t := 10\ kN \cdot \frac{m}{m}$

$$C_1 := \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.723$$

$$C_2 := -\frac{\sin(\theta_0 - \theta_1)^2}{2} + \frac{m \cdot \sin(\theta_0 - \theta_1)^2}{2} = 0.018$$

$$C_3 := -\left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \sin(\theta_0 - \theta_1) = -2.123$$

$$C_4 := \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - \frac{m \cdot \left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 0.018$$

$$C_5 := -\left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.781$$

$$C_6 := -\frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - m \cdot (\cos(\theta_0 - \theta_1) - 1) + m \cdot \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 2.071$$

Valori ipotizzati

$M_C := 1$ $T_C := 1$

Vincoli

$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 26 \cdot C_3 = 0$

$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 26 \cdot C_6 = 0$

Solutore

$\mathbf{Find}\left(M_C, T_C\right) = \begin{bmatrix} 204.092 \\ -194.976 \end{bmatrix}$

$M_C := 204.092\ (kN \cdot m)$ $T_C := -194.976\ (kN \cdot m)$

$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$

$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$

$M_\theta\left(0°\right) = 204.092\ kN \cdot m$	$T_\theta\left(0°\right) = -194.976\ kN \cdot m$
$M_\theta\left(20° - \theta_0\right) = 190.627\ kN \cdot m$	$T_\theta\left(20° - \theta_0\right) = -190.601\ kN \cdot m$
$M_\theta\left(30° - \theta_0\right) = 158.584\ kN \cdot m$	$T_\theta\left(30° - \theta_0\right) = -175.659\ kN \cdot m$
$M_\theta\left(40° - \theta_0\right) = 129.622\ kN \cdot m$	$T_\theta\left(40° - \theta_0\right) = -155.38\ kN \cdot m$
$M_\theta\left(50° - \theta_0\right) = 104.621\ kN \cdot m$	$T_\theta\left(50° - \theta_0\right) = -130.379\ kN \cdot m$
$M_\theta\left(60° - \theta_0\right) = 84.341\ kN \cdot m$	$T_\theta\left(60° - \theta_0\right) = -101.417\ kN \cdot m$
$M_\theta\left(70° - \theta_0\right) = 69.399\ kN \cdot m$	$T_\theta\left(70° - \theta_0\right) = -69.373\ kN \cdot m$
$M_\theta\left(80° - \theta_0\right) = 60.248\ kN \cdot m$	$T_\theta\left(80° - \theta_0\right) = -35.222\ kN \cdot m$
$M_\theta\left(90° - \theta_0\right) = 57.167\ kN \cdot m$	$T_\theta\left(90° - \theta_0\right) = 0\ kN \cdot m$
$M_\theta\left(100° - \theta_0\right) = 60.248\ kN \cdot m$	$T_\theta\left(100° - \theta_0\right) = 35.221\ kN \cdot m$
$M_\theta\left(110° - \theta_0\right) = 69.399\ kN \cdot m$	$T_\theta\left(110° - \theta_0\right) = 69.373\ kN \cdot m$
$M_\theta\left(120° - \theta_0\right) = 84.341\ kN \cdot m$	$T_\theta\left(120° - \theta_0\right) = 101.416\ kN \cdot m$
$M_\theta\left(130° - \theta_0\right) = 104.62\ kN \cdot m$	$T_\theta\left(130° - \theta_0\right) = 130.378\ kN \cdot m$
$M_\theta\left(140° - \theta_0\right) = 129.621\ kN \cdot m$	$T_\theta\left(140° - \theta_0\right) = 155.379\ kN \cdot m$
$M_\theta\left(150° - \theta_0\right) = 158.583\ kN \cdot m$	$T_\theta\left(150° - \theta_0\right) = 175.659\ kN \cdot m$
$M_\theta\left(160° - \theta_0\right) = 190.627\ kN \cdot m$	$T_\theta\left(160° - \theta_0\right) = 190.601\ kN \cdot m$

$M_\theta\left(\theta_1-\theta_0\right)=204.091\ kN\cdot m$

$T_\theta\left(\theta_1-\theta_0\right)=194.976\ kN\cdot m$

Girder radius = 27 m

$r:=2700\ cm$

$$M_\theta:=M_C\cdot cos(\theta)+T_C\cdot sin(\theta)+\int_{\theta_0}^{\theta} t\cdot r\cdot sin(\theta-\phi)\,d\phi$$

$$T_\theta:=-M_C\cdot sin(\theta)+T_C\cdot cos(\theta)+\int_{\theta_0}^{\theta} t\cdot r\cdot cos(\theta-\phi)\,d\phi$$

$$\int_{\theta_0}^{\theta_1} m\cdot M_\theta\cdot cos(\theta)-T_\theta\cdot sin(\theta)\,d\theta$$

$$\int_{\theta_0}^{\theta_1} m\cdot M_\theta\cdot sin(\theta)+T_\theta\cdot cos(\theta)\,d\theta$$

$E:=210000\ MPa$ $\quad \nu:=0.3$ $\quad G:=\dfrac{E}{2\,(1+\nu)}=80769.231\ MPa$ $\quad b:=4\ m$ $\quad h:=2\ m$

$I:=b\cdot\dfrac{h^3}{12}=4.36$ $\quad J:=\dfrac{1}{3}\cdot(b-0.53\cdot h)\cdot h^3=12.818$ $\quad \theta_0:=22\ deg$ $\quad \theta_1:=158\ deg$

$m:=\dfrac{(G\cdot J)}{(E\cdot I)}=1.131$ $\quad t:=10\ kN\cdot\dfrac{m}{m}$

$$C_1:=\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)-m\cdot\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)=2.496$$

$$C_2:=-\frac{\sin(\theta_0-\theta_1)^2}{2}+\frac{m\cdot\sin(\theta_0-\theta_1)^2}{2}=0.032$$

$$C_3:=-\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)+m\cdot\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)-m\cdot\sin(\theta_0-\theta_1)=-1.711$$

$$C_4:=\frac{\left(\cos(\theta_0-\theta_1)^2-1\right)}{2}-\frac{m\cdot\left(\cos(\theta_0-\theta_1)^2-1\right)}{2}=0.032$$

$$C_5:=-\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)+m\cdot\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin(2\cdot\theta_0-2\cdot\theta_1)}{4}\right)=2.562$$

$$C_6:=-\frac{\left(\cos(\theta_0-\theta_1)^2-1\right)}{2}-m\cdot\left(\cos(\theta_0-\theta_1)-1\right)+m\cdot\frac{\left(\cos(\theta_0-\theta_1)^2-1\right)}{2}=1.913$$

Valori ipotizzati

$M_C := 1$ $T_C := 1$

Vincoli

$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 27 \cdot C_3 = 0$

$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 27 \cdot C_6 = 0$

Solutore

$\textbf{Find}\left(M_C, \mathrm{T_C}\right) = \begin{bmatrix} 187.614 \\ -203.913 \end{bmatrix}$

$M_C := 187.614\ (kN \cdot m)$ $T_C := -203.913\ (kN \cdot m)$

$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$

$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$

$M_\theta(0°) = 187.614\ kN \cdot m$ $T_\theta(0°) = -203.913\ kN \cdot m$

$M_\theta(30° - \theta_0) = 160.037\ kN \cdot m$ $T_\theta(30° - \theta_0) = -190.463\ kN \cdot m$

$M_\theta(40° - \theta_0) = 128.634\ kN \cdot m$ $T_\theta(40° - \theta_0) = -168.474\ kN \cdot m$

$M_\theta(50° - \theta_0) = 101.526\ kN \cdot m$ $T_\theta(50° - \theta_0) = -141.367\ kN \cdot m$

$M_\theta(60° - \theta_0) = 79.538\ kN \cdot m$ $T_\theta(60° - \theta_0) = -109.964\ kN \cdot m$

$M_\theta(70° - \theta_0) = 63.336\ kN \cdot m$ $T_\theta(70° - \theta_0) = -75.22\ kN \cdot m$

$M_\theta(80° - \theta_0) = 53.414\ kN \cdot m$ $T_\theta(80° - \theta_0) = -38.19\ kN \cdot m$

$M_\theta(90° - \theta_0) = 50.073\ kN \cdot m$ $T_\theta(90° - \theta_0) = 0\ kN \cdot m$

$M_\theta(100° - \theta_0) = 53.414\ kN \cdot m$ $T_\theta(100° - \theta_0) = 38.19\ kN \cdot m$

$M_\theta(110° - \theta_0) = 63.336\ kN \cdot m$ $T_\theta(110° - \theta_0) = 75.219\ kN \cdot m$

$M_\theta(120° - \theta_0) = 79.537\ kN \cdot m$ $T_\theta(120° - \theta_0) = 109.963\ kN \cdot m$

$M_\theta(130° - \theta_0) = 101.526\ kN \cdot m$ $T_\theta(130° - \theta_0) = 141.366\ kN \cdot m$

$M_\theta(140° - \theta_0) = 128.633\ kN \cdot m$ $T_\theta(140° - \theta_0) = 168.474\ kN \cdot m$

$M_\theta(150° - \theta_0) = 160.036\ kN \cdot m$ $T_\theta(150° - \theta_0) = 190.462\ kN \cdot m$

$M_\theta\left(\theta_1-\theta_0\right)=187.614\ kN\cdot m$ $\quad T_\theta\left(\theta_1-\theta_0\right)=203.913\ kN\cdot m$

Girder radius = 28 m $\quad r:=2800\ cm$

$$M_\theta:=M_C\cdot cos(\theta)+T_C\cdot sin(\theta)+\int_{\theta_0}^{\theta} t\cdot r\cdot sin(\theta-\phi)\,d\phi$$

$$T_\theta:=-M_C\cdot sin(\theta)+T_C\cdot cos(\theta)+\int_{\theta_0}^{\theta} t\cdot r\cdot cos(\theta-\phi)\,d\phi$$

$$\int_{\theta_0}^{\theta_1} m\cdot M_\theta\cdot cos(\theta)-T_\theta\cdot sin(\theta)\,d\theta$$

$$\int_{\theta_0}^{\theta_1} m\cdot M_\theta\cdot sin(\theta)+T_\theta\cdot cos(\theta)\,d\theta$$

$E:=210000\ MPa$ $\quad \nu:=0.3$ $\quad G:=\frac{E}{2(1+\nu)}=80769.231\ MPa$ $\quad b:=4\ m$ $\quad h:=2\ m$

$I:=b\cdot\frac{h^3}{12}=4.36$ $\quad J:=\frac{1}{3}\cdot(b-0.53\cdot h)\cdot h^3=12.818$ $\quad \theta_0:=27\ deg$ $\quad \theta_1:=153\ deg$

$m:=\frac{(G\cdot J)}{(E\cdot I)}=1.131$ $\quad t:=10\ kN\cdot\frac{m}{m}$

$$C_1:=\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)-m\cdot\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)=2.312$$

$$C_2:=-\frac{\sin\left(\theta_0-\theta_1\right)^2}{2}+\frac{m\cdot\sin\left(\theta_0-\theta_1\right)^2}{2}=0.043$$

$$C_3:=-\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)+m\cdot\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)-m\cdot\sin\left(\theta_0-\theta_1\right)=-1.397$$

$$C_4:=\frac{\left(\cos\left(\theta_0-\theta_1\right)^2-1\right)}{2}-\frac{m\cdot\left(\cos\left(\theta_0-\theta_1\right)^2-1\right)}{2}=0.043$$

$$C_5:=-\left(\frac{\theta_0}{2}-\frac{\theta_1}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)+m\cdot\left(\frac{\theta_1}{2}-\frac{\theta_0}{2}+\frac{\sin\left(2\cdot\theta_0-2\cdot\theta_1\right)}{4}\right)=2.374$$

$$C_6:=-\frac{\left(\cos\left(\theta_0-\theta_1\right)^2-1\right)}{2}-m\cdot\left(\cos\left(\theta_0-\theta_1\right)-1\right)+m\cdot\frac{\left(\cos\left(\theta_0-\theta_1\right)^2-1\right)}{2}=1.753$$

Valori ipotizzati

$M_C := 1$ $T_C := 1$

Vincoli

$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 28 \cdot C_3 = 0$

$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 28 \cdot C_6 = 0$

Solutore

$\mathbf{Find}\left(M_C, \mathrm{T_C}\right) = \begin{bmatrix} 173.085 \\ -209.833 \end{bmatrix}$

$M_C := 173.085\ (kN \cdot m)$ $T_C := -209.833\ (kN \cdot m)$

$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$

$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$

$M_\theta(30° - \theta_0) = 162.25\ kN \cdot m$ $T_\theta(30° - \theta_0) = -203.95\ kN \cdot m$

$M_\theta(33.5° - \theta_0) = 150.018\ kN \cdot m$ $T_\theta(33.5° - \theta_0) = -196.381\ kN \cdot m$

$M_\theta(40° - \theta_0) = 128.623\ kN \cdot m$ $T_\theta(40° - \theta_0) = -180.404\ kN \cdot m$

$M_\theta(50° - \theta_0) = 99.596\ kN \cdot m$ $T_\theta(50° - \theta_0) = -151.377\ kN \cdot m$

$M_\theta(60° - \theta_0) = 76.05\ kN \cdot m$ $T_\theta(60° - \theta_0) = -117.751\ kN \cdot m$

$M_\theta(70° - \theta_0) = 58.702\ kN \cdot m$ $T_\theta(70° - \theta_0) = -80.546\ kN \cdot m$

$M_\theta(80° - \theta_0) = 48.077\ kN \cdot m$ $T_\theta(80° - \theta_0) = -40.895\ kN \cdot m$

$M_\theta(90° - \theta_0) = 44.499\ kN \cdot m$ $T_\theta(90° - \theta_0) = 0\ kN \cdot m$

$M_\theta(100° - \theta_0) = 48.077\ kN \cdot m$ $T_\theta(100 - \theta_0) = -203.077\ kN \cdot m$

$M_\theta(110° - \theta_0) = 58.701\ kN \cdot m$ $T_\theta(110° - \theta_0) = 80.546\ kN \cdot m$

$M_\theta(120° - \theta_0) = 76.05\ kN \cdot m$ $T_\theta(120° - \theta_0) = 117.75\ kN \cdot m$

$M_\theta(130° - \theta_0) = 99.596\ kN \cdot m$ $T_\theta(130° - \theta_0) = 151.377\ kN \cdot m$

$M_\theta(140° - \theta_0) = 128.623\ kN \cdot m$ $T_\theta(140° - \theta_0) = 180.404\ kN \cdot m$

$M_\theta(150° - \theta_0) = 162.249\ kN \cdot m$ $T_\theta(150° - \theta_0) = 203.95\ kN \cdot m$

$M_\theta(\theta_1 - \theta_0) = 173.085\ kN \cdot m$ $T_\theta(\theta_1 - \theta_0) = 209.833\ kN \cdot m$

Girder radius = 29 m

$r := 2900\ cm$

$$M_\theta := M_C \cdot cos(\theta) + T_C \cdot sin(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot sin(\theta - \phi)\, d\phi$$

$$T_\theta := -M_C \cdot sin(\theta) + T_C \cdot cos(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot cos(\theta - \phi)\, d\phi$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot cos(\theta) - T_\theta \cdot sin(\theta)\, d\theta$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot sin(\theta) + T_\theta \cdot cos(\theta)\, d\theta$$

$E := 210000\ MPa$ $\qquad \nu := 0.3$ $\qquad G := \dfrac{E}{2(1+\nu)} = 80769.231\ MPa$ $\qquad b := 4\ m$

$I := b \cdot \dfrac{h^3}{12} = 4.36$ $\qquad J := \dfrac{1}{3} \cdot (b - 0.53 \cdot h) \cdot h^3 = 12.818$ $\qquad \theta_0 := 30.5\ deg$ $\qquad \theta_1 := 149.5\ deg$

$m := \dfrac{(G \cdot J)}{(E \cdot I)} = 1.131$ $\qquad t := 10\ kN \cdot \dfrac{m}{m}$

$$C_1 := \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.185$$

$$C_2 := -\frac{\sin(\theta_0 - \theta_1)^2}{2} + \frac{m \cdot \sin(\theta_0 - \theta_1)^2}{2} = 0.05$$

$$C_3 := -\left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \sin(\theta_0 - \theta_1) = -1.196$$

$$C_4 := \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - \frac{m \cdot \left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 0.05$$

$$C_5 := -\left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.24$$

$$C_6 := -\frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - m \cdot (\cos(\theta_0 - \theta_1) - 1) + m \cdot \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 1.629$$

alori ipotizzati

$M_C := 1$ $\qquad T_C := 1$

$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 29 \cdot C_3 = 0$

$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 29 \cdot C_6 = 0$

$\mathbf{Find}\left(M_C, \mathrm{T_C}\right) = \begin{bmatrix} 163.649 \\ -214.502 \end{bmatrix}$

$M_C := 163.649\ (kN \cdot m)$ $\qquad$ $T_C := -214.502\ (kN \cdot m)$

$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$

$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$

$M_\theta(0°) = 163.649\ kN \cdot m$	$T_\theta(0°) = -214.502\ kN \cdot m$
$M_\theta(33.5° - \theta_0) = 152.596\ kN \cdot m$	$T_\theta(35.5° - \theta_0) = -202.674\ kN \cdot m$
$M_\theta(40° - \theta_0) = 129.979\ kN \cdot m$	$T_\theta(40° - \theta_0) = -190.706\ kN \cdot m$
$M_\theta(50° - \theta_0) = 99.294\ kN \cdot m$	$T_\theta(50° - \theta_0) = -160.022\ kN \cdot m$
$M_\theta(60° - \theta_0) = 74.404\ kN \cdot m$	$T_\theta(60° - \theta_0) = -124.475\ kN \cdot m$
$M_\theta(70° - \theta_0) = 56.064\ kN \cdot m$	$T_\theta(70° - \theta_0) = -85.146\ kN \cdot m$
$M_\theta(80° - \theta_0) = 44.833\ kN \cdot m$	$T_\theta(80° - \theta_0) = -43.23\ kN \cdot m$
$M_\theta(90° - \theta_0) = 41.051\ kN \cdot m$	$T_\theta(90° - \theta_0) = 0\ kN \cdot m$
$M_\theta(\theta_1 - \theta_0) = 163.649\ kN \cdot m$	$T_\theta(\theta_1 - \theta_0) = 214.502\ kN \cdot m$

Girder radius = 30 m $\qquad$ $r := 3000\ cm$

$$M_\theta := M_C \cdot cos(\theta) + T_C \cdot sin(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot sin(\theta - \phi)\, \mathrm{d}\phi$$

$$T_\theta := -M_C \cdot sin(\theta) + T_C \cdot cos(\theta) + \int_{\theta_0}^{\theta} t \cdot r \cdot cos(\theta - \phi)\, \mathrm{d}\phi$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot cos(\theta) - T_\theta \cdot sin(\theta)\, \mathrm{d}\theta$$

$$\int_{\theta_0}^{\theta_1} m \cdot M_\theta \cdot sin(\theta) + T_\theta \cdot cos(\theta)\, \mathrm{d}\theta$$

$E := 210000\ MPa$ $\nu := 0.3$ $G := \frac{E}{2\ (1+\nu)} = 80769.231\ MPa$ $b := 4\ m$

$I := b \cdot \frac{h^3}{12} = 4.36$ $J := \frac{1}{3} \cdot (b - 0.53 \cdot h) \cdot h^3 = 12.818$

$\theta_0 := 33.5\ deg$ $\theta_1 := 146.5\ deg$

$m := \frac{(G \cdot J)}{(E \cdot I)} = 1.131$ $t := 10\ kN \cdot \frac{m}{m}$

$$C_1 := \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.078$$

$$C_2 := -\frac{\sin(\theta_0 - \theta_1)^2}{2} + \frac{m \cdot \sin(\theta_0 - \theta_1)^2}{2} = 0.055$$

$$C_3 := -\left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) - m \cdot \sin(\theta_0 - \theta_1) = -1.037$$

$$C_4 := \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - \frac{m \cdot \left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 0.055$$

$$C_5 := -\left(\frac{\theta_0}{2} - \frac{\theta_1}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) + m \cdot \left(\frac{\theta_1}{2} - \frac{\theta_0}{2} + \frac{\sin(2 \cdot \theta_0 - 2 \cdot \theta_1)}{4}\right) = 2.125$$

$$C_6 := -\frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} - m \cdot (\cos(\theta_0 - \theta_1) - 1) + m \cdot \frac{\left(\cos(\theta_0 - \theta_1)^2 - 1\right)}{2} = 1.517$$

Valori ipotizzati

$M_C := 1$ $T_C := 1$

Vincoli

$$M_C \cdot C_1 + T_C \cdot C_2 + 10 \cdot 30 \cdot C_3 = 0$$

$$M_C \cdot C_4 + T_C \cdot C_5 + 10 \cdot 30 \cdot C_6 = 0$$

Solutore

$$\mathbf{Find}\ (M_C, T_C) = \begin{bmatrix} 155.525 \\ -218.278 \end{bmatrix}$$

$M_C := 155.525\ (kN \cdot m)$ $T_C := -218.278\ (kN \cdot m)$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$$

$$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$$

$M_\theta\,(0°) = 155.525\ kN \cdot m$ $T_\theta\,(0) = -218.278\ kN \cdot m$

$M_\theta\,(40° - \theta_0) = 131.744\ kN \cdot m$ $T_\theta\,(40° - \theta_0) = -200.52\ kN \cdot m$

$M_\theta\,(50° - \theta_0) = 99.48\ kN \cdot m$ $T_\theta\,(50° - \theta_0) = -168.256\ kN \cdot m$

$M_\theta\,(60° - \theta_0) = 73.309\ kN \cdot m$ $T_\theta\,(60° - \theta_0) = -130.88\ kN \cdot m$

$M_\theta\,(70° - \theta_0) = 54.026\ kN \cdot m$ $T_\theta\,(70° - \theta_0) = -89.527\ kN \cdot m$

$M_\theta\,(80° - \theta_0) = 42.217\ kN \cdot m$ $T_\theta\,(80° - \theta_0) = -45.454\ kN \cdot m$

$M_\theta\,(90° - \theta_0) = 38.24\ kN \cdot m$ $T_\theta\,(90° - \theta_0) = 0\ kN \cdot m$

Girder restraints: vertical support at each cable-deck joint

Distributed torque: $t := 10\ kN$

Girder radius: $r := 2500\ cm$

Number of vertical pins: $n := 4$

Pin relative angular distance : $\beta := \dfrac{\pi}{(n-1)} \cdot rad = 1.047$

Initial and final angles: $\theta_0 := 0\ rad$ $\theta_1 := \pi\ rad$

Girder angular length: $\alpha := (\theta_1 - \theta_0) = 3.142$

Girder center of gravity: $y_G := r \cdot \left(\dfrac{\sin\left(\dfrac{\alpha}{2}\right)}{\dfrac{\alpha}{2}} \right) = 15.915\ m$ s

$$d := r \cdot \left(\sin(\beta) - \dfrac{\sin\left(\dfrac{\alpha}{2}\right)}{\dfrac{\alpha}{2}} \right) = 5.735\ m$$

Vertical reactions (1,right end support, 4 left end support):

$$R_1 := \dfrac{-\int_{\theta_0}^{\theta_1} t \cdot r\, \mathrm{d}\theta}{2\,(y_G + d)} = -18.138\ kN \qquad R_4 := R_1 = -18.138\ kN$$

(Reaction>0 if upwards)

$$R_2 := -R_1 = 18.138\ kN \qquad R_3 := R_2 = 18.138\ kN$$

0< θ < β (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, \mathrm{d}\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, \mathrm{d}\phi$$

$\beta < \theta < \frac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) - R_2 \cdot r \cdot \sin(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, d\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta - \theta_0)) + R_2 \cdot r \cdot (1 - \cos(\theta - \beta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, d\phi$$

$M_{\theta 1}(0) = 0 \; kN \cdot m$ — $T_{\theta 1}(0) = 0 \; kN \cdot m$

$M_{\theta 1}(10°) = 82.539 \; kN \cdot m$ — $T_{\theta 1}(10°) = 36.523 \; kN \cdot m$

$M_{\theta 1}(20°) = 170.166 \; kN \cdot m$ — $T_{\theta 1}(20°) = 58.159 \; kN \cdot m$

$M_{\theta 1}(30°) = 260.219 \; kN \cdot m$ — $T_{\theta 1}(30°) = 64.249 \; kN \cdot m$

$M_{\theta 1}(40°) = 349.961 \; kN \cdot m$ — $T_{\theta 1}(40°) = 54.61 \; kN \cdot m$

$M_{\theta 1}(50°) = 436.666 \; kN \cdot m$ — $T_{\theta 1}(50°) = 29.533 \; kN \cdot m$

$M_{\theta 1}(60°) = 517.699 \; kN \cdot m$ — $T_{\theta 1}(60°) = -10.219 \; kN \cdot m$

$M_{\theta 2}(70°) = 511.858 \; kN \cdot m$ — $T_{\theta 2}(70°) = -56.549 \; kN \cdot m$

$M_{\theta 2}(80°) = 498.06 \; kN \cdot m$ — $T_{\theta 2}(80°) = -101.161 \; kN \cdot m$

$M_{\theta 2}(90°) = 476.725 \; kN \cdot m$ — $T_{\theta 2}(90°) = -142.699 \; kN \cdot m$

Girder radius: $r := 2600 \; cm$

Initial and final angles: $\theta_0 := 16 \; deg$ $\quad \theta_1 := 164 \; deg$

Pin relative angular distance : $\beta := \frac{\theta_1 - \theta_0}{(n-1)} \cdot rad = 0.861$ $\quad \beta \cdot \frac{180}{\pi} = 49.333$

Girder angular length: $\alpha := (\theta_1 - \theta_0) = 2.583$

Girder center of gravity: $y_G := r \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) - r \cdot \sin(\theta_0) = 12.185 \; m$

$$d := r \cdot \left(\sin(\beta + \theta_0) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) = 4.276 \; m$$

Vertical reactions (1,right end support, 4 left end support):

$$R_1 := \frac{-\int_{\theta_0}^{\theta_1} t \cdot r \, d\theta}{2\,(y_G + d)} = -20.4\ kN \qquad R_4 := R_1 = -20.4\ kN$$

(Reaction>0 if upwards)

$$R_2 := -R_1 = 20.4\ kN \qquad R_3 := R_2 = 20.4\ kN$$

$\theta_0 < \theta < \beta$ (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi$$

$\beta < \theta < \frac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) - R_2 \cdot r \cdot \sin(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta)) + R_2 \cdot r \cdot (1 - \cos(\theta - \beta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi$$

$M_{\theta 1}(\theta_0) = 0\ kN \cdot m$ | $T_{\theta 1}(\theta_0) = 0\ kN \cdot m$

$M_{\theta 1}(20°) = 37.632\ kN \cdot m$ | $T_{\theta 1}(20°) = 16.845\ kN \cdot m$

$M_{\theta 1}(30°) = 136.038\ kN \cdot m$ | $T_{\theta 1}(30°) = 47.145\ kN \cdot m$

$M_{\theta 1}(40°) = 238.21\ kN \cdot m$ | $T_{\theta 1}(40°) = 59.896\ kN \cdot m$

$M_{\theta 1}(50°) = 341.044\ kN \cdot m$ | $T_{\theta 1}(50°) = 54.712\ kN \cdot m$

$M_{\theta 1}(60°) = 441.416\ kN \cdot m$ | $T_{\theta 1}(60°) = 31.75\ kN \cdot m$

$M_{\theta 2}(70°) = 493.123\ kN \cdot m$ | $T_{\theta 2}(70°) = -136.887\ kN \cdot m$

$M_{\theta 2}(80°) = 488.447\ kN \cdot m$ | $T_{\theta 2}(80°) = -187.325\ kN \cdot m$

$M_{\theta 2}(90°) = 476.829\ kN \cdot m$ | $T_{\theta 2}(90°) = -232.07\ kN \cdot m$

Girder radius: $r := 2700\ cm$

Initial and final angles: $\theta_0 := 22\ deg$ $\qquad \theta_1 := 158\ deg$

Pin relative angular distance : $\beta := \frac{\theta_1 - \theta_0}{(n-1)} \cdot rad = 0.791$

Girder angular length: $\alpha := (\theta_1 - \theta_0) = 2.374$

Girder center of gravity:

$$y_G := r \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) - r \cdot \sin\left(\theta_0\right) = 10.979\ m$$

$$d := r \cdot \left(\sin(\beta) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) = -1.891\ m$$

Vertical reactions (1,right end support, 4 left end support):

$$R_1 := \frac{-\int_{\theta_0}^{\theta_1} t \cdot r \, d\theta}{2\left(y_G + d\right)} = -35.259\ kN \qquad R_4 := R_1 = -35.259\ kN$$

(Reaction>0 if upwards)

$$R_2 := -R_1 = 35.259\ kN \qquad R_3 := R_2 = 35.259\ kN$$

$\theta_0 < \theta < \beta$ (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin\left(\theta - \theta_0\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, d\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot \left(1 - \cos\left(\theta - \theta_0\right)\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, d\phi$$

$\beta < \theta < \frac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin\left(\theta - \theta_0\right) - R_2 \cdot r \cdot \sin\left(\theta - \beta - \theta_0\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, d\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta)) + R_2 \cdot r \cdot \left(1 - \cos\left(\theta - \beta - \theta_0\right)\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, d\phi$$

$M_{\theta 1}\left(\theta_0\right) = 0\ kN \cdot m$ $\qquad$ $T_{\theta 1}\left(\theta_0\right) = 0\ kN \cdot m$

$M_{\theta 1}(30°) = 135.119\ kN \cdot m$ $\qquad$ $T_{\theta 1}(30°) = 28.312\ kN \cdot m$

$M_{\theta 1}(40°) = 307.397\ kN \cdot m$ $\qquad$ $T_{\theta 1}(40°) = 36.841\ kN \cdot m$

$M_{\theta 1}(50°) = 478.537\ kN \cdot m$ $\qquad$ $T_{\theta 1}(50°) = 15.324\ kN \cdot m$

$M_{\theta 1}(60°) = 643.342\ kN \cdot m$ $\qquad$ $T_{\theta 1}(60°) = -35.584\ kN \cdot m$

$M_{\theta 2}(70°) = 752.511\ kN \cdot m$ $\qquad$ $T_{\theta 2}(70°) = -424.712\ kN \cdot m$

$M_{\theta 2}(80°) = 725.505\ kN \cdot m$ $\qquad$ $T_{\theta 2}(80°) = -534.538\ kN \cdot m$

$M_{\theta 2}(90°) = 684.659\ kN \cdot m$ $\qquad$ $T_{\theta 2}(90°) = -628.123\ kN \cdot m$

Girder radius: $r := 2800\ cm$

Initial and final angles: $\theta_0 := 27\ deg$ $\theta_1 := 153\ deg$

Pin relative angular distance : $\beta := \frac{\theta_1 - \theta_0}{(n-1)} \cdot rad = 0.733$

Girder angular length: $\alpha := (\theta_1 - \theta_0) = 2.199$

Girder center of gravity: $y_G := r \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) - r \cdot \sin(\theta_0) = 9.978\ m$

$$d := r \cdot \left(\sin(\beta) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) = -3.954\ m$$

Vertical reactions (1,right end support, 4 left end support): $R_1 := \frac{-\int_{\theta_0}^{\theta_1} t \cdot r\, d\theta}{2\,(y_G + d)} = -51.109\ kN$ $R_4 := R_1 = -51.109\ kN$

(Reaction>0 if upwards)

$R_2 := -R_1 = 51.109\ kN$ $R_3 := R_2 = 51.109\ kN$

$\theta_0 < \theta < \beta$ (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi$$

$\beta < \theta < \frac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) - R_2 \cdot r \cdot \sin(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi)\, d\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta)) + R_2 \cdot r \cdot (1 - \cos(\theta - \beta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi)\, d\phi$$

$M_{\theta 1}(\theta_0) = 0\ kN \cdot m$ $T_{\theta 1}(\theta_0) = 0\ kN \cdot m$ $(\theta_0 + \beta) \cdot \frac{180}{\pi} = 69$

$M_{\theta 1}(30°) = 75.279\ kN \cdot m$ $T_{\theta 1}(30°) = 12.693\ kN \cdot m$

$M_{\theta 1}(40°) = 329.092\ kN \cdot m$ $T_{\theta 1}(40°) = 26.309\ kN \cdot m$

$M_{\theta 1}(50°) = 581.414\ kN \cdot m$ $T_{\theta 1}(50°) = -4.357\ kN \cdot m$

$M_{\theta 1}(60°) = 824.578\ kN \cdot m$ $T_{\theta 1}(60°) = -78.371\ kN \cdot m$

$M_{\theta 2}\left(70°\right) = 1026.219\ kN \cdot m$ $T_{\theta 2}\left(70°\right) = -750.424\ kN \cdot m$

$M_{\theta 2}\left(80°\right) = 981.321\ kN \cdot m$ $T_{\theta 2}\left(80°\right) = -932.64\ kN \cdot m$

$M_{\theta 2}\left(90°\right) = 915.115\ kN \cdot m$ $T_{\theta 2}\left(90°\right) = -1086.518\ kN \cdot m$

Girder radius: $r := 2900\ cm$

Initial and final angles: $\theta_0 := 30.5\ deg$ $\theta_1 := 149.5\ deg$

Pin relative angular distance : $\beta := \dfrac{\theta_1 - \theta_0}{(n-1)} \cdot rad = 0.692$

Girder angular length: $\alpha := \left(\theta_1 - \theta_0\right) = 2.077$

Girder center of gravity:

$$y_G := r \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) - r \cdot \sin\left(\theta_0\right) = 9.343\ m$$

$$d := r \cdot \left(\sin(\beta) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) = -5.55\ m$$

Vertical reactions (1,right end support, 4 left end support):

$$R_1 := \frac{-\int_{\theta_0}^{\theta_1} t \cdot r \, \mathrm{d}\theta}{2\left(y_G + d\right)} = -79.405\ kN \qquad R_4 := R_1 = -79.405\ kN$$

(Reaction>0 if upwards)

$$R_2 := -R_1 = 79.405\ kN \qquad R_3 := R_2 = 79.405\ kN$$

$\theta_0 < \theta < \beta$ (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin\left(\theta - \theta_0\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, \mathrm{d}\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot \left(1 - \cos\left(\theta - \theta_0\right)\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, \mathrm{d}\phi$$

$\beta < \theta < \dfrac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin\left(\theta - \theta_0\right) - R_2 \cdot r \cdot \sin\left(\theta - \beta - \theta_0\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, \mathrm{d}\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta)) + R_2 \cdot r \cdot \left(1 - \cos\left(\theta - \beta - \theta_0\right)\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, \mathrm{d}\phi$$

$M_{\theta 1}\left(\theta_0\right) = 0\ kN \cdot m$ $T_{\theta 1}\left(\theta_0\right) = 0\ kN \cdot m$ $\left(\theta_0 + \beta\right) \cdot \dfrac{180}{\pi} = 70.167$

$M_{\theta 1}\left(40°\right) = 384.039\ kN \cdot m$ $T_{\theta 1}\left(40°\right) = 16.283\ kN \cdot m$

$M_{\theta 1}\left(50°\right) = 785.305\ kN \cdot m$ $T_{\theta 1}\left(50°\right) = -35.278\ kN \cdot m$

$M_{\theta 1}\left(60°\right) = 1171.521\ kN \cdot m$ $T_{\theta 1}\left(60°\right) = -155.735\ kN \cdot m$

$M_{\theta 1}\left(70°\right) = 1530.953\ kN \cdot m$ $T_{\theta 1}\left(70°\right) = -341.427\ kN \cdot m$

$M_{\theta 2}\left(80°\right) = 1459.41\ kN \cdot m$ $T_{\theta 2}\left(80°\right) = -1648.527\ kN \cdot m$

$M_{\theta 2}\left(90°\right) = 1345.637\ kN \cdot m$ $T_{\theta 2}\left(90°\right) = -1916.279\ kN \cdot m$

Girder radius: $r := 3000\ cm$

Pin relative angular distance : $\beta := \dfrac{\theta_1 - \theta_0}{(n-1)} \cdot rad = 0.692$

Initial and final angles: $\theta_0 := 33.5\ deg$ $\theta_1 := 146.5\ deg$

Girder angular length: $\alpha := \left(\theta_1 - \theta_0\right) = 1.972$

Girder center of gravity:

$$y_G := r \cdot \left(\frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) - r \cdot \sin\left(\theta_0\right) = 8.811\ m$$

$$d := r \cdot \left(\sin(\beta) - \frac{\sin\left(\frac{\alpha}{2}\right)}{\frac{\alpha}{2}} \right) = -6.219\ m$$

Vertical reactions (1,right end support, 4 left end support):

$$R_1 := \frac{-\int_{\theta_0}^{\theta_1} t \cdot r \, \mathrm{d}\theta}{2\left(y_G + d\right)} = -114.156\ kN \qquad R_4 := R_1 = -114.156\ kN$$

(Reaction>0 if upwards)

$$R_2 := -R_1 = 114.156\ kN \qquad R_3 := R_2 = 114.156\ kN$$

$\theta_0 < \theta < \beta$ (b/w first and second support)

$$M_{\theta 1}(\theta) := -R_1 \cdot r \cdot \sin\left(\theta - \theta_0\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, \mathrm{d}\phi$$

$$T_{\theta 1}(\theta) := R_1 \cdot r \cdot \left(1 - \cos\left(\theta - \theta_0\right)\right) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, \mathrm{d}\phi$$

$\beta < \theta < \frac{\pi}{2}$ (b/w second and mid-span --------> use symmetry)

$$M_{\theta 2}(\theta) := -R_1 \cdot r \cdot \sin(\theta - \theta_0) - R_2 \cdot r \cdot \sin(\theta - \beta - \theta_0) + \int_{\theta_0}^{\theta} t \cdot r \cdot \sin(\theta - \phi) \, d\phi$$

$$T_{\theta 2}(\theta) := R_1 \cdot r \cdot (1 - \cos(\theta)) + R_2 \cdot r \cdot (1 - \cos(\theta - \beta - \theta_0)) + \int_{\theta_0}^{\theta} t \cdot r \cdot \cos(\theta - \phi) \, d\phi$$

$M_{\theta 1}(\theta_0) = 0 \ kN \cdot m$ $T_{\theta 1}(\theta_0) = 0 \ kN \cdot m$

$M_{\theta 1}(40°) = 389.612 \ kN \cdot m$ $T_{\theta 1}(40°) = 11.947 \ kN \cdot m$

$M_{\theta 1}(50°) = 985.012 \ kN \cdot m$ $T_{\theta 1}(50°) = -55.824 \ kN \cdot m$

$M_{\theta 1}(60°) = 1559.598 \ kN \cdot m$ $T_{\theta 1}(60°) = -225.955 \ kN \cdot m$

$M_{\theta 1}(70°) = 2095.912 \ kN \cdot m$ $T_{\theta 1}(70°) = -493.278 \ kN \cdot m$

$M_{\theta 2}(80°) = 2170.186 \ kN \cdot m$ $T_{\theta 2}(80°) = -2588.04 \ kN \cdot m$

$M_{\theta 2}(90°) = 1998.455 \ kN \cdot m$ $T_{\theta 2}(90°) = -3027.758 \ kN \cdot m$

Girder restraints: encastre at one end

Girder radius: $r := 2500 \ cm$ $\theta_0 := 0°$ $\theta_1 := 180°$

$$M_C := \int_{\theta_0}^{\theta_1} t \cdot r \cdot \sin(\phi) \, d\phi = 500 \ kN \cdot m$$

$$T_C := \int_{\theta_0}^{\theta_1} t \cdot r \cdot \cos(\phi) \, d\phi = 0 \ kN \cdot m$$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) + T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta - \theta_0) - 1)$$

$$T_\theta(\theta) := T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta - \theta_0)$$

$M_\theta(0) = 500 \ kN \cdot m$ $T_\theta(0) = 0 \ kN \cdot m$

$M_\theta(10°) = 496.202 \ kN \cdot m$ $T_\theta(10°) = -43.412 \ kN \cdot m$

$M_\theta(16°) = 490.315 \ kN \cdot m$ $T_\theta(16°) = -68.909 \ kN \cdot m$

$M_\theta(20°) = 484.923 \ kN \cdot m$ $T_\theta(20°) = -85.505 \ kN \cdot m$

$M_\theta(22°) = 481.796 \ kN \cdot m$ $T_\theta(22°) = -93.652 \ kN \cdot m$

$M_\theta(27°) = 472.752 \ kN \cdot m$ $T_\theta(27°) = -113.498 \ kN \cdot m$

$M_\theta(30°) = 466.506 \ kN \cdot m$ $T_\theta(30°) = -125 \ kN \cdot m$

$M_\theta\langle 30.5°\rangle = 465.407\ kN \cdot m$ $\quad T_\theta\langle 30.5°\rangle = -126.885\ kN \cdot m$

$M_\theta\langle 33.5°\rangle = 458.471\ kN \cdot m$ $\quad T_\theta\langle 33.5°\rangle = -137.984\ kN \cdot m$

$M_\theta\langle 40°\rangle = 441.511\ kN \cdot m$ $\quad T_\theta\langle 40°\rangle = -160.697\ kN \cdot m$

$M_\theta\langle 50°\rangle = 410.697\ kN \cdot m$ $\quad T_\theta\langle 50°\rangle = -191.511\ kN \cdot m$

$M_\theta\langle 60°\rangle = 375\ kN \cdot m$ $\quad T_\theta\langle 60°\rangle = -216.506\ kN \cdot m$

$M_\theta\langle 70°\rangle = 335.505\ kN \cdot m$ $\quad T_\theta\langle 70°\rangle = -234.923\ kN \cdot m$

$M_\theta\langle 80°\rangle = 293.412\ kN \cdot m$ $\quad T_\theta\langle 80°\rangle = -246.202\ kN \cdot m$

$M_\theta\langle 90°\rangle = 250\ kN \cdot m$ $\quad T_\theta\langle 90°\rangle = -250\ kN \cdot m$

$M_\theta\langle 100°\rangle = 206.588\ kN \cdot m$ $\quad T_\theta\langle 100°\rangle = -246.202\ kN \cdot m$

$M_\theta\langle 110°\rangle = 164.495\ kN \cdot m$ $\quad T_\theta\langle 110°\rangle = -234.923\ kN \cdot m$

$M_\theta\langle 120°\rangle = 125\ kN \cdot m$ $\quad T_\theta\langle 120°\rangle = -216.506\ kN \cdot m$

$M_\theta\langle 130°\rangle = 89.303\ kN \cdot m$ $\quad T_\theta\langle 130°\rangle = -191.511\ kN \cdot m$

$M_\theta\langle 140°\rangle = 58.489\ kN \cdot m$ $\quad T_\theta\langle 140°\rangle = -160.697\ kN \cdot m$

$M_\theta\langle 146.5°\rangle = 41.529\ kN \cdot m$ $\quad T_\theta\langle 146.5°\rangle = -137.984\ kN \cdot m$

$M_\theta\langle 149.5°\rangle = 34.593\ kN \cdot m$ $\quad T_\theta\langle 149.5°\rangle = -126.885\ kN \cdot m$

$M_\theta\langle 150°\rangle = 33.494\ kN \cdot m$ $\quad T_\theta\langle 150°\rangle = -125\ kN \cdot m$

$M_\theta\langle 153°\rangle = 27.248\ kN \cdot m$ $\quad T_\theta\langle 153°\rangle = -113.498\ kN \cdot m$

$M_\theta\langle 158°\rangle = 18.204\ kN \cdot m$ $\quad T_\theta\langle 158°\rangle = -93.652\ kN \cdot m$

$M_\theta\langle 160°\rangle = 15.077\ kN \cdot m$ $\quad T_\theta\langle 160°\rangle = -85.505\ kN \cdot m$

$M_\theta\langle 164°\rangle = 9.685\ kN \cdot m$ $\quad T_\theta\langle 164°\rangle = -68.909\ kN \cdot m$

$M_\theta\langle 170°\rangle = 3.798\ kN \cdot m$ $\quad T_\theta\langle 170°\rangle = -43.412\ kN \cdot m$

$M_\theta\langle 180°\rangle = 0\ kN \cdot m$ $\quad T_\theta\langle 180°\rangle = 0\ kN \cdot m$

Girder radius: $\quad r := 2600\ cm \quad \theta_0 := 16° \quad \theta_1 := 164°$

$$M_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \sin(\phi)\, d\phi = 480.493\ kN \cdot m$$

$$T_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \cos(\phi)\, d\phi = 137.779\ kN \cdot m$$

$M_\theta(\theta) := M_C \cdot \cos(\theta) - T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$

$T_\theta(\theta) := -T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$

$M_\theta(0°) = 480.493\ kN \cdot m$ $T_\theta(0°) = -137.779\ kN \cdot m$

$M_\theta(20° - \theta_0) = 470.344\ kN \cdot m$ $T_\theta(20° - \theta_0) = -152.824\ kN \cdot m$

$M_\theta(22° - \theta_0) = 464.883\ kN \cdot m$ $T_\theta(22° - \theta_0) = -160.072\ kN \cdot m$

$M_\theta(27° - \theta_0) = 450.152\ kN \cdot m$ $T_\theta(27° - \theta_0) = -177.32\ kN \cdot m$

$M_\theta(30° - \theta_0) = 440.611\ kN \cdot m$ $T_\theta(30° - \theta_0) = -187.028\ kN \cdot m$

$M_\theta(30.5° - \theta_0) = 438.972\ kN \cdot m$ $T_\theta(30.5° - \theta_0) = -188.597\ kN \cdot m$

$M_\theta(33.5° - \theta_0) = 428.856\ kN \cdot m$ $T_\theta(33.5° - \theta_0) = -197.706\ kN \cdot m$

$M_\theta(40° - \theta_0) = 405.39\ kN \cdot m$ $T_\theta(40° - \theta_0) = -215.55\ kN \cdot m$

$M_\theta(50° - \theta_0) = 365.752\ kN \cdot m$ $T_\theta(50° - \theta_0) = -237.522\ kN \cdot m$

$M_\theta(60° - \theta_0) = 322.9\ kN \cdot m$ $T_\theta(60° - \theta_0) = -252.277\ kN \cdot m$

$M_\theta(70° - \theta_0) = 278.137\ kN \cdot m$ $T_\theta(70° - \theta_0) = -259.367\ kN \cdot m$

$M_\theta(80° - \theta_0) = 232.823\ kN \cdot m$ $T_\theta(80° - \theta_0) = -258.576\ kN \cdot m$

$M_\theta(90° - \theta_0) = 188.334\ kN \cdot m$ $T_\theta(90° - \theta_0) = -249.928\ kN \cdot m$

$M_\theta(100° - \theta_0) = 146.024\ kN \cdot m$ $T_\theta(100° - \theta_0) = -233.686\ kN \cdot m$

$M_\theta(110° - \theta_0) = 107.176\ kN \cdot m$ $T_\theta(110° - \theta_0) = -210.344\ kN \cdot m$

$M_\theta(120° - \theta_0) = 72.972\ kN \cdot m$ $T_\theta(120° - \theta_0) = -180.611\ kN \cdot m$

$M_\theta(130° - \theta_0) = 44.45\ kN \cdot m$ $T_\theta(130° - \theta_0) = -145.39\ kN \cdot m$

$M_\theta(140° - \theta_0) = 22.478\ kN \cdot m$ $T_\theta(140° - \theta_0) = -105.752\ kN \cdot m$

$M_\theta(146.5° - \theta_0) = 12.034\ kN \cdot m$ $T_\theta(146.5° - \theta_0) = -78.184\ kN \cdot m$

$M_\theta(149.5° - \theta_0) = 8.282\ kN \cdot m$ $T_\theta(149.5° - \theta_0) = -65.099\ kN \cdot m$

$M_\theta(150° - \theta_0) = 7.723\ kN \cdot m$ $T_\theta(150° - \theta_0) = -62.9\ kN \cdot m$

$M_\theta(153° - \theta_0) = 4.777\ kN \cdot m$ $T_\theta(153° - \theta_0) = -49.61\ kN \cdot m$

$M_\theta(158° - \theta_0) = 1.424\ kN \cdot m$ $T_\theta(158° - \theta_0) = -27.177\ kN \cdot m$

$M_\theta(160° - \theta_0) = 0.633\ kN \cdot m$ $T_\theta(160° - \theta_0) = -18.137\ kN \cdot m$

$M_\theta\left(164° - \theta_0\right) = 0\ kN \cdot m$ $\qquad T_\theta\left(164° - \theta_0\right) = 0\ kN \cdot m$

Girder radius: $\quad r := 2700\ cm \qquad \theta_0 := 22° \qquad \theta_1 := 158°$

$$M_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \sin(\phi)\ \mathrm{d}\phi = 464.222\ kN \cdot m$$

$$T_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \cos(\phi)\ \mathrm{d}\phi = 187.558\ kN \cdot m$$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) - T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$$

$$T_\theta(\theta) := -T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$$

$M_\theta\left(22° - \theta_0\right) = 464.222\ kN \cdot m$ $\qquad T_\theta\left(22° - \theta_0\right) = -187.558\ kN \cdot m$

$M_\theta\left(27° - \theta_0\right) = 447.136\ kN \cdot m$ $\qquad T_\theta\left(27° - \theta_0\right) = -203.772\ kN \cdot m$

$M_\theta\left(30° - \theta_0\right) = 436.229\ kN \cdot m$ $\qquad T_\theta\left(30° - \theta_0\right) = -212.763\ kN \cdot m$

$M_\theta\left(30.5° - \theta_0\right) = 434.366\ kN \cdot m$ $\qquad T_\theta\left(30.5° - \theta_0\right) = -214.205\ kN \cdot m$

$M_\theta\left(33.5° - \theta_0\right) = 422.93\ kN \cdot m$ $\qquad T_\theta\left(33.5° - \theta_0\right) = -222.514\ kN \cdot m$

$M_\theta\left(40° - \theta_0\right) = 396.757\ kN \cdot m$ $\qquad T_\theta\left(40° - \theta_0\right) = -238.396\ kN \cdot m$

$M_\theta\left(50° - \theta_0\right) = 353.435\ kN \cdot m$ $\qquad T_\theta\left(50° - \theta_0\right) = -256.785\ kN \cdot m$

$M_\theta\left(60° - \theta_0\right) = 307.577\ kN \cdot m$ $\qquad T_\theta\left(60° - \theta_0\right) = -267.372\ kN \cdot m$

$M_\theta\left(70° - \theta_0\right) = 260.577\ kN \cdot m$ $\qquad T_\theta\left(70° - \theta_0\right) = -269.836\ kN \cdot m$

$M_\theta\left(80° - \theta_0\right) = 213.864\ kN \cdot m$ $\qquad T_\theta\left(80° - \theta_0\right) = -264.1\ kN \cdot m$

$M_\theta\left(90° - \theta_0\right) = 168.856\ kN \cdot m$ $\qquad T_\theta\left(90° - \theta_0\right) = -250.34\ kN \cdot m$

$M_\theta\left(100° - \theta_0\right) = 126.922\ kN \cdot m$ $\qquad T_\theta\left(100° - \theta_0\right) = -228.973\ kN \cdot m$

$M_\theta\left(110° - \theta_0\right) = 89.335\ kN \cdot m$ $\qquad T_\theta\left(110° - \theta_0\right) = -200.649\ kN \cdot m$

$M_\theta\left(120° - \theta_0\right) = 57.237\ kN \cdot m$ $\qquad T_\theta\left(120° - \theta_0\right) = -166.229\ kN \cdot m$

$M_\theta\left(130° - \theta_0\right) = 31.604\ kN \cdot m$ $\qquad T_\theta\left(130° - \theta_0\right) = -126.757\ kN \cdot m$

$M_\theta\left(140° - \theta_0\right) = 13.215\ kN \cdot m$ $\qquad T_\theta\left(140° - \theta_0\right) = -83.435\ kN \cdot m$

$M_\theta\left(146.5° - \theta_0\right) = 5.42\ kN \cdot m$ $\qquad T_\theta\left(146.5° - \theta_0\right) = -53.829\ kN \cdot m$

$M_\theta\left(149.5° - \theta_0\right) = 2.966\ kN \cdot m$ $\qquad T_\theta\left(149.5° - \theta_0\right) = -39.909\ kN \cdot m$

$M_\theta\left(150° - \theta_0\right) = 2.628\ kN \cdot m$ $T_\theta\left(150° - \theta_0\right) = -37.577\ kN \cdot m$

$M_\theta\left(153° - \theta_0\right) = 1.027\ kN \cdot m$ $T_\theta\left(153° - \theta_0\right) = -23.532\ kN \cdot m$

$M_\theta\left(158° - \theta_0\right) = 0\ kN \cdot m$ $T_\theta\left(158° - \theta_0\right) = 0\ kN \cdot m$

Girder radius: $r := 2800\ cm$ $\theta_0 := 27°$ $\theta_1 := 153°$

$$M_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \sin(\phi)\, \mathrm{d}\phi = 444.58\ kN \cdot m$$

$$T_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \cos(\phi)\, \mathrm{d}\phi = 226.525\ kN \cdot m$$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) - T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$$

$$T_\theta(\theta) := -T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$$

$M_\theta\left(27° - \theta_0\right) = 444.58\ kN \cdot m$ $T_\theta\left(27° - \theta_0\right) = -226.525\ kN \cdot m$

$M_\theta\left(30° - \theta_0\right) = 432.499\ kN \cdot m$ $T_\theta\left(30° - \theta_0\right) = -234.828\ kN \cdot m$

$M_\theta\left(30.5° - \theta_0\right) = 430.444\ kN \cdot m$ $T_\theta\left(30.5° - \theta_0\right) = -236.15\ kN \cdot m$

$M_\theta\left(33.5° - \theta_0\right) = 417.879\ kN \cdot m$ $T_\theta\left(33.5° - \theta_0\right) = -243.7\ kN \cdot m$

$M_\theta\left(40° - \theta_0\right) = 389.405\ kN \cdot m$ $T_\theta\left(40° - \theta_0\right) = -257.741\ kN \cdot m$

$M_\theta\left(50° - \theta_0\right) = 342.986\ kN \cdot m$ $T_\theta\left(50° - \theta_0\right) = -272.824\ kN \cdot m$

$M_\theta\left(60° - \theta_0\right) = 294.654\ kN \cdot m$ $T_\theta\left(60° - \theta_0\right) = -279.616\ kN \cdot m$

$M_\theta\left(70° - \theta_0\right) = 245.877\ kN \cdot m$ $T_\theta\left(70° - \theta_0\right) = -277.913\ kN \cdot m$

$M_\theta\left(80° - \theta_0\right) = 198.136\ kN \cdot m$ $T_\theta\left(80° - \theta_0\right) = -267.765\ kN \cdot m$

$M_\theta\left(90° - \theta_0\right) = 152.883\ kN \cdot m$ $T_\theta\left(90° - \theta_0\right) = -249.482\ kN \cdot m$

$M_\theta\left(100° - \theta_0\right) = 111.492\ kN \cdot m$ $T_\theta\left(100° - \theta_0\right) = -223.618\ kN \cdot m$

$M_\theta\left(110° - \theta_0\right) = 75.221\ kN \cdot m$ $T_\theta\left(110° - \theta_0\right) = -190.96\ kN \cdot m$

$M_\theta\left(120° - \theta_0\right) = 45.172\ kN \cdot m$ $T_\theta\left(120° - \theta_0\right) = -152.499\ kN \cdot m$

$M_\theta\left(130° - \theta_0\right) = 22.259\ kN \cdot m$ $T_\theta\left(130° - \theta_0\right) = -109.405\ kN \cdot m$

$M_\theta\left(140° - \theta_0\right) = 7.176\ kN \cdot m$ $T_\theta\left(140° - \theta_0\right) = -62.986\ kN \cdot m$

$M_\theta\left(146.5° - \theta_0\right) = 1.8\ kN \cdot m$ $T_\theta\left(146.5° - \theta_0\right) = -31.697\ kN \cdot m$

$M_\theta\left(149.5° - \theta_0\right) = 0.522\ kN \cdot m$ $T_\theta\left(149.5° - \theta_0\right) = -17.094\ kN \cdot m$

$M_\theta\left(150° - \theta_0\right) = 0.384\ kN \cdot m$ $\quad$ $T_\theta\left(150° - \theta_0\right) = -14.654\ kN \cdot m$

$M_\theta\left(153° - \theta_0\right) = 0\ kN \cdot m$ $\quad$ $T_\theta\left(153° - \theta_0\right) = 0\ kN \cdot m$

Girder radius: $\quad r := 2900\ cm \quad \theta_0 := 30.5° \quad \theta_1 := 149.5°$

$$M_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \sin(\phi)\, d\phi = 430.595\ kN \cdot m$$

$$T_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \cos(\phi)\, d\phi = 253.64\ kN \cdot m$$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) - T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$$

$$T_\theta(\theta) := -T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$$

$M_\theta\left(30.5° - \theta_0\right) = 430.595\ kN \cdot m$ $\quad$ $T_\theta\left(30.5° - \theta_0\right) = -253.64\ kN \cdot m$

$M_\theta\left(33.5° - \theta_0\right) = 417.128\ kN \cdot m$ $\quad$ $T_\theta\left(33.5° - \theta_0\right) = -260.65\ kN \cdot m$

$M_\theta\left(40° - \theta_0\right) = 386.804\ kN \cdot m$ $\quad$ $T_\theta\left(40° - \theta_0\right) = -273.366\ kN \cdot m$

$M_\theta\left(50° - \theta_0\right) = 337.864\ kN \cdot m$ $\quad$ $T_\theta\left(50° - \theta_0\right) = -286.023\ kN \cdot m$

$M_\theta\left(60° - \theta_0\right) = 287.469\ kN \cdot m$ $\quad$ $T_\theta\left(60° - \theta_0\right) = -289.989\ kN \cdot m$

$M_\theta\left(70° - \theta_0\right) = 237.152\ kN \cdot m$ $\quad$ $T_\theta\left(70° - \theta_0\right) = -285.144\ kN \cdot m$

$M_\theta\left(80° - \theta_0\right) = 188.44\ kN \cdot m$ $\quad$ $T_\theta\left(80° - \theta_0\right) = -271.635\ kN \cdot m$

$M_\theta\left(90° - \theta_0\right) = 142.814\ kN \cdot m$ $\quad$ $T_\theta\left(90° - \theta_0\right) = -249.872\ kN \cdot m$

$M_\theta\left(100° - \theta_0\right) = 101.66\ kN \cdot m$ $\quad$ $T_\theta\left(100° - \theta_0\right) = -220.518\ kN \cdot m$

$M_\theta\left(110° - \theta_0\right) = 66.229\ kN \cdot m$ $\quad$ $T_\theta\left(110° - \theta_0\right) = -184.463\ kN \cdot m$

$M_\theta\left(120° - \theta_0\right) = 37.597\ kN \cdot m$ $\quad$ $T_\theta\left(120° - \theta_0\right) = -142.803\ kN \cdot m$

$M_\theta\left(130° - \theta_0\right) = 16.634\ kN \cdot m$ $\quad$ $T_\theta\left(130° - \theta_0\right) = -96.804\ kN \cdot m$

$M_\theta\left(140° - \theta_0\right) = 3.977\ kN \cdot m$ $\quad$ $T_\theta\left(140° - \theta_0\right) = -47.864\ kN \cdot m$

$M_\theta\left(146.5° - \theta_0\right) = 0.397\ kN \cdot m$ $\quad$ $T_\theta\left(146.5° - \theta_0\right) = -15.177\ kN \cdot m$

$M_\theta\left(149.5° - \theta_0\right) = 0\ kN \cdot m$ $\quad$ $T_\theta\left(149.5° - \theta_0\right) = 0\ kN \cdot m$

Girder radius: $\quad r := 3000\ cm \quad \theta_0 := 33.5° \quad \theta_1 := 146.5°$

$$M_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \sin(\phi)\, d\phi = 417.219\ kN \cdot m$$

$$T_C := \int_0^{\theta_1 - \theta_0} t \cdot r \cdot \cos(\phi)\, d\phi = 276.151\ kN \cdot m$$

$$M_\theta(\theta) := M_C \cdot \cos(\theta) - T_C \cdot \sin(\theta) - r \cdot t \cdot (\cos(\theta) - 1)$$

$$T_\theta(\theta) := -T_C \cdot \cos(\theta) - M_C \cdot \sin(\theta) + r \cdot t \cdot \sin(\theta)$$

$M_\theta(33.5° - \theta_0) = 417.219\ kN \cdot m$ $T_\theta(33.5° - \theta_0) = -276.151\ kN \cdot m$

$M_\theta(40° - \theta_0) = 385.205\ kN \cdot m$ $T_\theta(40° - \theta_0) = -287.646\ kN \cdot m$

$M_\theta(50° - \theta_0) = 333.961\ kN \cdot m$ $T_\theta(50° - \theta_0) = -298.072\ kN \cdot m$

$M_\theta(60° - \theta_0) = 281.685\ kN \cdot m$ $T_\theta(60° - \theta_0) = -299.44\ kN \cdot m$

$M_\theta(70° - \theta_0) = 229.966\ kN \cdot m$ $T_\theta(70° - \theta_0) = -291.711\ kN \cdot m$

$M_\theta(80° - \theta_0) = 180.375\ kN \cdot m$ $T_\theta(80° - \theta_0) = -275.118\ kN \cdot m$

$M_\theta(90° - \theta_0) = 134.419\ kN \cdot m$ $T_\theta(90° - \theta_0) = -250.166\ kN \cdot m$

$M_\theta(100° - \theta_0) = 93.494\ kN \cdot m$ $T_\theta(100° - \theta_0) = -217.612\ kN \cdot m$

$M_\theta(110° - \theta_0) = 58.843\ kN \cdot m$ $T_\theta(110° - \theta_0) = -178.447\ kN \cdot m$

$M_\theta(120° - \theta_0) = 31.52\ kN \cdot m$ $T_\theta(120° - \theta_0) = -133.859\ kN \cdot m$

$M_\theta(130° - \theta_0) = 12.354\ kN \cdot m$ $T_\theta(130° - \theta_0) = -85.205\ kN \cdot m$

$M_\theta(140° - \theta_0) = 1.928\ kN \cdot m$ $T_\theta(140° - \theta_0) = -33.961\ kN \cdot m$

$M_\theta(146.5° - \theta_0) = 0\ kN \cdot m$ $T_\theta(146.5° - \theta_0) = 0\ kN \cdot m$

B. Load-bearing system optimization

<u>Procedure for obtaining cables' prestresses that are mutually equal and in equilibrium with external dead loads (from simplified continuous beam model)</u>

$R_1 := 343.6$ $\quad R_2 := 400.3$ $\quad R_3 := 377.7$ $\quad R_4 := 115$ $\quad R_5 := 275.5$ $\quad R_6 := 221.9$ $\quad R_7 := 331.4$ $\quad R_8 := 163.2$ $\quad R_9 := 213.4$

$x_1 := 38.69$ $\quad y_1 := 25.11$ $\quad z_1 := -8.15$ $\quad z := 8$

$x_2 := 40.28$ $\quad y_2 := 19.47$ $\quad z_2 := -8.073$

$x_3 := 42.68$ $\quad y_3 := 14.17$ $\quad z_3 := -7.954$

$x_4 := 45.95$ $\quad y_4 := 9.345$ $\quad z_4 := -7.791$

$x_5 := 50.01$ $\quad y_5 := 5.253$ $\quad z_5 := -7.592$

$x_6 := 54.95$ $\quad y_6 := 2.291$ $\quad z_6 := -7.363$

$x_7 := 60.41$ $\quad y_7 := 0.273$ $\quad z_7 := -7.147$

$x_8 := 66.14$ $\quad y_8 := -0.79$ $\quad z_8 := -6.975$

$x_9 := 72.03$ $\quad y_9 := -1.034$ $\quad z_9 := -6.874$

$$f_1(x,y) := \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_1)}{\sqrt{|x-x_1|^2+|y-y_1|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_2)}{\sqrt{|x-x_2|^2+|y-y_2|^2}}\right)\right)} - \frac{R_1}{R_2} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_2)}{\sqrt{|x-x_2|^2+|y-y_2|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_3)}{\sqrt{|x-x_3|^2+|y-y_3|^2}}\right)\right)} - \frac{R_2}{R_3} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_3)}{\sqrt{|x-x_3|^2+|y-y_3|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_4)}{\sqrt{|x-x_4|^2+|y-y_4|^2}}\right)\right)} - \frac{R_3}{R_4} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_4)}{\sqrt{|x-x_4|^2+|y-y_4|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_5)}{\sqrt{|x-x_5|^2+|y-y_5|^2}}\right)\right)} - \frac{R_4}{R_5} \right|$$

$$f_2(x,y) := \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_5)}{\sqrt{|x-x_5|^2+|y-y_5|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_6)}{\sqrt{|x-x_6|^2+|y-y_6|^2}}\right)\right)} - \frac{R_5}{R_6} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_6)}{\sqrt{|x-x_6|^2+|y-y_6|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_7)}{\sqrt{|x-x_7|^2+|y-y_7|^2}}\right)\right)} - \frac{R_6}{R_7} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_7)}{\sqrt{|x-x_7|^2+|y-y_7|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_8)}{\sqrt{|x-x_8|^2+|y-y_8|^2}}\right)\right)} - \frac{R_7}{R_8} \right| + \left| \frac{\sin\left(\operatorname{atan}\left(\frac{(z-z_8)}{\sqrt{|x-x_8|^2+|y-y_8|^2}}\right)\right)}{\sin\left(\operatorname{atan}\left(\frac{(z-z_9)}{\sqrt{|x-x_9|^2+|y-y_9|^2}}\right)\right)} - \frac{R_8}{R_9} \right|$$

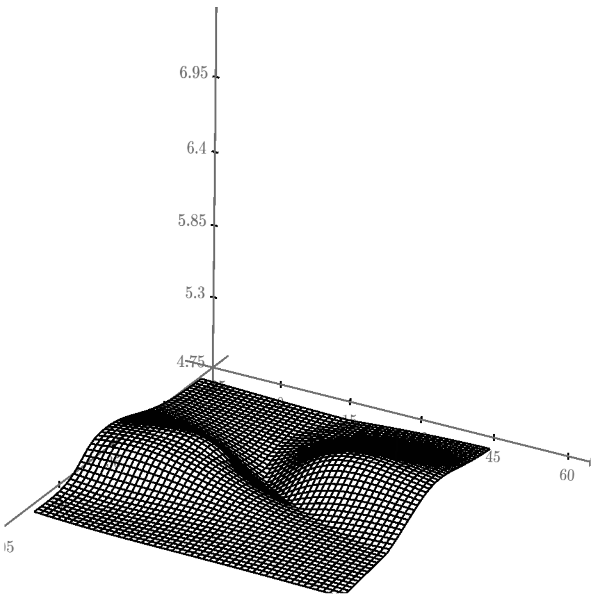
$f(x,y) := f_1(x,y) + f_2(x,y)$

$x := 1$ $\quad y := 1$

$f(x,y) = 0$

$\boxed{\text{find}(x,y)} = ?$

As expected, a solution could not be found. The function's absolute minimum is obtained instead.



f

$x := 1 \qquad y := 1$

$$\textbf{minimize}(f, x, y) = \begin{bmatrix} 36.95 \\ 17.866 \end{bmatrix}$$

$f(35.828, 18.228) = 4.795$ (representing a solution that is not compatible with the architectural requirements)

Procedure for obtaining the coordinates of the top mast node so that cables' stresses are in equilibrium with external dead loads and the minimum strain energy configuration is achieved

Young modulus: $E := 205000\ MPa$

Cable cross-sectional area: $A := 0.007854\ m^2$

Continuous beam model vertical reactions

$R_1 := 102.3\ kN \qquad R_2 := 459.9\ kN \qquad R_3 := 318.7\ kN \qquad R_4 := 306.1\ kN \qquad R_5 := 352.8\ kN$

$R_6 := 341.6\ kN \qquad R_7 := 278.9\ kN \qquad R_8 := 374.5\ kN \qquad R_9 := 366.4\ kN \qquad R_{10} := 314.1\ kN$

Hinged nodes coordinates

Node#			
584	$x_1 := 2.980\ m$	$y_1 := 1.020\ m$	$z_1 := 0.2095\ m$
629	$x_2 := 7.858\ m$	$y_2 := 1.356\ m$	$z_2 := 0.5111\ m$
674	$x_3 := 12.62\ m$	$y_3 := 2.307\ m$	$z_3 := 0.7845\ m$
719	$x_4 := 17.20\ m$	$y_4 := 3.924\ m$	$z_4 := 1.017\ m$
764	$x_5 := 21.49\ m$	$y_5 := 6.178\ m$	$z_5 := 1.200\ m$
809	$x_6 := 25.36\ m$	$y_6 := 9.062\ m$	$z_6 := 1.332\ m$
854	$x_7 := 28.70\ m$	$y_7 := 12.55\ m$	$z_7 := 1.416\ m$
899	$x_8 := 31.42\ m$	$y_8 := 16.54\ m$	$z_8 := 1.486\ m$
944	$x_9 := 33.44\ m$	$y_9 := 20.92\ m$	$z_9 := 1.486\ m$
1622	$x_{10} := 34.26\ m$	$y_{10} := 25.55\ m$	$z_{10} := 1.493\ m$
1612	$x_{11} := 34.49\ m$	$y_{11} := 30.40\ m$	$z_{11} := 1.494\ m$

Fixed mast's height: $z := 15\ m$

$$f_1(x, y) := \left(\frac{R_1}{\sin\left(\operatorname{atan}\left(\frac{(z - z_1)}{\sqrt{|x - x_1|^2 + |y - y_1|^2}}\right)\right)} \right)^2 \cdot \frac{(z - z_1)}{\sqrt{|x - x_1|^2 + |y - y_1|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_2(x, y) := \left(\frac{R_2}{\sin\left(\operatorname{atan}\left(\frac{(z - z_2)}{\sqrt{|x - x_2|^2 + |y - y_2|^2}}\right)\right)} \right)^2 \cdot \frac{(z - z_2)}{\sqrt{|x - x_2|^2 + |y - y_2|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_3(x,y) := \left(\frac{R_3}{\sin\left(\operatorname{atan}\left(\frac{(z-z_3)}{\sqrt{\left|x-x_3\right|^2+\left|y-y_3\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_3)}{\sqrt{\left|x-x_3\right|^2+\left|y-y_3\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_4(x,y) := \left(\frac{R_4}{\sin\left(\operatorname{atan}\left(\frac{(z-z_4)}{\sqrt{\left|x-x_4\right|^2+\left|y-y_4\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_4)}{\sqrt{\left|x-x_4\right|^2+\left|y-y_4\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_5(x,y) := \left(\frac{R_5}{\sin\left(\operatorname{atan}\left(\frac{(z-z_5)}{\sqrt{\left|x-x_5\right|^2+\left|y-y_5\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_5)}{\sqrt{\left|x-x_5\right|^2+\left|y-y_5\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_6(x,y) := \left(\frac{R_6}{\sin\left(\operatorname{atan}\left(\frac{(z-z_6)}{\sqrt{\left|x-x_6\right|^2+\left|y-y_6\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_6)}{\sqrt{\left|x-x_6\right|^2+\left|y-y_6\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_7(x,y) := \left(\frac{R_7}{\sin\left(\operatorname{atan}\left(\frac{(z-z_7)}{\sqrt{\left|x-x_7\right|^2+\left|y-y_7\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_7)}{\sqrt{\left|x-x_7\right|^2+\left|y-y_7\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_8(x,y) := \left(\frac{R_8}{\sin\left(\operatorname{atan}\left(\frac{(z-z_8)}{\sqrt{\left|x-x_8\right|^2+\left|y-y_8\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_8)}{\sqrt{\left|x-x_8\right|^2+\left|y-y_8\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_9(x,y) := \left(\frac{R_9}{\sin\left(\operatorname{atan}\left(\frac{(z-z_9)}{\sqrt{\left|x-x_9\right|^2+\left|y-y_9\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_9)}{\sqrt{\left|x-x_9\right|^2+\left|y-y_9\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f_{10}(x,y) := \left(\frac{R_{10}}{\sin\left(\operatorname{atan}\left(\frac{(z-z_{10})}{\sqrt{\left|x-x_{10}\right|^2+\left|y-y_{10}\right|^2}}\right)\right)}\right)^2 \cdot \frac{(z-z_{10})}{\sqrt{\left|x-x_{10}\right|^2+\left|y-y_{10}\right|^2}} \cdot \frac{1}{E \cdot A}$$

$$f(x,y) := f_1(x,y)+f_2(x,y)+f_3(x,y)+f_4(x,y)+f_5(x,y)+f_6(x,y)+f_7(x,y)+f_8(x,y)+f_9(x,y)+f_{10}(x,y)$$

Valori ipotizzati

$x := 1\ m \qquad y := 1\ m$

Vincoli

Solutore

$$\mathbf{minimize}(f,x,y) = \begin{bmatrix} 19.522 \\ 20.01 \end{bmatrix}$$

-----------> Mast's top node coordinates: (x,y,z) = (17.265m , 17.696m , 15m)

Mast's tilt angle

Top mast nodal forces from a linear model (top mast node is hinged)

$F_x := 1775\ kN$ $\quad$ $F_y := 1231\ kN$ $\quad$ $F_z := 3203\ kN$

Angle b/w net top mast nodal force and x-axis (in the horizontal plane)

$$\alpha_{rad} := \operatorname{atan}\left(\frac{F_y}{F_x}\right) = 0.606 \qquad \alpha_{deg} := \alpha_{rad} \cdot \frac{180}{\pi} = 34.742$$

Angle b/w net top mast nodal force and y-axis (in the horizontal plane)

$$\gamma_{deg} := 90 - \alpha_{deg} = 55.258$$

Angle of top mast nodal net force in the vertical plane

$$\beta_{rad} := \operatorname{atan}\left(\frac{F_z}{\sqrt{F_x^{\ 2} + F_y^{\ 2}}}\right) = 0.977 \qquad \beta_{deg} := \beta_{rad} \cdot \frac{180}{\pi} = 56.004$$

The mast length is fixed and equal to 21.42m so that the base mast node falls into a region that was choosen according the river flood level.

$L_m := 21.42\ m$ $\quad$ $x := 17.265\ m$ $\quad$ $y := 17.696\ m$ $\quad$ $z = 16.962$

-----------> Mast's base node coordinates:

$$x_{base} := x - L_m \cdot \cos\left(\beta_{rad}\right) \cdot \cos\left(\alpha_{rad}\right) = 8.394$$

$$y_{base} := y - L_m \cdot \cos\left(\beta_{rad}\right) \cdot \sin\left(\alpha_{rad}\right) = 12.292$$

$$z_{base} := z - L_m \cdot \sin\left(\beta_{rad}\right) = -3.12$$

(refer to the GSA model global axes)

Horizontal angle b/w mast and global x-axis

$$\beta_{mast} := \alpha_{rad} = 0.606$$

Mast vertical angle

$$\alpha_{mast} := \beta_{rad} = 0.977 \qquad 0.977 \cdot \frac{180}{\pi} = 55.978 \text{ deg}$$

Cinematic analysis for assessing mast's position under dead loads and prestresses

Horizontal angle b/w cables and x-axis (undeformed geometry)

$$\beta_1 := \operatorname{atan}\left(\frac{\left(y - y_1\right)}{\left(x - x_1\right)}\right) = 0.862 \qquad \beta_2 := \operatorname{atan}\left(\frac{\left(y - y_2\right)}{\left(x - x_2\right)}\right) = 1.048 \qquad \beta_3 := \operatorname{atan}\left(\frac{\left(y - y_3\right)}{\left(x - x_3\right)}\right) = 1.278 \qquad \beta_4 := \operatorname{atan}\left(\frac{\left(y - y_4\right)}{\left(x - x_4\right)}\right) = 1.566$$

$$\beta_5 := \left|\operatorname{atan}\left(\frac{\left(y - y_5\right)}{\left(x - x_5\right)}\right)\right| = 1.219 \qquad \beta_6 := \left|\operatorname{atan}\left(\frac{\left(y - y_6\right)}{\left(x - x_6\right)}\right)\right| = 0.818 \qquad \beta_7 := \left|\operatorname{atan}\left(\frac{\left(y - y_7\right)}{\left(x - x_7\right)}\right)\right| = 0.423 \qquad \beta_8 := \left|\operatorname{atan}\left(\frac{\left(y - y_8\right)}{\left(x - x_8\right)}\right)\right| = 0.081$$

$$\beta_9 := \operatorname{atan}\left(\frac{\left(y - y_9\right)}{\left(x - x_9\right)}\right) = 0.197 \qquad \beta_{10} := \operatorname{atan}\left(\frac{\left(y - y_{10}\right)}{\left(x - x_{10}\right)}\right) = 0.433 \qquad \beta_{11} := \operatorname{atan}\left(\frac{\left(y - y_{11}\right)}{\left(x - x_{11}\right)}\right) = 0.635$$

Horizontal angle b/w cables and x-axis (dead loads + prestresses configuration)

$x := 17.7024\ m$ $\quad y := 17.5170\ m$ $\quad z := 15.1048\ m$

$\beta'_1 := \operatorname{atan}\left(\frac{(y-y_1)}{(x-x_1)}\right) = 0.842$ $\quad \beta'_2 := \operatorname{atan}\left(\frac{(y-y_2)}{(x-x_2)}\right) = 1.024$ $\quad \beta'_3 := \operatorname{atan}\left(\frac{(y-y_3)}{(x-x_3)}\right) = 1.248$ $\quad \beta'_4 := \operatorname{atan}\left(\frac{(y-y_4)}{(x-x_4)}\right) = 1.534$

$\beta'_5 := \left|\operatorname{atan}\left(\frac{(y-y_5)}{(x-x_5)}\right)\right| = 1.248$ $\quad \beta'_6 := \left|\operatorname{atan}\left(\frac{(y-y_6)}{(x-x_6)}\right)\right| = 0.835$ $\quad \beta'_7 := \left|\operatorname{atan}\left(\frac{(y-y_7)}{(x-x_7)}\right)\right| = 0.424$ $\quad \beta'_8 := \left|\operatorname{atan}\left(\frac{(y-y_8)}{(x-x_8)}\right)\right| = 0.071$

$\beta'_9 := \operatorname{atan}\left(\frac{(y-y_9)}{(x-x_9)}\right) = 0.213$ $\quad \beta'_{10} := \operatorname{atan}\left(\frac{(y-y_{10})}{(x-x_{10})}\right) = 0.452$ $\quad \beta'_{11} := \operatorname{atan}\left(\frac{(y-y_{11})}{(x-x_{11})}\right) = 0.655$

Cables' horizontal rotation

$|\beta_1-\beta'_1| = 0.02$ $\quad |\beta_2-\beta'_2| = 0.025$ $\quad |\beta_3-\beta'_3| = 0.029$ $\quad |\beta_4-\beta'_4| = 0.032$ $\quad |\beta_5-\beta'_5| = 0.029$ $\quad |\beta_6-\beta'_6| = 0.017$

$|\beta_7-\beta'_7| = 0.001$ $\quad |\beta_8-\beta'_8| = 0.01$ $\quad |\beta_9-\beta'_9| = 0.016$ $\quad |\beta_{10}-\beta'_{10}| = 0.019$ $\quad |\beta_{11}-\beta'_{11}| = 0.019$

Vertical angle b/w cables and x-axis (undeformed geometry)

$x := 17.68\ m$ $\quad y := 17.55\ m$ $\quad z := 15.09\ m$

$\alpha_1 := \operatorname{atan}\left(\frac{(z-z_1)}{\sqrt{|x-x_1|^2+|y-y_1|^2}}\right)$ $\quad \alpha_2 := \operatorname{atan}\left(\frac{(z-z_2)}{\sqrt{|x-x_2|^2+|y-y_2|^2}}\right)$ $\quad \alpha_3 := \operatorname{atan}\left(\frac{(z-z_3)}{\sqrt{|x-x_3|^2+|y-y_3|^2}}\right)$ $\quad \alpha_4 := \operatorname{atan}\left(\frac{(z-z_4)}{\sqrt{|x-x_4|^2+|y-y_4|^2}}\right)$

$\alpha_5 := \operatorname{atan}\left(\frac{(z-z_5)}{\sqrt{|x-x_5|^2+|y-y_5|^2}}\right)$ $\quad \alpha_6 := \operatorname{atan}\left(\frac{(z-z_6)}{\sqrt{|x-x_6|^2+|y-y_6|^2}}\right)$ $\quad \alpha_7 := \operatorname{atan}\left(\frac{(z-z_7)}{\sqrt{|x-x_7|^2+|y-y_7|^2}}\right)$ $\quad \alpha_8 := \operatorname{atan}\left(\frac{(z-z_8)}{\sqrt{|x-x_8|^2+|y-y_8|^2}}\right)$

$\alpha_9 := \operatorname{atan}\left(\frac{(z-z_9)}{\sqrt{|x-x_9|^2+|y-y_9|^2}}\right)$ $\quad \alpha_{10} := \operatorname{atan}\left(\frac{(z-z_{10})}{\sqrt{|x-x_{10}|^2+|y-y_{10}|^2}}\right)$ $\quad \alpha_{11} := \operatorname{atan}\left(\frac{(z-z_{11})}{\sqrt{|x-x_{11}|^2+|y-y_{11}|^2}}\right)$

Horizontal angle b/w cables and x-axis (dead loads + prestresses configuration)

$x := 17.7024\ m$ $\quad y := 17.5170\ m$ $\quad z := 15.1048\ m$

$\alpha'_1 := \operatorname{atan}\left(\frac{(z-z_1)}{\sqrt{|x-x_1|^2+|y-y_1|^2}}\right)$ $\quad \alpha'_2 := \operatorname{atan}\left(\frac{(z-z_2)}{\sqrt{|x-x_2|^2+|y-y_2|^2}}\right)$ $\quad \alpha'_3 := \operatorname{atan}\left(\frac{(z-z_3)}{\sqrt{|x-x_3|^2+|y-y_3|^2}}\right)$ $\quad \alpha'_4 := \operatorname{atan}\left(\frac{(z-z_4)}{\sqrt{|x-x_4|^2+|y-y_4|^2}}\right)$

$\alpha'_5 := \operatorname{atan}\left(\frac{(z-z_5)}{\sqrt{|x-x_5|^2+|y-y_5|^2}}\right)$ $\quad \alpha'_6 := \operatorname{atan}\left(\frac{(z-z_6)}{\sqrt{|x-x_6|^2+|y-y_6|^2}}\right)$ $\quad \alpha'_7 := \operatorname{atan}\left(\frac{(z-z_7)}{\sqrt{|x-x_7|^2+|y-y_7|^2}}\right)$ $\quad \alpha'_8 := \operatorname{atan}\left(\frac{(z-z_8)}{\sqrt{|x-x_8|^2+|y-y_8|^2}}\right)$

$\alpha'_9 := \operatorname{atan}\left(\dfrac{(z-z_9)}{\sqrt{|x-x_9|^2+|y-y_9|^2}}\right)$ $\alpha'_{10} := \operatorname{atan}\left(\dfrac{(z-z_{10})}{\sqrt{|x-x_{10}|^2+|y-y_{10}|^2}}\right)$ $\alpha'_{11} := \operatorname{atan}\left(\dfrac{(z-z_{11})}{\sqrt{|x-x_{11}|^2+|y-y_{11}|^2}}\right)$

Cables's vertical rotation

$|\alpha_1-\alpha'_1|=0.001$ $|\alpha_2-\alpha'_2|=0.001$ $|\alpha_3-\alpha'_3|=0.001$ $|\alpha_4-\alpha'_4|=0.002$ $|\alpha_5-\alpha'_5|=0.002$ $|\alpha_6-\alpha'_6|=0.002$

$|\alpha_7-\alpha'_7|=0.002$ $|\alpha_8-\alpha'_8|=0.001$ $|\alpha_9-\alpha'_9|=0.001$ $|\alpha_{10}-\alpha'_{10}|=0.001$ $|\alpha_{11}-\alpha'_{11}|=0$

Horizontal and vertical rotations are approximately of the same order

Mast's tilt angles

$x_1 := 17.68\ m$ $y_1 := 17.55\ m$ $z_1 := 15.09\ m$ $x_2 := 26.08\ m$ $y_2 := 11.49\ m$ $z_2 := -6.127\ m$

$\gamma_{mast} := \operatorname{atan}\left(\dfrac{(z_1-z_2)}{\sqrt{|x_1-x_2|^2+|y_1-y_2|^2}}\right)=1.117$ $1.117 \cdot \dfrac{180}{\pi}=63.999$ deg

Horizontal angle b/w cables and mast

$\gamma_1 := \pi-\beta_1-\beta_{mast}=1.673$ $\gamma_2 := \pi-\beta_2-\beta_{mast}=1.487$ $\gamma_3 := \pi-\beta_3-\beta_{mast}=1.258$ $\gamma_4 := \pi-\beta_4-\beta_{mast}=0.969$

$\gamma_5 := \beta_5-\beta_{mast}=0.613$ $\gamma_6 := \beta_6-\beta_{mast}=0.211$ $\gamma_7 := -\beta_7+\beta_{mast}=0.183$ $\gamma_8 := -\beta_8+\beta_{mast}=0.525$

$\gamma_9 := \beta_9+\beta_{mast}=0.803$ $\gamma_{10} := \beta_{10}+\beta_{mast}=1.039$ $\gamma_{11} := \beta_{11}+\beta_{mast}=1.242$